RADIATION FROM A FLANGED WAVEGUIDE COVERED WITH A DIELECTRIC OR PLASMA SLAB

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The problem of an antenna radiating into a dielectric or plasma medium is of interest in the areas of plasma diagnostics, space communication, radome design, and so on. In this paper we consider the geometry of a flanged waveguide radiating into a dielectric or plasma layer (Fig. 1) and present a solution to this problem using an extension of the newly developed Modified Residue Calculus Technique.[1,2] The method has the advantages that it exhibits rapid numerical convergence, guarantees the satisfaction of the edge condition, and provides a built-in convergence check, not conveniently available in some of the other methods conventionally employed for solving problems of this type. One important feature of the construction of the solution is that an advantageous use is made of the known exact solution of the associated canonical problem, that of radiation from an open-ended waveguide.

Assume that a TM_{po} wave is incident in the guide from the left. The problem is formulated by expressing the fields in terms of appropriate spectral representations, which are discrete or continuous depending on whether the transverse dimension of the region in question is finite or infinite. Matching and Fourier analyzing the field at each int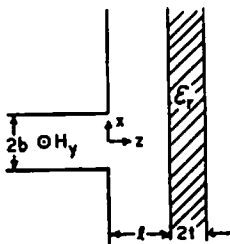erface, and performing some al-gebraic manipulation, we get the following equations for the unknown weight co-efficient $E(\alpha)$ in the spectral representation of the H_y field for $z > \ell + 2t$:

$$(-1)^P b \gamma_{pb} (1+\delta_p^0) \delta_m^P$$

$$= \int_0^\infty E(\alpha) \sin \alpha b \left[\frac{R(\alpha)}{\omega - \gamma_{mb}} - \frac{Q(\alpha)}{\omega + \gamma_{mb}} \right] \alpha d\alpha \quad (1)$$

$$\pi \Gamma E(\beta) \cos \beta b [R(\beta) - Q(\beta)]$$

$$= \int_0^\infty E(\alpha) \sin \alpha b \left[\frac{R(\alpha)}{\omega - \Gamma} - \frac{Q(\alpha)}{\omega + \Gamma} + \frac{R(\alpha)}{\omega + \Gamma} \right.$$

$$\left. - \frac{Q(\alpha)}{\omega - \Gamma} \right] \alpha d\alpha, \quad \omega = \sqrt{\alpha^2 - k^2} \quad (2)$$

where $\Gamma = \sqrt{\beta^2 - k^2}$ and the integral in (2) is a Cauchy principal value. $1/R$ and Q/R are the transmission and the reflection coefficients from the slab.

The solution is constructed by considering a complex function $f(\omega)$ with the following properties:
(i) $f(\omega)$ has zeros at $\omega = \gamma_{nb}$ $n \neq p$, $n = 1, 2, \ldots, \infty$ and at $\omega = \gamma_{ob} \delta_p^0$; (ii) $f(\omega)$ has two branch points $\omega = \pm \gamma_{ob}$; (iii) $f(\omega) \sim |\omega|^{1/2 - \nu}$, $\nu = 1/6$ as $|\omega| \to \infty$; (iv) $f(\gamma_{pb}) = b \gamma_{pb} (-1)^P (1+\delta_p^0)$.

We consider the integrals

$$\frac{1}{2\pi j} \int_P \frac{f(\omega) d\omega}{\omega - \gamma_{mb}} , \quad m = 0, 1, 2, \ldots, \text{ and}$$

$$\frac{1}{2\pi j} \int_P \left\{ \frac{f(\omega)}{\omega - \Gamma} + \frac{f(\omega)}{\omega + \Gamma} \right\} d\omega$$

where the path P is shown in Fig. 2, and expand them using complex variable theory. After some manipulations and the use of the properties (i)-(iv), one can show that the solutions of (1) and (2) can be obtained from $f(\omega)$ if it is made to satisfy the equations:
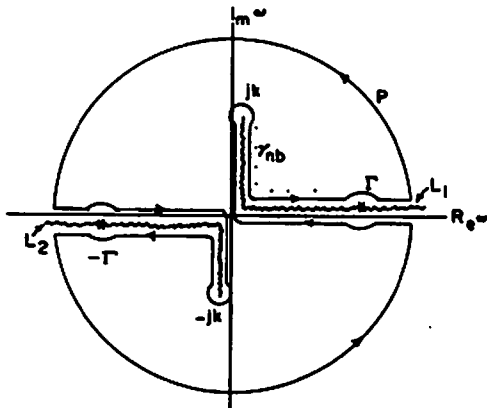

Fig. 1. Radiating structure.

Fig. 2. Contour P in the complex ω-plane.

$$f^+(-\omega)-f^-(-\omega)=-\frac{Q(\alpha)}{R(\alpha)}\,[f^+(\omega)-f^-(\omega)] \quad (3a)$$

$$f^+(\omega)e^{-j\alpha b}-f^-(\omega)e^{j\alpha b}=$$

$$-[f^+(-\omega)e^{-j\alpha b}-f^-(-\omega)e^{j\alpha b}] \quad (3b)$$

where ω is on the branch cut L_1. $E(\alpha)$ and all other spectral weight coefficients are expressible in terms of the resultant $f(\omega)$.

On the basis of the experience with the flanged waveguide[2] we choose a form for $f(\omega)$ appropriate for the present open region geometry that reads

$$f(\omega)=K\,X(\omega)\,\{\frac{1}{\omega-\gamma_{pb}}+\frac{\omega-2\gamma_{ob}}{\omega-\gamma_{ob}}$$

$$[\int_{L_1}\frac{F(\zeta)d\zeta}{\zeta-\omega}\quad \int_{L_1}\frac{G(\zeta)d\zeta}{\zeta+\omega}\,] \quad (4)$$

$$X(\omega)=(\omega-\gamma_{ob})[\ \prod_{n=1}^{\infty}(1-\frac{\omega}{\gamma_{nb}})\exp(\frac{\omega b}{n\pi})]h(\omega)$$

$$h(\omega)=\exp[\frac{b\omega}{\pi}(1-C_e+\ln\frac{2\pi}{kb})-j\frac{b\omega}{2}$$

$$+b\sqrt{\omega^2+k^2}\ \ln\frac{\omega-\sqrt{\omega^2+k^2}}{-jk}\,]$$

where C_e is the Euler's constant. It should be noted that $X(\omega)/(\omega-\gamma_{pb})$ is the generating function for the canonical problem. Substituting (4) into

(3), two coupled integral equations are derived which are to be solved for F and G.

Since the asymptotic nature of F and G are known a priori from the edge condition for the field components, it is possible to truncate the two infinite integrals and consider them only for a finite range of ζ after incorporating the asymptotic solution. It is this feature that contributes to the numerical efficiency of the method and allows guaranteeing the edge condition as well.

Fig. 3 shows the magnitude of the reflection coefficient in the waveguide for a TEM incident wave for several different choices of ϵ_r.

References

1. R. Mittra and T. Itoh, "Charge and potential distributions in shielded striplines," IEEE Trans. MTT, vol. MTT-18, pp. 149-156, March 1970.

2. T. Itoh and R. Mittra, "A new method of solution for radiation from a flanged waveguide," Proc. IEEE (Lett.), vol. 59, May 1971.
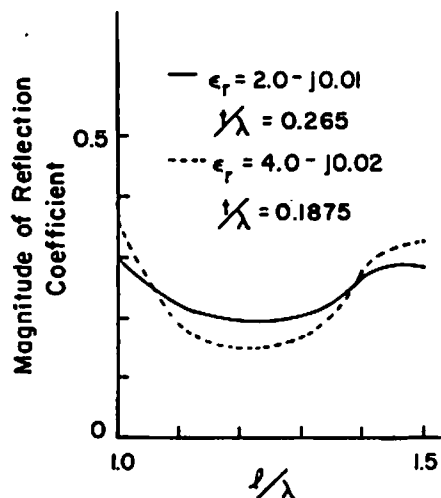
Fig. 3. Magnitude of the reflection coefficient vs. layer distance ℓ. ($b=0.2\lambda$).