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FDTD ANALYSIS OF ELECTROMAGNETIC SCATTERING BY A PEC OBJECT WITH THIN ANISOTROPIC COATING

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1 Introduction

The Finite-different time-domain (FDTD) method is an efficient and reliable technique for the analysis of electromagnetic problems [1]. However it meets a difficulty while an object is coated by an anisotropic medium of thin thickness, because the constitutive parameters are of 3×3 matrix form for anisotropic material and the thin thickness requires a fine grid in FDTD computation. To overcome this difficulty, we first extended FDTD algorithm to be capable of treating the scattering problems involving an anisotropic medium. Based on the dyadic surface impedance concept introduced by Huang and Yin [2], an equivalence scheme is then proposed, with which one may change both the thickness and anisotropic constitutive parameters of a coating, but keeping the dyadic surface impedance invariant. Using the proposed scheme we may construct a new anisotropic coating with different constitutive parameters and thickness to fit the FDTD cell size, but having the same dyadic surface impedance. Finally, computed results are given to exemplify the feasibility and applicability of this equivalence scheme.

2 FDTD algorithm for anisotropic medium

Suppose the constitutive relation of an anisotropic medium is

$$\vec{D} = \overline{\overline{\varepsilon}} \cdot \vec{E}\,, \qquad \vec{J} = \overline{\overline{\sigma}} \cdot \vec{E} \tag{1}$$

where $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\sigma}}$ are 3×3 matrices. For simplicity we assume the medium is isotropic for magnetic characteristics. The Maxwell equations are therefore of the following form:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} - \vec{J}_m = -\mu\frac{\partial\vec{H}}{\partial t} - \sigma_m\vec{H} \tag{2}$$

$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \vec{J} = \overline{\overline{\varepsilon}}\cdot\frac{\partial\vec{E}}{\partial t} + \overline{\overline{\sigma}}\cdot\vec{E} \tag{3}$$

Eq.(2) is discretised and reorganized as

$$\vec{H}^{n+1/2} = \frac{2\mu-\sigma_m\Delta t}{2\mu+\sigma_m\Delta t}\vec{H}^{n-1/2} - \frac{2\Delta t}{2\mu+\sigma_m\Delta t}\nabla\times\vec{E}^{n} \tag{4}$$

which is the time advancing formula for magnetic field $\vec{H}$. To obtain the time advancing formula for electric field, Eq. (3) is then discretised and reorganized as

$$\vec{E}^{n+1} = \overline{\overline{P}}\cdot\vec{E}^{n} + \overline{\overline{Q}}\cdot\nabla\times\vec{H}^{n+1/2} \tag{5}$$

where

$$\begin{aligned}\overline{\overline{P}} &= \left(\frac{\overline{\overline{\varepsilon}}}{\Delta t}+\frac{\overline{\overline{\sigma}}}{2}\right)^{-1}\cdot\left(\frac{\overline{\overline{\varepsilon}}}{\Delta t}-\frac{\overline{\overline{\sigma}}}{2}\right)\\ \overline{\overline{Q}} &= \left(\frac{\overline{\overline{\varepsilon}}}{\Delta t}+\frac{\overline{\overline{\sigma}}}{2}\right)^{-1}\end{aligned} \tag{6}$$

It is worth noting that each component such as E_x^{n+1} is related to all three components E_x^n, E_y^n and E_z^n in Eq.(5), in which the components E_y, E_z are however not located at their corresponding sampling points in Yee's cell. Therefore it is necessary to take a spatial average to specify E_y, E_z and other related field components in Eq.(5) to their sampling points in Yee's cell, respectively, such as

$$E_y\left(i+\frac{1}{2},j,k\right)=\frac{1}{4}\left[E_y\left(i,j+\frac{1}{2},k\right)+E_y\left(i,j-\frac{1}{2},k\right)+E_y\left(i+1,j+\frac{1}{2},k\right)+E_y\left(i+1,j-\frac{1}{2},k\right)\right] \tag{7}$$

3 Dyadic surface impedance of an anisotropic medium coating on a PEC substrate

Consider an anisotropic medium coating on a PEC substrate, as shown in Fig.1. It can be deduced from Maxwell equations that the equivalent electric and magnetic currents are related with the incident waves as follows [2]:

$$\vec{J}_s=\hat{n}\times\vec{H}\Big|_{z=0}=\frac{1}{\eta_0}\left\{\hat{e}_\perp\left[-s_{21}E_{//}^i\cos\theta^i+(1-s_{22})E_\perp^i\right]\cos\theta^i+(\hat{n}\times\hat{e}_\perp)\left[(1-s_{11})E_{//}^i-\frac{s_{12}}{\cos\theta^i}E_\perp^i\right]\right\} \tag{8}$$

$$\vec{J}_{ms}=\vec{E}\times\hat{n}\Big|_{z=0}=\hat{e}_\perp\left[(1+s_{11})E_{//}^i\cos\theta^i+s_{12}E_\perp^i\right]-(\hat{n}\times\hat{e}_\perp)\left[s_{21}E_{//}^i\cos\theta^i+(1+s_{22})E_\perp^i\right] \tag{9}$$

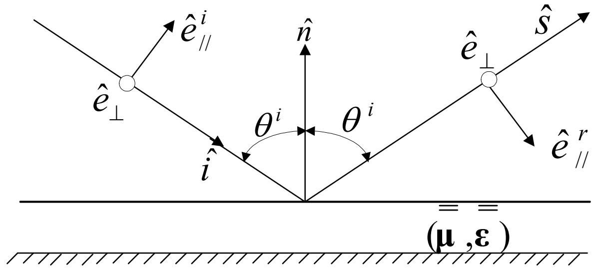

Fig.1 Geometry of reflection from anisotropic medium coated on a PEC plane

where $E_{//}^i=\vec{E}^i\cdot\hat{e}_{//}^i$ and $E_\perp^i=\vec{E}^i\cdot\hat{e}_\perp$ are the components of incident electric field parallel and perpendicular to the incidence plane, respectively. Eq.(8) and Eq.(9) can also be rewritten as

$$\vec{J}_{ms}=-\eta_0\hat{n}\times\bar{\bar{Z}}_s\cdot\vec{J}_s \tag{10}$$

in which $\bar{\bar{Z}}_s=Z_{\perp\perp}\hat{e}_\perp\hat{e}_\perp+Z_{\perp//}\hat{e}_\perp\hat{e}_w+Z_{//\perp}\hat{e}_w\hat{e}_\perp+Z_{////}\hat{e}_w\hat{e}_w$ is the dyadic surface impedance normalized by the free space characteristic impedance η_0 and $\hat{e}_w=\hat{n}\times\hat{e}_\perp$ the tangential unit vector. The four elements of dyadic surface impedance are

$$\begin{aligned}
Z_{\perp\perp}&=\frac{(1-s_{11})(1+s_{22})+s_{12}s_{21}}{[(1-s_{11})(1-s_{22})-s_{12}s_{21}]\cos\theta^i},\\
Z_{\perp//}&=\frac{2s_{21}\cos\theta^i}{(1-s_{11})(1-s_{22})-s_{12}s_{21}}\\
Z_{//\perp}&=\frac{2s_{12}}{[(1-s_{11})(1-s_{22})-s_{12}s_{21}]\cos\theta^i},\\
Z_{////}&=\frac{[(1+s_{11})(1-s_{22})+s_{12}s_{21}]\cos\theta^i}{(1-s_{11})(1-s_{22})-s_{12}s_{21}}
\end{aligned} \tag{11}$$

in which $s_{11}, s_{12}, s_{21}, s_{22}$ are related to the anisotropic material properties, the detailed expressions of which are not included here because of limited space.

4 Equivalence technique for thin anisotropic medium coating on a PEC substrate

In FDTD computations, one meets a difficulty if the anisotropic coating thickness is less than the size of Yee's cell. To overcome this difficulty, we may invoke an equivalence technique, trying to construct a new coating possessing the same surface impedance, but its thickness is larger than original one. This equivalence technique was implemented to the isotropic coating case in [3]. Suppose the thickness of these two equivalent coatings is denoted by d and d', respectively. Their constitutive parameters are also different, however their dyadic surface impedance, $\overline{\overline{Z}}$ and $\overline{\overline{Z}}'$ are identical, i.e. $\overline{\overline{Z}} = \overline{\overline{Z}}'$. Now we are in a position to determine the anisotropic constitutive parameters for the new coating of thickness d'

$$\overline{\overline{\mu}}' = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}, \quad \overline{\overline{\varepsilon}}' = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \tag{12}$$

Note that $\overline{\overline{\mu}}'$ and $\overline{\overline{\varepsilon}}'$ consist of $9 \times 2 = 18$ elements. Obviously, it is impossible to determine $\overline{\overline{\mu}}'$ and $\overline{\overline{\varepsilon}}'$ by solving Eq.(11), which consist of only 4 equations. By observing the deduction process in details we found that the dyadic surface impedance, $\overline{\overline{Z}}$ and $\overline{\overline{Z}}'$ may keep invariant, i.e. $\overline{\overline{Z}} = \overline{\overline{Z}}'$, if the following identities are fulfilled:

$$\frac{\varepsilon_{33}}{\varepsilon_{33}'} = \frac{\mu_{33}}{\mu_{33}'} = \frac{d}{d'} \text{ and } \frac{\varepsilon_{11}'}{\varepsilon_{11}} = \frac{\varepsilon_{12}'}{\varepsilon_{12}} = \frac{\varepsilon_{21}'}{\varepsilon_{21}} = \frac{\varepsilon_{22}'}{\varepsilon_{22}} = \frac{\mu_{11}'}{\mu_{11}} = \frac{\mu_{12}'}{\mu_{12}} = \frac{\mu_{21}'}{\mu_{21}} = \frac{\mu_{22}'}{\mu_{22}} = \frac{d}{d'} \tag{13}$$

in which all quantities with prime belong to the new equivalent anisotropic coating. By using Eq.(13) we can easily construct a new coating with designated thickness.

5 Numerical example

Consider a PEC sphere of radius R=0.03 m. The wavelength of incident wave is λ=0.03m. We take δ=1mm and $\Delta t = \delta/(2c)$ in FDTD computation. Suppose the front illuminated half sphere is coated by an anisotropic medium of thickness of d=1mm, as shown in Fig.2, and

$$\overline{\overline{\varepsilon}}_r = \begin{bmatrix} 7.39 & 2.6 & 2.5 \\ 1.8 & 1.39 & 1.7 \\ 3.0 & 1.7 & 2.3 \end{bmatrix}, \quad \overline{\overline{\mu}}_r = \begin{bmatrix} 8.19 & 7.5 & 0.0 \\ 1.5 & 10.4 & 0.0 \\ 0.0 & 0.0 & 6.5 \end{bmatrix}, \quad \overline{\overline{\sigma}}_E = \begin{bmatrix} 3.34 & 0.16 & 0.56 \\ 0.62 & 0.52 & 0.67 \\ 0.67 & 0.72 & 1.12 \end{bmatrix}, \quad \overline{\overline{\sigma}}_m = \begin{bmatrix} 3355.1 & 2724.0 & 434.3 \\ 513.2 & 3379.3 & 513.2 \\ 513.2 & 434.3 & 2230.6 \end{bmatrix} \tag{14}$$

We construct two other equivalent coatings of $d' = 2mm$ and 3mm, respectively, for this sphere. The corresponding constitutive parameters can be determined by Eq.(13) providing all three coatings possess the identical dyadic surface impedance. The computed bistatic RCS of coated spheres are shown in Fig.3, where □ stands for the case of 1mm coating, ○ for 2mm coating, and △ for 3mm coating. For comparison, the RCS of a bare PEC sphere is also depicted by solid line in Fig.3. This example together with other computations that are not shown in this paper validates our proposed scheme.

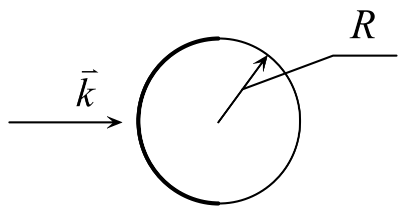

Fig.2 PEC sphere with anisotropic coating and on the illuminated half sphere

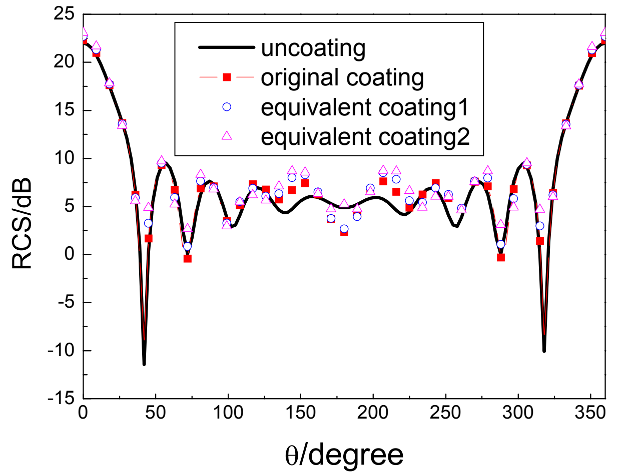

Fig.3 Bistatic RCS of a PEC sphere with without anisotropic coating

6 Conclusions

To treat the scattering by a PEC object coated with an anisotropic medium the commonly used FDTD algorithm is first extended to anisotropic medium case. An equivalence technique based on the dyadic surface impedance concept for anisotropic coating is proposed, with which one may construct a new anisotropic coating of different thickness providing the original coating thickness is less than the designated cell size in FDTD. Computed results verify our proposed scheme and exemplify its feasibility and applicability.

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8 References

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