PROCEEDINGS OF ISAP '92, SAPPORO, JAPAN

EFFECT OF RANDOM WEIGHT ERRORS ON THE PERFORMANCE OF PARTIALLY ADAPTIVE ARRAY BEAMFORMERS

Ju-Hong Lee and Shiann-Jeng Yu

Department of Electrical Engineering
National Taiwan University
Taipei, TAIWAN 10764

I. INTRODUCTION

Although adaptive array beamformers have been successfully used for the purpose of beamforming, their performance is very sensitive to the mismatch between the actual weights and the ideal weights. The mismatch may be due to the quantization of weight values. Nitzberg [1] and Godara [2] have analyzed the effect of weight errors on array performance. They show that a fully adaptive array beamformer is very sensitive to the random weight errors. To reduce the array sensitivity, Jablon [3] and Zahm [4] have injected an artificial noise to increase the received noise level when computing the adaptive weights. This prevents the beamformer from nulling out the desired signal, while the array's ability of suppressing interference is also reduced.

This paper investigates the effect of random errors induced by the adaptive weights on the performance of adaptive array beamformers. Analytical formulas representing the array output SINR (signal-to-interference plus noise power ratio) in the presence of random weight errors are derived for partially adaptive and fully adaptive beamformers, respectively. It is shown that the partially adaptive beamformer is less sensitive to random weight errors when compared to the fully adaptive beamformer with the same size. Computer simulations confirm the theoretical work.

II. FORMULATION OF ADAPTIVE ARRAY BEAMFORMING

Consider an adaptive array beamformer based on the GSC structure with N elements as shown in Fig.1. Let X(t) be the received signal data vector and W_q be the quiescent weight vector. Then W_q is given as $W_q = C(C^HC)^{-1}f$, where C denotes the constraint matrix with size NxL and f is an Nx1 response vector satisfying $C^HW_q = f$, H is the Hermitian operation. The overall array weight vector $W = W_q - BW_p$. The optimal adaptive weight vector W_p is given as $W_p = R_u^{-1}P$ which is found by minimizing the array output power = $E[|y(t)|^2] = E[|W_q^HX(t)-W_p^HU(t)|^2]$, where $R_u = B^HR_xB$ is the correlation matrix of the output $U(t) = B^HX(t)$ of the signal blocking matrix B, $P = B^HR_xW_q$, and $R_x = E\{X(t)X^H(t)\}$ is correlation matrix of the received signal vector $X(t) = S(t) + S_i(t) + N(t)$, where S(t) is the desired signal vector with power p_s, $S_i(t)$ is the interference signal vector with correlation matrix R_i, and N(t) is the white noise vector with correlation matrix σ^2I. Then the array output power is given as

$$\zeta_f = E[|W_q^HX(t)|^2] - P^HR_u^{-1}P \quad (1)$$

Next, a partially adaptive beamformer uses a part of U(t) to find the optimal adaptive weights. Suppose that M (M<N-L) signals of the signal vector U(t) are used. Let the matrix T with size (N-L)xM spans the signal subspace associated with the M signals and G with size (N-L)x(N-L-M) spans the unused signal subspace. Hence, rank of [T|G] = (N-L). Then the optimal weight vector W_p is obtained by solving the following minimization problem

$$\text{Minimize } E[|y(t)|^2] \text{ subject to } G^H W_p = 0 . \tag{2}$$

Its solution is given as

$$W_p = W_f - R_u^{-1} G (G^H R_u^{-1} G)^{-1} G^H W_f . \tag{3}$$

Subsittuting (3) into $y(t) = d(t) - W_f^H U(t)$ and computing the power of y(t) yields

$$\zeta_p = \zeta_f + P^H R_u^{-1} G (G^H R_u^{-1} G)^{-1} G^H R_u^{-1} P . \tag{4}$$

Comparing (1) and (4), we note that $\zeta_p > \zeta_f$.

III. EFFECT OF RANDOM WEIGHT ERRORS

Here, we assume that the computed adaptive weights are different from the optimal adaptive weights as follows

$$\widehat{W}_p = W_p + \Gamma_w , \tag{5}$$

where Γ_w denotes the weight error vector and its elements γ_i , i = 1, 2,..., N-L, are statistically independent random variables with mean zero and variance σ_w^2. The overall array weight vector is given as $\widehat{W} = W_q - B\widehat{W}_p = W - B\Gamma_w$. Assuming S(t), $S_i(t)$, and N(t) are mutually uncorrelated. Then the array output power is given as

$$\widehat{\zeta} = \widehat{W}^H R_x \widehat{W} = \widehat{\zeta}_s + \widehat{\zeta}_i + \widehat{\zeta}_n , \tag{6}$$

where $\widehat{\zeta}_s = \widehat{W}^H E\{S(t)S(t)^H\}\widehat{W} = p_s$ is the output signal power, $\widehat{\zeta}_i$ is the output interference power given as

$$\widehat{\zeta}_i = \widehat{W}^H E\{S_i(t)S_i(t)^H\}\widehat{W} = (W - B\Gamma_w)^H R_i (W - B\Gamma_w) = W^H R_i W + \Gamma_w^H B^H R_i B \Gamma_w - \Gamma_w^H B R_i W - W^H R_i B \Gamma_w . \tag{7}$$

$\widehat{\zeta}_n$ is the output noise power given as

$$\widehat{\zeta}_n = \sigma^2 (W^H W + \Gamma_w^H B^H B \Gamma_w - W^H B \Gamma_w - \Gamma_w^H B^H W) . \tag{8}$$

Taking the expectations for $\widehat{\zeta}_i$ and $\widehat{\zeta}_n$, we obtain

$$E\{ \widehat{\zeta}_i \} = \widehat{\zeta}_i' = W^H R_i W + E\{ \Gamma_w^H B^H R_i B \Gamma_w^H \}) , \tag{9}$$

and

$$E\{ \widehat{\zeta}_n \} = \widehat{\zeta}_n' = \sigma^2 (W^H W + E\{ \Gamma_w^H B^H B \Gamma_w^H \}) , \tag{10}$$

respectively. Since the term $W^H R_i W$ represents the output interference power, it can be neglected when the random weight errors are not present. The output SINR is then approximately given as

$$\mathrm{SINR} = \frac{P_s}{\sigma^2 W^H W}\left[1+\frac{\mathrm{Tr}(B^H B E\{\Gamma_w \Gamma_w^H\})+\sigma^{-2}\mathrm{Tr}(B^H R_i B E\{\Gamma_w \Gamma_w^H\})}{W^H W}\right]^{-1} \tag{11}$$

where $P_s/\sigma^2 W^H W$ is the output SINR of the adaptive array without random weight errors. 'Tr' denotes the trace operation. The inverse term in (11) is called the weight degrading factor (WDF). Next, we evaluate the sensitivity of adaptive array beamformers to the random weight error. The $W^H W$ is given as

$$W^H W = f^H (C^H C)^{-1} f + W_f^H B^H B W_f \tag{12}$$

for fully adaptive beamformers, while it is approximately given as

$$W^H W = f^H (C^H C)^{-1} f + W_f^H B^H B W_f + P^H R_u^{-1} G (G^H R_u^{-1} G)^{-1} G^H R_u^{-1} P \tag{13}$$

for partially adaptive beamformers. Obviously, the value of (13) is greater than that of (12). Therefore, we note from (11) that the WDF of a partilly adaptive beamformer is greater than that of a fully adaptive beamformer, i.e., a partially adaptive beamformer is less sensitive to the random weight error.

IV. SENSITIVITY OF ELEMENT-SPACE PARTIALLY ADAPTIVE BEAMFORMERS

In this section, we consider the case of element-space partially adaptive beamformers which have been widely considered in the literature. Since the signal blocking matrix B must be orthogonal to the constraint matrix C, one may choose the eigenvectors corresponding to zero eigenvalues of $C(C^H C)^{-1} C^H$ as the columns of B. In the case of unit gain constraint, B can be of the form

$$B^H = \begin{bmatrix} 1 & -1 & 0\ldots & 0..0 \\ 0..0 & 1 & -1 & 0..0 \\ 0..0 & 0..0 & 1 & -1 \end{bmatrix}. \tag{14}$$

Assume that there are J interferers. The covarinace matrix R_i can be written as

$$R_i = \sum_{i=1}^{J} p_i S_i S_i^H, \tag{15}$$

where p_i and S_i are the power and phase vector of the ith interferer, respectively. From (14) and (15), we obtain

$$B^H R_i B = \sum_{i=1}^{J} \alpha_i p_i S_i' S_i'^H, \tag{16}$$

where $\alpha_i = |1 - e^{j\varphi_i}|^2$ and ϕ_i is the phase associated with the ith interferer. S_i' denotes the phase vector associated with the ith interferer with size less than that of S_i by one. Furthermore, we have

$$\mathrm{Tr}(B^H B E\{\Gamma_w \Gamma_w^H\}) = 2M\sigma_w^2, \tag{17}$$

where M is the degrees of freedom for adaptation. From (16), we have

$$\mathrm{Tr}(B^H R_i B E\{\Gamma_w \Gamma_w^H\}) = M\sigma_w^2 \sum_{j=1}^{J} p_i \alpha_i. \tag{18}$$

Substituting (17) and (18) into the WDF of (11) yields

$$WDF = \left[1 + M\sigma_w^2 \left(\frac{2 + \sum_{j=1}^{J} p_i \alpha_i}{W^H W} \right) \right]^{-1} \qquad (19)$$

In the case of arrays with full adaptivity, M = (N-1) in (19). Moreover, (19) reveals that the WDF is independent of the elements of U(t) chosen for adaptation.

V. COMPUTER SIMULATIONS

An example of element-space partially adaptive beamformers is presented. Consider an equally spaced linear array with 10 elements and half wavelength for interelement spacing. A desired signal is at broadside with SNR=10 dB and an incoherent interferer is incident from 30^0 off broadside with INR=20 dB. Figure 2 shows the WDF in dB versus the variance σ_w^2 of the random weight error. This comfirms that the fully adaptive array is very sensitive to random weight error and partially adaptive arrays are less sensitive than the fully adaptive array with the same array size.

REFERENCES

[1] R. Nitzberg,"Effect of errors in adaptive wights", IEEE Trans. Aerospace and Electronic Systems, Vol.AES-12, May 1976, pp.369-373.

[2] L.C. Godara,"Error analysis of the optimal antenna array processors", IEEE Trans. Aerospace and Electronic Systems, Vol.AES-22, July 1986, pp.395-408.

[3] N.K. Jablon,"Adaptive beamforming with the generalized sidelobe canceller in the presence of array imperfections", IEEE Trans. Antennas and Propagation, Vol.AP-34, Aug. 1988, pp.996-1012.

[4] C.L. Zahm,"Effects of errors in the direction of incidence on the performance of an adaptive array", Proc. of IEEE, Vol.54, Aug. 1972, pp.1008-1009.

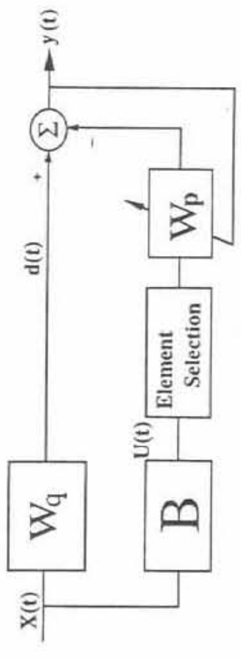

Fig. 1 The Configuration of The Partially Adaptive Beamformer

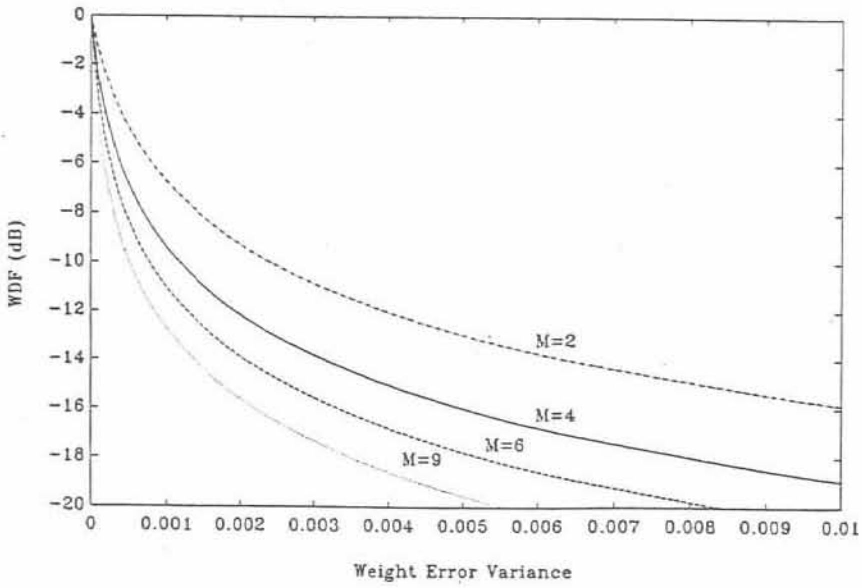

Fig. 2 The WDF versus The Weight Error Variance