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COMPARISON OF VARIOUS WAVELET BASIS FUNCTIONS IN SOLVING THIN-WIRE EFIE FOR WIDE-BAND APPLICATIONS

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1. Introduction

The problem of time harmonic radiation of thin-wire antennas involves the solution of the Electric Field Integral Equation (EFIE) [1]. To numerically solve the EFIE for its unknown current distribution along the antenna, one can employ the method of moments (MoM). In this method, the unknown current can be expanded by various basis functions, including the conventional triangular, sinusoidal, and piecewise constant functions [1]. The application of appropriate boundary conditions reduces the problem to a matrix equation. The matrix, known as the Impedance Matrix (IM), is generally dense.

The use of wavelet basis functions in the MoM has recently attracted much attention. This is due to the fact that they can weaken the mutual coupling of non-overlapping basis, causing the IM off-diagonal elements become much smaller than their respective diagonal elements [2]. The omission of relatively small elements leads to a sparse IM whose inversion is more desired in terms of computation time and memory requirement. It is noted that the inversion of the new approximated IM does not significantly affect the accuracy of the final solution [2]-[5].

Several wavelet basis functions have been proposed for solving the EFIE by the MoM. These are categorized in two groups namely, the Ortho-Normal (ON) bases and Semi-Orthogonal (SO) bases. The ON bases are generally used for cases where the approximated function is defined from $-\infty$ to $+\infty$. Haar and Battle-Lemarie (BL) bases are examples of this group [2], [3]. The SO bases, on the other hand, are designed for finite intervals. Examples of their group are compactly supported linear and cubic spline wavelets [4], [5]. Although each of these bases has its own advantages, it may not be appropriate for the problem in hand.

The objective of this paper is to study the merits of various wavelet bases in solving thin-wire EFIE by the MoM. The paper follows by a brief description of the problem, introducing a criterion for evaluation of the degree of sparsity obtained in the approximated IM without losing a predefined accuracy in the final solution. Numerical results are also presented to compare several wavelet bases for their merits in solving the problem in hand.

2. Theory

The schematic of a straight thin-wire antenna in free space is depicted in Fig. 1. The thin-wire approximation assumes that the wire radius a is much smaller than the wavelength and the wire length is

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much greater than a. Applying the zero-tangential electric field boundary condition along the wire leads to the EFIE expressed as follows [1]:

$$\int_{-l/2}^{l/2} K(z-z')I(z') = E(z) \tag{1}$$

where $K(z-z')$ and $E(z)$, respectively, denote the kernel and the excitation function of the EFIE.

In the MoM discretization scheme, one needs first expand the unknown current $I(z')$ in a set of basis functions. In order to effectively approximate the unknown current by wavelet bases, the concept of multiresolution analysis is used [5]. In the wavelet expansion technique, the unknown current in the EFIE can be expanded as a twofold summation of shifted and dilated forms of a properly chosen basis function [5]. This arrangement causes the localized bases to concentrate in the vicinity of sharp local variations while those with more spatially diffused ones distribute over the smooth part of the current. One can therefore expand the unknown current in a set of scaling functions $\phi_{s_0,k}$ (approximation at the lowest resolution 2^{-s_0}), and a multiresolution set of wavelets $\psi_{s,k}$ at 2^{-s_0} and finer resolution,

$$I(z') = \sum_k c_{s_0,k}\phi_{s_0,k}(z') + \sum_{s=s_0}^{s_u}\sum_k d_{s,k}\psi_{s,k}(z') \tag{2}$$

where the couple subscripts s and k ,respectively, are octave level (scale) and position (shift) parameter, and $c_{s_0,k}$ and $d_{s,k}$ represent, respectively, the approximation and detail coefficients [5].

To determine the unknown coefficients $c_{s_0,k}$, $d_{s,k}$, one can test (1) with similar scaling functions and wavelets in (2) as weighting functions (i.e., the Galerkin procedure), transforming the EFIE into a set of linear equations system as $ZJ=V$. Z, J, and V denote the IM, unknown current expansion coefficients, and excitation vector, respectively.

Those elements of IM $Z_{N\times N}$ which are smaller than specific threshold $Z_\delta = \delta.\max_j \sum_i |Z(i,j)|/N$ have been discarded. To set a uniform criterion for all basis functions relative errors ε_r in current distribution (defined by L^2 norm as in [4]) and e_r in input impedance are assumed as the desired convergence criteria during the selection of proper individual octave level s_0 and threshold value Z_δ.

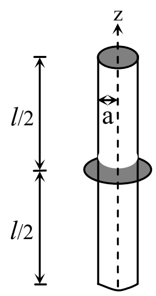


Fig. 1. The schematic of a center-fed z-directed thin-wire antenna excited by a voltage source.

3. Numerical Results

To evaluate the performance of various wavelet bases, the input impedance of a center-fed straight thin-wire antenna with a=5mm radius has been evaluated using the Magnetic Frill source model [1]. It is assumed that the operating frequency ranges from 30 MHz to 600 MHz. Also It is assumed that ε_r and e_r are, respectively, 1 % and 10 % throughout our investigation.

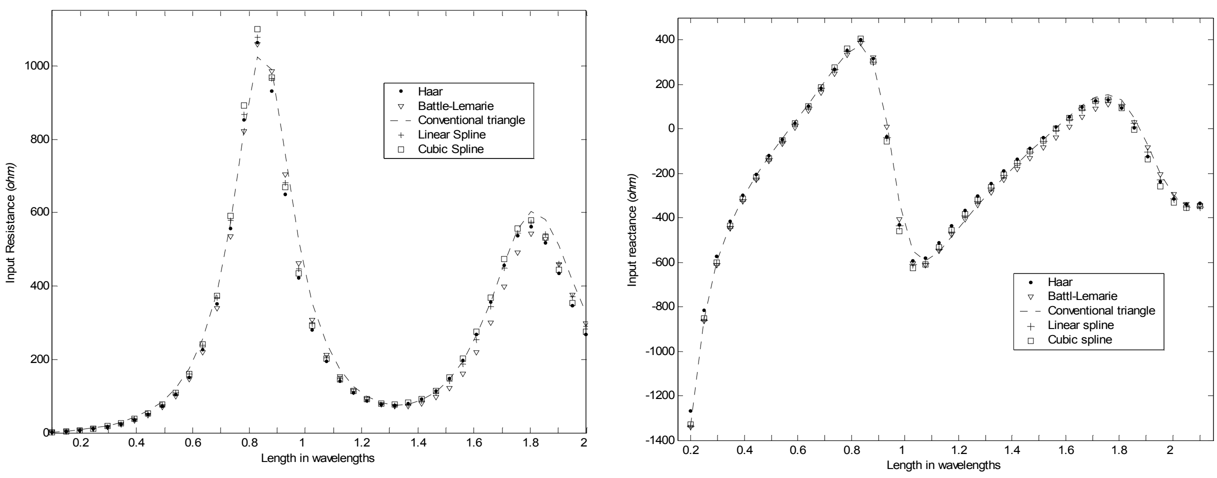
Fig. 2 (a) and (b) respectively show the real and imaginary parts of linear antenna input impedance by the classical triangle basis as well as Haar, BL, linear and cubic spline wavelet bases for several electric lengths. Table I compares the performance of all aforementioned wavelet bases in terms of number of unknowns (indicating the convergence speed), the relative computational time, and the average percentage sparsity achieved in the IM. As seen in the Table I, the SO wavelet expansion extracts the spatial variation of current more rapidly than the other cases. In particular, the SO linear spline appears to be the most appropriate choice for solving the thin-wire EFIE. In fact, there is no need to use such smoother bases as cubic spline. Table I also reveals that Haar wavelet accelerates the determination of IM elements. This is sought to be due to its simple formulation. In addition, although BL gives the highest sparsity in the IM, it produces large number of unknowns which, in turn, increases the computation time. Here truncation of infinite support BL scaling functions and wavelets two discretization steps away from their centers at each octave level *s* is proposed.

In order to search adequate specific threshold value which permits us the maximum allowable sparsity S_δ one should set it for IM which has more sensitive solution to data error (has largest condition number). To reach average sparsity to S_δ in evaluation responses of frequency domain EFIE for several electrical lengths, we recommend to start the analysis from the largest electrical length (frequency) of antenna. Because decreasing the electrical length of antenna increase the mutual coupling between segments. Consequently, whenever the sparsity is going to exceed S_δ, a little real time reduction in threshold value is sufficient to retain the sparsity a little less than S_δ and prevent losing some important information during threshold process.

Fig. 3 shows the sparsity pattern of most ill-conditioned IM for various wavelet basis functions after threshold process. By means of Fig. 3, priori locations of non significant matrix elements have been estimated and evaluation of small IM elements has been avoided.

4. Conclusions

It has been shown that compactly supported SO spline wavelets extract spatial variation of the



(a) (b)

Fig. 2. Input (a) Resistance and (b) Reactance of center-fed straight thin-wire antenna

current especially in vicinity of antenna feed with least number of unknown, since they specially constructed for bounded interval. Although ON wavelet bases produce discontinuity in end point current, they have some advantages over conventional basis choices. In fact simple definition of Haar accelerates determination of the IM elements and BL provide high sparsity despite its truncation.

To reach maximum allowable sparsity of the IM in analysis of wide frequency range, a real time threshold process is proposed, which only needs determination of most ill-conditioned IM.

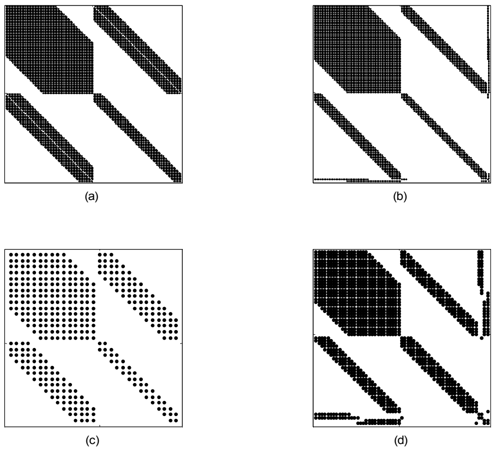


Fig. 3. Sparsity pattern of diagonalized impedance matrix with largest condition number in solving linear antenna EFIE for several electric lengths by (a) Haar, (b) Battle-Lemarie, (c) Linear, and (d) Cubic Spline wavelet bases.

Tabel I: Comparison of the number of unknown (convergence speed), relative computation time, and average sparsity for conventional triangle, ON (Haar and BL) and SO (linear and cubic Spline) wavelet bases in precisely characterizing current on dipole antenna in free space for (30-600) *MHz* frequency range.

Type of Basis	Category	s_0,s_u	Unknowns Number	Computation Time Ratio	Av. Sparsity
Haar	Orthogonal Wavelet	6,6	128	0.21	61.5 %
Battle-Lemarie		6,6	123	3.54	68.7 %
Conventional Triangle	Orthonormal	-	80	1	2.5 %
Linear Spline	Semi-Orthogonal Wavelet	4,4	29	0.53	54.3 %
Cubic Spline		5,5	59	3.53	60.3 %

Acknowledgment

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