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APPLICATION OF MULTIGRID MOMENT METHOD TO SCATTERING OF A GAUSSIAN BEAM BY A NONLINEAR DIELECTRIC CYLINDER

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1. Introduction

Scattering problem is one of the important issues in electromagnetic theory. So far, many books [1]–[3] and papers have been published. Recently, the problems for the scattering of the large size compared to the wavelength and of the random medium are interested from the practical point of view. In these cases, the large size matrix equation has to be solved as fast as possible. FMM (Fast Multipole Method) is one candidate to solve these problems [4]. Multigrid method [5]–[7] is another fast solver. This method is applied to various problems such as the analysis of corner reflector antenna [8], the analysis of the eigenvalue problem in anisotropic microstrip line [9], [10], and the scattering from the rough surface [11] and so on.

So far, the scattering of EM wave by weakly nonlinear cylinders have been examined by using an iterative method [12], [13]. In this article, the moment method with multigrid is applied to the scattering of a Gaussian beam by a nonlinear dielectric cylinder. The scattered field is obtained by the volume equivalent theorem. The integral form is converted into the matrix equation by the moment method. Full Approximation Scheme (FAS) of the multigrid method, which is the scheme to solve the nonlinear problem, is applied to the matrix equation to obtain the solution as fast as possible. The effect of the number of cycle indices and multigrid cycles on the residual norm are examined numerically. Also, the scattered near fields are calculated for various parameters of the multigrid method.

2. Scattering by a Nonlinear Dielectric Cylinder

Consider the scattering of a Gaussian beam by a nonlinear dielectric cylinder as shown in Fig. 1. The nonlinearity is assumed to be of the Kerr type whose refractive index depends on the light intensity. The incident electric field is polarized in the z axis. In this case, the scattered field $E_z^s(\mathbf{r})$ is expressed in the integral form as follows [3]:

$$E_z^s(\mathbf{r}) = -j\omega\mu_0 \int_{S'} G(\mathbf{r},\mathbf{r}') J_{eq}(\mathbf{r}')\, dS' \tag{1}$$

where the equivalent current $J_{eq}(\mathbf{r})$ is given by

$$J_{eq}(\mathbf{r}) = j\omega\varepsilon_0 \left(\varepsilon_r - 1 + \alpha|E_z(\mathbf{r})|^2\right) E_z(\mathbf{r}) \tag{2}$$

and Green's function $G(\mathbf{r},\mathbf{r}')$ is given by the Hankel function of the second kind of order 0. ε_r and α are the linear relative permittivity and the nonlinear coefficient, respectively. After adding the incident field to the both side in Eq. (1) and applying the moment method, the following equation is obtained.

$$\sum_{n=1}^{N} C_{mn}\, E_n = E_m^i, \quad m = 1, \cdots, N \tag{3}$$

where

$$C_{mn} = \delta_{mn} + \frac{jk^2}{4}\left\{\varepsilon_r(n) - 1 + \alpha|E_n|^2\right\} \int_{cell\ \ n} H_0^{(2)}(k\rho)\, dx'\, dy' \tag{4}$$

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and E_n and $\varepsilon_r(n)$ are the total electric field and the relative permittivity at nth cell. N is the total number of unknowns, $\rho = \sqrt{(x_m - x')^2 + (y_m - y')^2}$, and E_m^i is the incident field at (x_m, y_m), which is the center of the cell m.

3. Multigrid Method

The matrix equation (3) is solved by FAS of the multigrid method with the Kaczmarz iteration method. Since the matrix equation obtained here is nonlinear and is not the diagonal dominance, the Gauss-Seidel method is not suitable. Kaczmarz method has been proved to converge for any system of linear equations [14].

The outline of the two-grid cycle of FAS, which is the basis for any multigrid algorithm, is as follows [5]–[7]. The schematic diagram is illustrated in Fig. 2. Equation (3) is expressed in the matrix form by

$$C^q(\mathbf{E}^q)\mathbf{E}^q = \mathbf{E}_i^q \tag{5}$$

where q denotes the fineness of the grid G^q and $\mathbf{E}^q$ is the rigorous solution on this grid. By providing an appropriate initial guess $\mathbf{e}_0^q$, applying a few Kaczmarz iterations gives the approximate solution $\mathbf{e}_1^q$. In the case of nonlinear equations, the unknown electric field is contained in the coefficient of a matrix. A correction equation taking this unknown into account has to be used. Due to this, the following correction equation can be obtained:

$$C^q(\mathbf{E}^q)\mathbf{E}^q = \mathbf{R}^q + C^q(\mathbf{e}_1^q)\mathbf{e}_1^q \tag{6}$$

where the residual $\mathbf{R}^q$ is defined as

$$\mathbf{R}^q = C^q(\mathbf{E}^q)\mathbf{E}^q - C^q(\mathbf{e}_1^q)\mathbf{e}_1^q = \mathbf{E}_i^q - C^q(\mathbf{e}_1^q)\mathbf{e}_1^q \tag{7}$$

In order to reduce the errors of the low-frequency components by using coarser grids, the correction equation (6) is solved directly due to nonlinearity of C^q after restriction to coarser grids. By using the restriction operator $\mathscr{R}$ as shown in Fig. 3, the following correction equation on the grid G^{q-1} is obtained.

$$C^{q-1}(\mathbf{E}^q)\mathbf{E}^{q-1} = \mathbf{R}^{q-1} + C^{q-1}(\mathbf{e}_1^q)\mathbf{e}_1^{q-1} \tag{8}$$

where

$$\mathbf{R}^{q-1} = \mathscr{R}\mathbf{R}^q, \qquad \text{and} \qquad \mathbf{e}_1^{q-1} = \mathscr{R}\mathbf{e}_1^q \tag{9}$$

Correction equation (8) is solved by Kaczmarz method and the solution $\mathbf{e}_2^{q-1}$ is obtained. The correction on this grid is given by

$$\mathbf{v}^{q-1} = \mathbf{e}_2^{q-1} - \mathbf{e}_1^{q-1} \tag{10}$$

This correction is prolongated by using the operation $\mathscr{P}$ as shown in Fig. 4.

$$\mathbf{v}^q = \mathscr{P}\mathbf{v}^{q-1} \tag{11}$$

Only the correction factor is interpolated and returned to finer grids.

The approximate solution on the grid G^q is corrected as follows:

$$\mathbf{e}_2^q = \mathbf{e}_1^q + \mathbf{v}^q \tag{12}$$

Subsequently, relaxation calculations are performed l times and the final approximate solution $\mathbf{e}_3^q$ is obtained. By applying the above cycle recursively, a multilevel multigrid cycle can be constructed.

4. Numerical Results

As numerical examples, the scattering of a Gaussian beam by a nonlinear dielectric circular cylinder is examined. The smallest spot size of the incident beam is $w_0 = \lambda$ and its location is $x_0 = -2\lambda$, $y_0 = 0$. The wavelength λ is 1.55μm and the nonlinear coefficient α is 6.377×10^{-12}[m^2/V^2]. The incident Gaussian beam is expressed by the complex-source-point method. The radius of the circular cylinder is $a = \lambda$ and the relative permittivity is $\varepsilon_r = 4.0$. The total number of unknowns N is 48×48. Three-level scheme of the multigrid method is applied. The amplitude of the incident field is set as the nonlinear part of the relative permittivity is about 10% of the linear part.

In order to check the accuracy of the scheme, the residual norm is defined as follows

$$\text{residual norm } = \sqrt{\sum_{s=1}^{N}\left|E_s^i - \sum_{m=1}^{N} C_{sm}E_m\right|^2} \tag{13}$$

At first, the effect of the initial field (condition) for FAS on the convergence speed is examined. Three cases are considered here. The first case is to use the electric field by a linear dielectric circular cylinder, and the second one is to use the incident field and the last one is null.

Figure 5 shows the residual norm for three cases as functions of the number of multigrid cycles. The electric field by a linear dielectric field is obtained by Correction Scheme (CS) of the multigrid method [15]. As the parameters for CS, cycle index is 30, and the number of the multigrid cycles is 2 and the number of iterations for each relaxation is 5. On the other hand, the number of iterations is 5 for FAS. It is found that the convergence speed is improved when the initial field is close to the approximate solution as much as possible. Figure 6 shows the normalized scattered near field at $x/\lambda = 2$ for three initial conditions. The peak near $(0, 2\lambda)$ is emphasized due to the nonlinearity.

The contour map of the electric fields by a nonlinear dielectric cylinder is indicated in Fig. 7. The electric field is focused in the cylinder. Figure 8 shows the contour map of the nonlinear part of the relative permittivity. It is turned out that the nonlinear permittivity is emphasized near the region $(0, 0.8\lambda)$ according to the influence of the electric field.

5. Conclusions

The scattering of a Gaussian beam by a nonlinear dielectric cylinder has been examined by the moment method with multigrid. The effect of the initial field on the residual norm has been examined and it is desirable that the initial field is close to the approximate solution as much as possible. Also, the contour map of the electric field and the nonlinear permittivity have been shown. The examination of the scattered far field and the scattering by many objects are future works.

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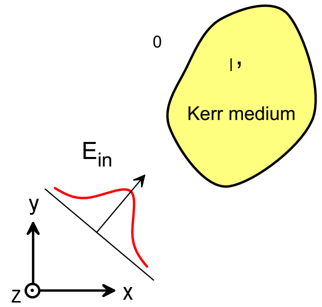

Fig. 1 Geometry of problem

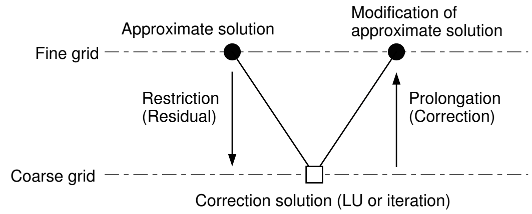

Fig. 2 Two-grid cycle (•: smoothing, □:exact solution or approximate solution by iterative method)

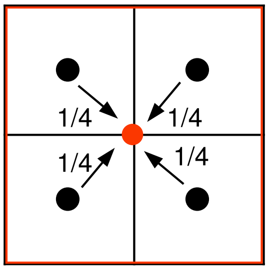

Fig. 3 Interpolation from fine grids to coarse grids (Restriction)

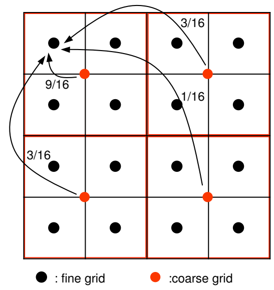

Fig. 4 Interpolation from coarse grids to fine grids (Prolongation)

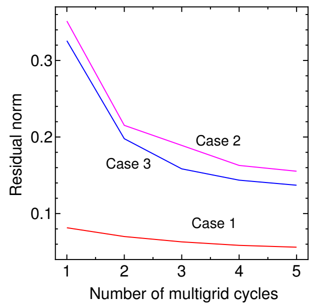

Fig. 5 Residual norm for various initial conditions

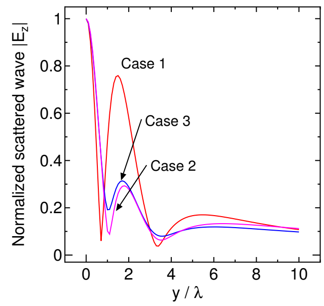

Fig. 6 Scattered near field for various initial conditions

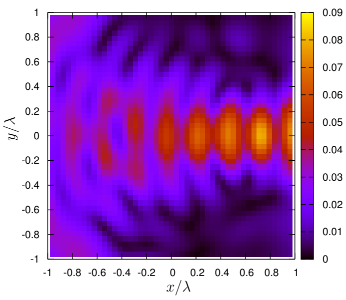

Fig. 7 Total electric field for a nonlinear dielectric cylinder in a cylinder

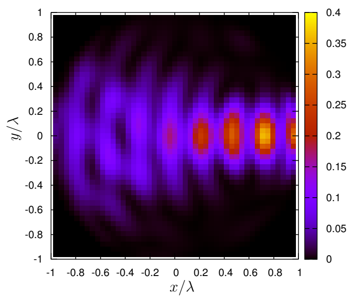

Fig. 8 Nonlinear part of the relative permittivity in a cylinder