NUMERICAL ANALYSIS OF GRATINGS WITH PROFILES REPRESENTED BY SUPERPOSITION OF SINUSOIDAL WAVES

Toyonori MATSUDA
Kumamoto Radio Technical College, Nishigoshi 861-11, Japan

Yoichi OKUNO
Department of Electronic Engineering, Kumamoto University, Kumamoto 860, Japan

1. INTRODUCTION

In a previous paper[1], the authors presented an efficient numerical technique based on the mode-matching method with the smoothing procedure (MMM with the SP)[2,3] for analysing the problem of plane-wave diffraction by a sinusoidal grating. In the present paper the authors extend the technique of Ref. 1 to analyse the problem of gratings whose profiles are represented by superposition of sinusoidal waves. We present the efficiency of a quasi-triangular (or trapezoidal) grating as numerical examples. The time factor $\exp(-i\omega t)$ is suppressed throughout this paper.

2. FORMULATION OF THE PROBLEM

Let us consider an infinite grating made of a perfect conductor. Figure 1 shows the cross section C of the grating and coordinate system. The surface of the grating is uniform in the Z-direction and is periodic in the X-direction with a normalized period 1. We assume that the profile of the cross section of the grating is given by

$$y=f(x)=h[\sin 2\pi x + \alpha \sin(6\pi x + \delta)] \quad (1)$$

where x and y are the rectangular coordinates of point s on the cross section C. By varying the parameters h, α and δ, we can construct various profiles.

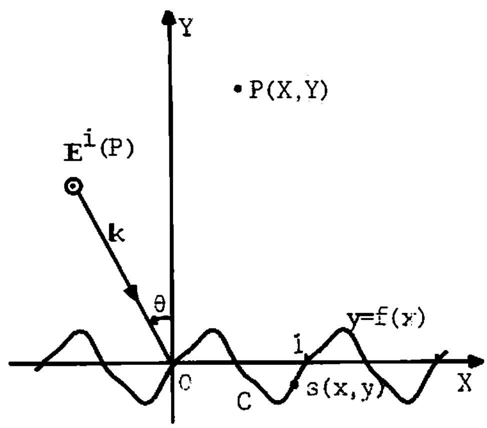


Fig. 1 Cross section of a grating and coordinate system

We deal with an E-polarized problem, i.e., an E-polarized plane-wave

$$\mathbb{E}^i(P) = \hat{a}_Z F(P), \quad F(P)=\exp(ikX\sin\theta - ikY\cos\theta) \quad (2)$$

hits on the surface of the grating. Here, k is the wave number and θ is the angle of incidence measured counterclockwise from Y axis. We seek the solution $E^d(P)$ satisfying the following conditions:

D1) The two-dimensional Helmholtz equation;

D2) The boundary condition $[E^d(s)=-F(s)]$; (3)

D3) The radiation condition in the Y-direction;

D4) The periodic condition $[E^d(X+1,Y)=\exp(ik\sin\theta)E(X,Y)]$. (4)

3. DESCRIPTION OF THE METHOD

In this section, we describe the numerical algorithm of the MMM with the SP [2,3]. We employ

$$\phi_m(P) = \exp(i\alpha_m X + i\beta_m Y) \quad m=0, \pm1, \pm2, \ldots$$
$$\alpha_m = k\sin\theta + 2m\pi,\ \beta_m{}^2 = k^2 - \alpha_m{}^2,\ \ \mathrm{Re}(\beta_m)\geq 0,\ \mathrm{Im}(\beta_m)\geq 0 \qquad (5)$$

as modal functions. Note that these are the solutions of D1) obtained by separation of variables to satisfy the conditions D3) and D4).

The approximate wave function for the diffracted field $E^d(P)$ is set up by putting

$$E_N^d(P) = \sum_{m=-N}^{N} A_m(N)\ \phi_m(P). \qquad (6)$$

It is clear that the approximate wave function $E_N^d(P)$ satisfies the conditions D1), D3) and D4).

The numerical algorithm of the MMM with the pth-order SP, where p is a positive integer, leads us to solve the following set of linear equations[1]:

$$\begin{cases} \sum_{n=-N}^{N} f_{mn}{}^p A_n{}^p(N) + [R_m(h\beta_m)]^* \lambda_N{}^p = g_m{}^p \quad (m=-N, -N+1, \ldots, N), \\ \sum_{n=-N}^{N} R_n(h\beta_n)\ A_n{}^p(N) = -R_0(-h\beta_0), \end{cases} \qquad (7)$$

where

$$f_{mn}{}^p = \sum_{\substack{\ell=-\infty \\ \ell\neq 0}}^{\infty} [R_{m-\ell}(h\beta_m)]^*\ R_{n-\ell}(h\beta_n)/(2\ell\pi)^{2p} \qquad (8)$$

and

$$g_m{}^p = -\sum_{\substack{\ell=-\infty \\ \ell\neq 0}}^{\infty} [R_{m-\ell}(h\beta_m)]^*\ R_{-\ell}(-h\beta_0)/(2\ell\pi)^{2p} \qquad (9)$$

with

$$R_m(z) = \int_0^1 \exp[i2m\pi x + izf(x)/h]dx. \qquad (10)$$

In Eq. (7), $\lambda_N{}^p$ stands for the Lagrange multiplier and the superscript p means that the quantity with this symbol is obtained by the pth-order SP.

As it is clear from Eq. (10), the function $R_m(z)$ depends on the profile of the cross section of gratings. For the profile given by Eq. (1), $R_m(z)$ is represented in terms of Bessel functions and can be evaluated rapidly. That is, the integral in the right-hand side of Eq. (10) can be evaluated by using the integral representation and the formula of generating function of the Bessel function $J_m(z)$. The result leads to

$$R_m(z) = \sum_{j=-\infty}^{\infty} \exp(ij\delta)\ J_{m+3j}(-z)\ J_j(\alpha z). \qquad (11)$$

The series appeared in Eqs. (8), (9) and (11) converge rapidly as we will see later.

The diffraction efficiency for the propagating mode [β_m: real] is given by

$$\eta_m{}^p = |A_m{}^p(N)|^2 \beta_m/\beta_0 \qquad (p=1,2,3). \qquad (12)$$

4. NUMERICAL RESULTS AND DISCUSSION

In numerical calculation, we truncate the series of Eq. (11) and (8) [or (9)]:

$$R_m(z) = \sum_{j=-K}^{K} \exp(i j \delta)\, J_{m+3j}(-z) J_j(\alpha z); \tag{13}$$

$$f_{mn}{}^{p} = \sum_{\ell=-L}^{L} [R_{m-\ell}(h\beta_m)]^{*}\, R_{n-\ell}(h\beta_n)/(2\ell\pi)^{2p}. \tag{14}$$

We decide the truncation sizes K and L through numerical experiments. Fig. 2(a) shows the variation of the expansion coefficients obtained by the method with the first-order SP as a function of the truncation size K while L is fixed. In this figure, the expansion coefficient becomes constant for K being not less than 6(=N). Therefore, in later numerical examples we choose the truncation size K such that K=N. On the other hand, from Fig. 2(b) we decide that L=2N+1. The convergence of expansion coefficients with respect to K or L becomes slow as the values of h and $|\alpha|$ increase. However, for the problems of ordinary commercial gratings (i.e., $h \leq 0.25$ and $|\alpha|<0.2$) the above choice of the truncation sizes [K=N and L=2N+1] is sufficient for finding the diffracted field.

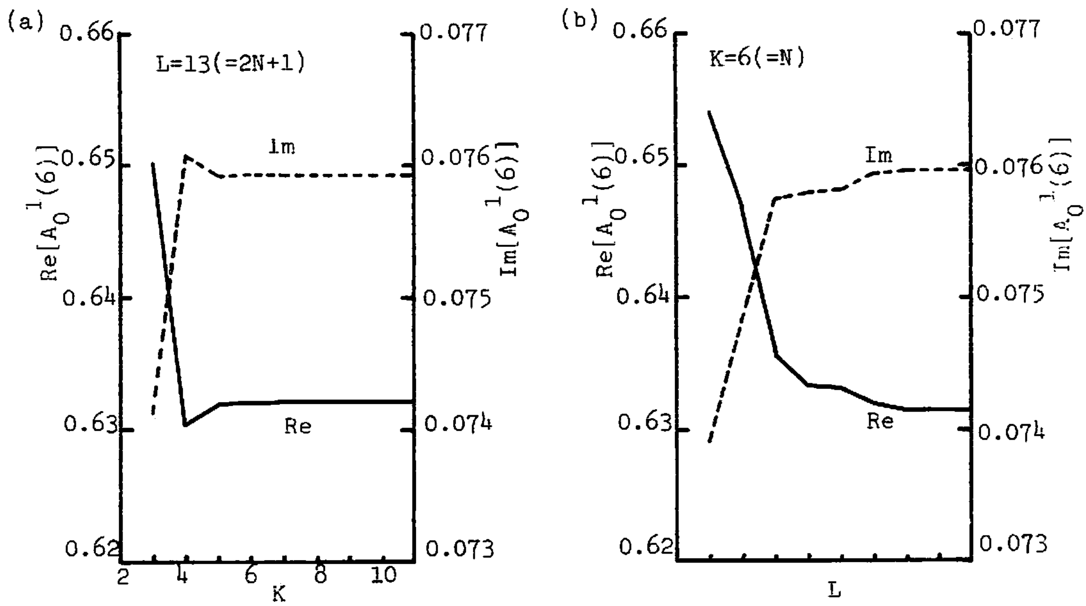
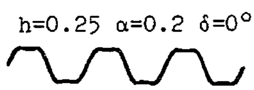
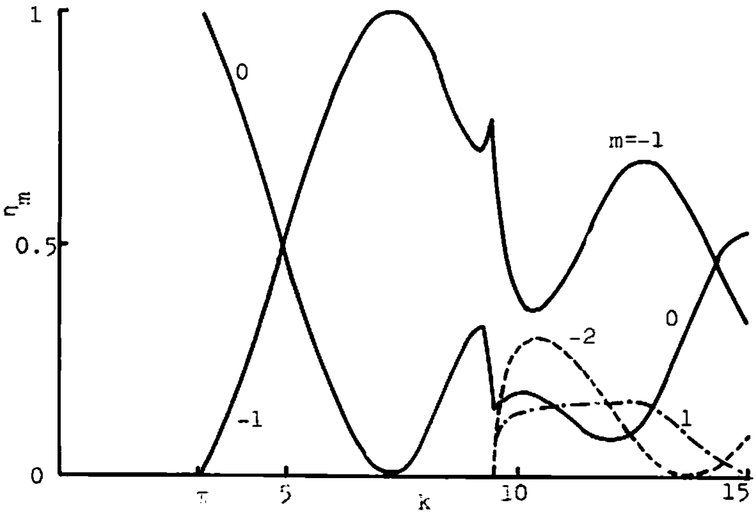


Fig. 2 Variation of the expansion coefficient as functions of truncation sizes K and L: $h=0.25$, $\alpha=0.2$, $\delta=0°$, $k=10$, $\theta=0°$, $N=6$ and $p=3$.



The following figures are numerical examples under -1 order Littrow mounting obtained by the MMM with the third-order SP. The number of modal functions (2N+1) is not more than 25 and the energy error is less than 1 %. Fig. 3 shows the efficiency of various spectral orders as functions of the wave number k. Fig. 4 shows the efficiency of -1 order diffracted mode as functions of α while h and δ are kept constant. In this figure we find the efficiency is almost independent of α and depends mostly on the fundamental amplitude h provided that there are only two propagating diffracted orders[4].



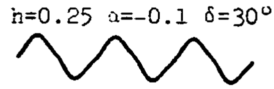


Fig. 3 Efficiency of a grating in -1 order Littrow mounting: h=0.25, α=-0.1, δ=30°, N=12 and p=3.

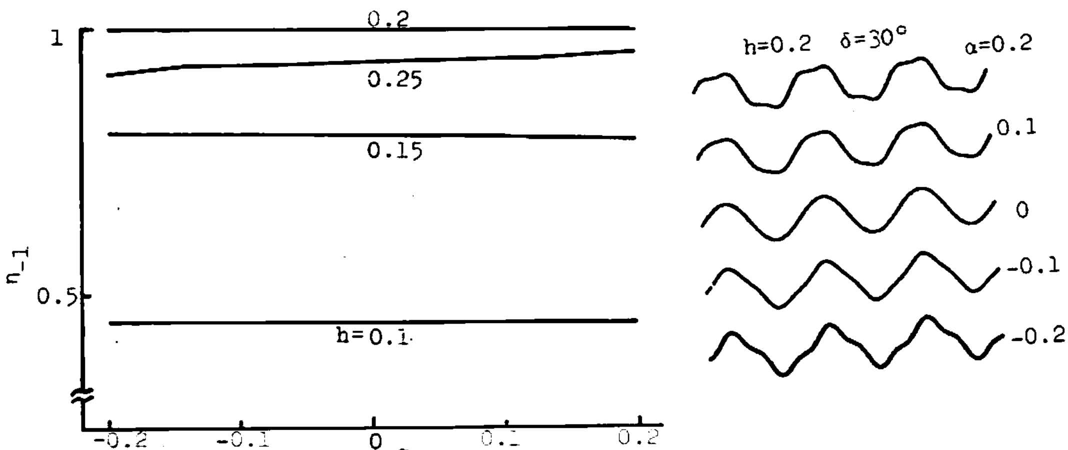


Fig. 4 Efficiency as functions of α: δ=30°, k=8, θ=23.12°, N=10 and p=3.

ACKNOWLEDGMENTS

The authors wish to express their thanks to K. Yasuura of Kyushu University and to T. Itakura of Kumamoto University for useful discussions and continuous encouragements.

REFRENCES

[1] Y. Okuno and T. Matsuda, "An efficient technique for numerical solution of diffraction by a sinusoidal grating," J. Opt. Soc. Amer. A, 2, 4 (to be published).
[2] K. Yasuura and Y. Okuno, "Numerical analysis of diffraction from a grating by the mode-matching method with a smoothing procedure," J. Opt. Soc. Am. 72, 847-852(1982).
[3] Y. Okuno and T. Matsuda, "Mode-matching method with a higer-order smoothing procedure for the numerical solution of diffraction by a grating," J. Opt. Soc. Amer. 73, 1305-1311(1983).
[4] R. Petit, ed., Electromagnetic Theory of Gratings (Springer-Verlag, Berlin, 1980).