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BOUNDARY PROBLEM BETWEEN VACUUM AND ANISOTROPIC VLASOV PLASMA

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The problem that is treated here is the reflection at the interface between vacuum and anisotropic one component (electron) plasma. The anisotropy is due to an externally applied static magnetic field applied perpendicularly to the interface. The electromagnetic wave impinges normally upon the interface between the vacuum and plasma. The plasma is considered to be homogeneous and warm. The procedure that is applied in this paper is somewhat similar to Felderhof's method[1]. But, because of the anisotropy present, all frequencies in the plasma are Doppler shifted by the electron cyclotron frequency ω_c.

This Doppler shift of the frequency destroys the symmetry which makes the problem more difficult to handle. For this case Felderhof's method has to be modified.

In order to obtain the first order field quantities, i.e., the first order electric and magnetic fields and the velocity distribution function, a more systematic procedure is used:

First, the dyadic Green's function is derived in the Fourier transformed space with respect to the position and time; viz. in the k-ω space (k: wave number, ω: angular frequency). Then using the wave number k as an eigenvalue the complete set of eigenvectors for the guided wave representation is obtained in terms of the spectral representation of the Green's function[2]. This spectral representation gives the normal modes or the Van Kampen-Case Modes[3,4,5,6] for this problem. Secondly, all the first order field quantities are obtained in terms of the complete set of eigenvectors by using superposition, once the expansion coefficients are determined. In deriving the superposition coefficients the speculum assumption is used, i.e., all particles at the boundary are reflected back into the plasma with the same magnitude of velocity as they come to the boundary, or equally one can state that the boundary at the interface between vacuum and plasma is abrupt. In addition the outgoing character of the waves and causality conditions are taken into account to get a unique solution.

The field quantities in the plasma (z>0) are given as:

Electric field, $E_{\substack{\ell\\r}}(z)$,

$$E_{\substack{\ell\\r}}(z) = \frac{\omega}{\pi\epsilon_0} H_{\substack{\ell\\r}}(0) \int_{-\infty}^{\infty} \frac{e^{ikz}}{\omega^2 - c^2k^2 + \omega_p^2 \int_{-\infty}^{\infty} \frac{\frac{\omega}{k}F(u') - I(u')}{u' - \frac{\Omega_{\substack{\ell\\r}}}{k}} du'} dk \qquad (1)$$

Electron distribution function, $f_{\substack{\ell\\r}}(u,z)$,

$$f_{\substack{\ell\\r}}(u,z) = \frac{\omega_p^2}{ie\pi} H_{\substack{\ell\\r}}(0) \left[\int_{-\infty}^{\infty} \frac{\left(\frac{\omega}{k}F(u) - I(u)\right)e^{ikz}}{\left(u - \frac{\Omega_{\substack{\ell\\r}}}{k}\right)\left(\omega^2 - c^2k^2 + \omega_p^2 \int_{-\infty}^{\infty} \frac{\frac{\omega}{k}F(u') - I(u')}{u' - \frac{\Omega_{\substack{\ell\\r}}}{k}} du'\right)} dk \right] \qquad (2)$$

where c: Speed of light in the vacuum

$H_{\substack{\ell\\r}}(0) = H_x(0) \pm iH_y(0)$: $H_x(0)$ and $H_y(0)$ are the x and y components of magnetic field at the interface between vacuum and Vlasov plasma. ℓ and r indicate the left and right hand polarized fields respectively.

$\Omega_{\substack{\ell\\r}} = \omega \pm \omega_c$: ω_c Electron cyclotron frequency.

ω_p: Electron plasma frequency.

$F(u')$: Background electron distribution function in z direction of velocity. (Maxwellian distribution is assumed),

$$F(u)=\int_{-\infty}^{\infty} f_o(\underset{\sim}{v})d\underset{\sim\perp}{v_\perp}, \quad \underset{\sim}{v}=v_x\underset{\sim}{x}_o+v_y\underset{\sim}{y}_o+u\underset{\sim}{z}_o,$$

$$d\underset{\sim}{v_\perp} = dv_x dv_y$$

$I(u)=uF(u)-G(u)$:

$$G(u)=-\frac{\partial}{\partial u}\int_{-\infty}^{\infty} v_x^2 f_o(\underset{\sim}{v})d\underset{\sim}{v_\perp}=-\frac{\partial}{\partial u}\int_{-\infty}^{\infty} v_y^2 f_o(\underset{\sim}{v})d\underset{\sim}{v_\perp}.$$

The integral with respect to k for Eqs. (1) and (2) must be interpreted as follows:

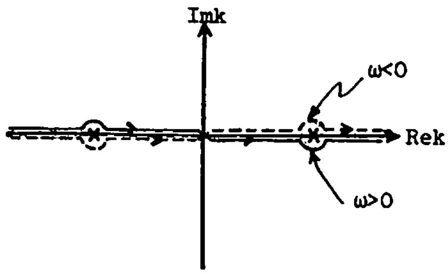

Fig. 1 Integral Path in the Complex k-Plane

The reflection coefficients for the left and right hand polarized field are given as:

$$R_{\substack{\ell\\r}} = \frac{S_{\substack{\ell\\r}} - \frac{\mu_o}{\varepsilon_o}}{S_{\substack{\ell\\r}} + \frac{\mu_o}{\varepsilon_o}} \qquad (3)$$

where $S_{\substack{\ell\\r}}$: Surface impedance at z=0,

$$S_{\substack{\ell\\r}} = \frac{E_x(0)}{H_y(0)}$$

$$S_{\substack{\ell\\r}} = \pm i\,\frac{\omega}{\pi\varepsilon_o}$$

$$\left[\int_{-\infty}^{\infty} \frac{e^{ikz}}{\omega^2-c^2k^2+\omega_p^2\int_{-\infty}^{\infty}\frac{\frac{\omega}{k}F(u')-I(u')}{u'-\frac{\Omega_{\substack{\ell\\r}}}{k}}du}\,dk\right] \qquad (4)$$

From Eq. (4), one can easily see the different surface impedance for the left and right hand polarized fields. Therefore the reflection coefficients for two different polarized fields are different.

Suppose a linearly polarized wave is incident upon the interface and if the frequency of the incident wave is in the range as specified in the following, the corresponding polarizations of the reflected and transmitted fields are given below (assuming $\omega_p > \omega_c$):

i. $\omega > \frac{\omega_c}{2} + \sqrt{(\frac{\omega_c}{2})^2 + \omega_p^2}$

The reflected wave is generally elliptically polarized. Both polarized waves can be transmitted with different phase velocities.

ii. $\frac{\omega_c}{2} + \sqrt{(\frac{\omega_c}{2})^2+\omega_p^2} > \omega > -\frac{\omega_c}{2} + \sqrt{(\frac{\omega_c}{2})^2+\omega_p^2}$

The reflected wave is an elliptically polarized wave. The transmitted wave is a left handed polarization wave.

iii. $-\frac{\omega_c}{2} + \sqrt{(\frac{\omega_c}{2})^2+\omega_p^2} > \omega > \omega_c$

The reflected wave is a linearly polarized wave. No transmitted wave.

iv. $\omega_c > \omega$

The reflected wave is an elliptically polarized wave. The transmitted wave is a right handed polarized wave.

References

1. B.U. Felderhof, "The Theory of Transverse Waves in Vlasov Plasma III", Physica 29, 662 (1963).
2. B. Friedman, "Principles and Techniques of Applied Mathematics", P. 213, John Wiley, New York, 1960.
3. K.M. Case and P.F. Zweifel, "Linear Transport Theory", Addison-Wesley, 1967
4. K.M. Case, "On the Boundary Value Problems of Linear Transport Theory", The University of Michigan, Ann Arbor, Michigan, April 1967.
5. N. Marcuvitz, "Complete Set of Eigenfunctions for a Collisionless Isotropic Maxwell-Vlasov Plasma", Memorandum M-3, New York University.
6. N.G. Van Kampen, "On the Theory of Stationary Waves in Plasma", Physica 21, 949 (1955).
7. K. Sakuda, "Boundary Problems at a Plane Interface Between Vacuum and Vlasov Plasma", Technical Report 402-5, New York University, January 1971.