ANALYSIS OF DIFFRACTION FROM A METAL GRATING

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Abstract

A numerical technique for analysing the diffraction from metal grating is presented. The boundary conditions are approximated by the surface impedance of metal. The Robin boundary-value problem is solved by the mode-matching method with smoothing procedure. It is shown numerically that this method gives a good approximate solution for a sinusoidal metal grating with somewhat deep groove.

1.Introduction

Many numerical methods have been presented for analysing the diffraction from gratings made of perfect conductor[1]-[6]. These results does not always give the practical behavior of metal gratings, because metal has lossy and dispersive properties. The two-media analysis, with respect to surrounding medium and metal, may give a realistic result but becomes very cumbersome. Moreover each of metal surfaces has the proper charactor different from the bulk. Then we will present a simple technique to solve the diffraction problem for metal gratings.

Boundary conditions are approximated so that the metal surface impedance Z_m equales to the ratio of the tangential electric and magnetic fields on the surface of metal. This assumption derives Robin's boundary condition for diffracted wave. The Robin condition makes the calculation to be much simpler than the two-media analysis.

In this paper, the discussion is restricted to the case of a sinusoidal grating. The numerical technique is the mode-matching method with smoothing procedure. By numerical results, it is shown that this boundary condition gives a good solution for the diffraction problem of sinusoidal metal grating with somewhat deep groove. The time factor $exp(-j\omega t)$ is suppressed throughout this paper.

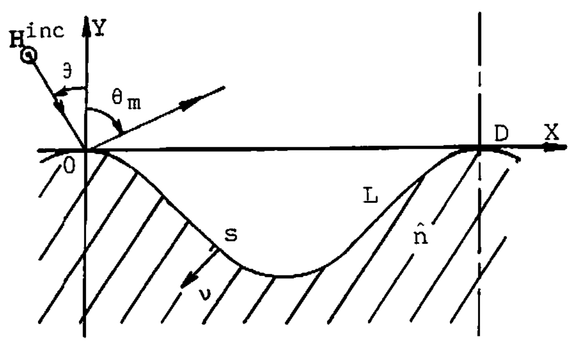

Fig.1. Cross section of sinusoidal grating

2.Robin's type boundary condition

The surface L of metal grating is a sinusoidal form, $y=h\cos(2\pi x/D)$. An arc length between $(x, h\cos(2\pi x/D))$ and $(0, h)$ is denoted by s and L is the surface length for one period. The incident plane wave is the P-polarized wave whose magnetic field is

$$H^{inc}(x,y)=i_zF(x,y)=i_z\exp\{jkx\sin\theta-jky\cos\theta\}, \qquad (1)$$

where $k(=2\pi/\lambda)$ is the wave number. The upper region of L is vacuum and the lower is filled with metal. The magnetic field of diffracted wave is

$$H^d(x,y)=i_z\Psi(x,y) . \qquad (2)$$

The radiative wave function $\Psi(x,y)$ satisfies the equation

$$(\partial_x^2+\partial_y^2+k^2)\Psi(x,y)=0 \qquad (3)$$

and the periodic property

$$\Psi(x+D)=\exp(jkD\sin\theta)\Psi(x) . \qquad (4)$$

The metal impedance is $Z_m=\sqrt{\mu_0/\varepsilon_0\hat{n}^2}$ where $\hat{n}$ is the complex refraction index of metal. We assume that the boundary condition on the surface L can be approximated by the Robin's type condition,

$$\partial_\nu\Psi(s)-j\omega\varepsilon_0Z_m\Psi(s)=f(s),$$

$$f(s)=-\{\partial_\nu F(s)-j\omega\varepsilon_0Z_mF(s)\} \qquad (5)$$

where ∂_ν is the normal derivative on the surface L.

For the case of the S-polarized wave, we have the similar Robin's type condition.

3.Mode-matching method with smoothing procedure

Let us introduce the modal functions

$$\varphi_m(x,y)=\exp\{j(k\sin\theta+2m\pi/D)x+j\kappa_my\}$$

$$\kappa_m^2=k^2-(k\sin\theta+2m\pi/D)^2=k^2\cos^2\theta_m$$

$$Re[\kappa_m]\geqq 0,\ I[\kappa_m]\geqq 0$$

$$m=0,\pm1,\pm2,\cdots . \qquad (6)$$

The Rayleigh principle is refined as follows[7],[8]; We have the sequence of approximate wave functions

$$\Psi_N(x,y)=\sum_{-N}^{N}b_m(N)\varphi_m(x,y) ,\ N=1,2,\cdots \qquad (7)$$

which converges to $\Psi(x,y)$ uniformly in the upper vacuum region. The coefficient $b_m(N)$ depends on the truncated number N.

The smoothing procedure is the technique to minimize an indefinite integration of residue for boundary condition. First, we introduce the smoothing operator K [5]; Under the constraint

$$\int_0^L u(s)ds=0 \ ,$$

K is defined as follows,

$$Ku(s)=\int_0^s u(s')ds'+\frac{1}{L}\int_0^L s'u(s')ds'=\int_0^L K(s,s')u(s')ds' \tag{8}$$

where the anti-symmetric kernel K(s,s') is

$$K(s,s')=\frac{1}{L}(s'-s)+\frac{1}{2}\ (s>s')\ ,\ \frac{1}{L}(s'-s)-\frac{1}{2}\ (s<s')\ . \tag{9}$$

The mode-matching method with smoothing procedure gives the following process; Under the constraint

$$\int_0^L [\ \{\partial_\nu\Psi_N(s)-j\omega\varepsilon_0 Z_m\Psi_N(s)-f(s)\}\ e^{-jkx(s)\sin\theta}]ds=0\ ,$$

let the mean square error defined on the surface L

$$\Omega_N=\int_0^L |K[\ \{\partial_\nu\Psi_N(s)-j\omega\varepsilon_0 Z_m\Psi_N(s)-f(s)\}\ e^{-jkx(s)\sin\theta}]\ |^2 ds \tag{10}$$

minimize. Then we have the set of linear equations whose solution gives the expansion coefficients $\{b_m(N)\}$ of the wave function $\Psi_N(x,y)$ in the vacuum region.

We can construct the similar process for the S-polarizaton.

4.Numerical results

The numerical computation is carried out for the sinusoidal grating made of gold whose refraction index is set in values of the bulk gold metal as shown in the table 1[9], although these values are not always suitable for metal surface. The period of the grating is given by D=0.83 μm and the depth of groove is 2h= 0.3D. In Fig.2, the efficiency of the P-polarized diffraction is shown for the sinusoidal grating which is settled in the -1st order Littrow mounting: $\theta=-\theta_{-1}=\arcsin(\pi/kD)$. For truncation number N=20, the relative mean square error $\Omega_N/\int_0^L |Kf(s)|^2 ds$ is less than 1%, and the power of total reflection and the blazing power are sufficiently stable for increasing N. The dotted curves in Fig.2 represent the results of the two-media analysis. The dip about $\lambda=0.72D$ in the curve of P_{-1}/P_i showes the plasmon anomaly of the metal. We can say that the difference of those solutions is sufficiently small to find an overall behavior of diffraction from the metal grating.

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λ(μm)	n_r	n_i
0.40	1.580	1.785
0.42	1.570	1.800
0.44	1.533	1.790
0.46	1.450	1.740
0.48	1.280	1.685
0.50	0.935	1.750
0.52	0.670	2.010
0.53	0.594	2.143
0.54	0.535	2.305
0.55	0.494	2.390
0.56	0.464	2.057
0.57	0.437	2.620
0.58	0.415	2.750
0.59	0.394	2.841
0.60	0.378	2.950
0.62	0.350	3.160
0.64	0.336	3.360
0.66	0.320	3.540
0.68	0.303	3.672
0.70	0.280	3.800

Table 1.
Complex refraction index of gold, $\hat{n}=n_r+jn_i$

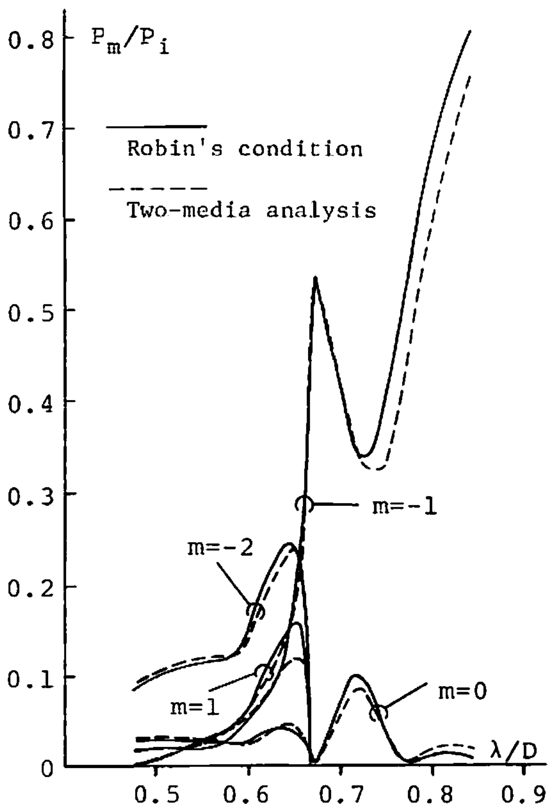

Fig.2.
Normalized diffracted power in the -1st order Littrow mounting for the P-polarization