A NEW APPROACH TO ROBUST BEAMFORMING AGAINST STEERING ERROR

M H ER[1] & B C NG[2]

[1] Centre for Signal Processing
School of Electrical & Electronic Engineering
Nanyang Technological University
Nanyang Avenue, Singapore 2263

[2] Defence Science Organisation
20 Science Park Drive
Singapore 0511

1. INTRODUCTION

Optimal beamforming with multiple linear constraints is now a well established technique. In the simplest case, a single linear constraint is imposed, namely unity gain response in the look direction [1]; the weight vector is then calculated by minimizing the beamformer output power subject to this constraint. In order for the beamformer to reject noise and interferences without rejecting signal, it is necessary to make some assumptions about the signal characteristics. In actual operating conditions, the assumptions of plane wave signals, perfectly matched sensor channels, exact knowledge of sensor locations and perfect match between assumed look direction and actual direction of arrival of the desired signal do not hold. Under these non-ideal conditions, the performance of the single linear constraint beamformer degrades rapidly [2]. To combat against beam-steer error, the use of multiple linear constraints [2, 3, 4] and quadratic inequality constraints [5] has been described in the literature. The method of artificial noise injections [6] and hybrid method [7] have also been proposed by various researchers. Recently, a robust adaptive array based on signal subspace approach has been proposed in [8].

In this paper, a new approach to array beamforming in the presence of steering error is presented. The basic idea is based on the fact that the output power of an optimized beamformer has a maximum if the steering vector coincides with that of the desired signal. The approach iteratively searches for the correct steering vector by maximizing the array mean output power with respect to the steering errors. A first order Taylor series approximation to the steering vector in terms of the steering error results in a simple one dimension optimization problem. This approach has several advantages. It is computionally inexpensive and is easy to implement. There is also no loss in the degrees of freedom in the beamformer for interference rejection. It is independent of the location of the reference origin.

2. PROPOSED APPROACH TO ROBUST BEAMFORMING AGAINST POINTING ERROR

In the linearly constrained minimum power beamforming technique [2], the processor weights are chosen to minimize the processor mean output power subject to a unity gain response in the look direction as an indirect way of rejecting noise and interferences incident on the array. Mathematically, the optimum weight vector $\underline{\mathbf{W}}$ is the solution to the following constrained optimization problem:

$$\min_{\underline{W}} \underline{W}^H R \underline{W} \quad \text{subject to } \underline{W}^H \underline{S}(f_o,\theta) = 1 \tag{1}$$

where $\mathbf{R}$ is the array covariance matrix defined by $\mathbf{R} = \mathcal{E}X(t)X^H(t)$ with $\mathcal{E}$ denoting the expectation operator, and $\underline{S}(f_o,\theta)$ is the N-dimensional steering vector $\underline{S}(f_o,\theta) = \left[e^{j2\pi f_o \tau_1}, \ldots, e^{j2\pi f_o \tau_N}\right]^T$ where $\{\tau_i, i= 1, 2, \ldots, N\}$ are the propagation delays between the plane wavefront and the antenna elements.

Using the method of Lagrange multiplier, it is easy to show that the optimum weight vector which solves (1) is given by

$$\hat{\underline{W}} = \mathbf{R}^{-1}\underline{S}(f_o,\theta)/\underline{S}^H(f_o,\theta)\mathbf{R}^{-1}\underline{S}(f_o,\theta) \tag{2}$$

and the optimum output power of the array is given by

$$P(\hat{\underline{W}}) = 1/\underline{S}^H(f_o,\theta)\mathbf{R}^{-1}\underline{S}(f_o,\theta) \tag{3}$$

It can be shown that the output power of an optimized beamformer achieves a maximum when the steering vector coincides with that of the desired signal. A simple and yet effective approach is proposed to search for the correct steering vector.

It is clear from (3) that maximizing the mean output power with respect to the steering vector is equivalent to

$$\min_{\underline{S}} \underline{S}^H R^{-1} \underline{S} \tag{4}$$

In a small region $\delta\theta$ about the assumed look direction θ_o, $\underline{S}$ can be approximated by a first order Taylor series,

$$\underline{S}(\theta) = \underline{S}(\theta_o + \delta\theta) \approx \underline{S}(\theta_o) + \delta\theta \underline{S}_1(\theta_o) \tag{5}$$

where $\underline{S}_1(\theta_o) = \left.\dfrac{d\underline{S}(\theta)}{d\theta}\right|_{\theta=\theta_o}$. Use of (5) in (4) results in

$$\min_{\delta\theta} (\underline{S}(\theta_o) + \delta\theta\underline{S}_1(\theta_o))^H R^{-1}(\underline{S}(\theta_o) + \delta\theta\underline{S}_1(\theta_o)) \tag{6}$$

which is a simple one-dimensional optimization problem. It can be easily shown that the optimum $\delta\theta_o$ is given by

$$\delta\theta_o = -\operatorname{Re}(\underline{S}^H(\theta_o)\mathbf{R}^{-1}\underline{S}_1(\theta_o))/\underline{S}_1^H(\theta_o)\mathbf{R}^{-1}\underline{S}_1(\theta_o) \tag{7}$$

where Re(.) denotes the real part of the complex quantity. The optimum steering vector can then be found by substituting (7) into (5).

If the pointing error is very small, then the steering vector found using (7) is a true steering vector in the sense that it points to a certain direction. However if the pointing error is quite large, then there is no guarantee that the vector found is a true steering vector. To overcome this, an iterative algorithm that updates the steering error $\delta\theta$ is proposed. Let θ_0 be the assumed (initial) look direction.

Algorithm: For i = 0, 1, 2, ...

$$\delta\theta_i = -\operatorname{Re}(\underline{S}^H(\theta_i)R^{-1}\underline{S}_1(\theta_i))/\underline{S}_1^H(\theta_i)R^{-1}\underline{S}_1(\theta_i)$$

If $\delta\theta_i \leq \epsilon$ then

Stop

Else

$$\theta_{i+1} = \theta_i + \delta\theta_i$$

Repeat.

where ϵ can be set to a desired precision (for example 0.01°) and $\delta\theta_i$ is the steering error at the i^{th} iteration.

3. NUMERICAL RESULTS

To illustrate the performance of the proposed technique, computer studies involving a uniform linear array of 10 sensors with a spacing of half-wavelength have been carried out. The source scenario was assumed to consist of a desired signal of 0 dB power, two interference signals at -25° and 25° with powers of 20 dB and 10 dB respectively and background white noise of -10 dB. The assumed look direction is at 0°.

Figure 1 shows the output signal-to-interference-plus-noise ratio (SINR) of the proposed algorithm (dash line) against steering angular error, where the steering angular error corresponds to the actual angle of arrival of the desired signal. The performance of the Capon's beamformer [5] (solid line) and derivative constrained approach [11] (dot-dash line) are also included for comparison. It can be seen that the proposed method maintains a flat output SINR identical to the ideal case while the Capon's beamformer degrades rapidly when the steering angular error is increased. For small error, the zero-plus-first derivative constrained approach works quite well but the performance also degrades rapidly as the steering angular error increases.

Figure 2 shows the respective beam patterns in the presence of a 5° steering angular error. Note the desired signal cancellation that results in the Capon's beamformer (solid line) and the derivative constrained approach (dot-dash line) while the beam pattern of the proposed approach (dash line) is remarkably similar to the ideal case (dotted line).

4. CONCLUSIONS

This paper has presented a simple and effective approach to array beamforming in the presence of steering angular error. The approach iteratively searches for the correct steering vector by maximizing the array mean output power with respect to the steering error. By a first order Taylor series approximation of the steering vector in terms of the steering error, the maximization process reduces to a simple one-dimensional optimization problem. Thge approach has the advantages of easy implementation and suffers no loss in the degrees of freedom for interference rejection. The approach can be extended to handle multiple errors.

REFERENCES

[1] J. Capon, "High-resolution frequency wavenumber spectrum analysis," *Proc. IEEE*, No. 8, pp 1408-1418, 1969.

[2] A.M. Vural, "Effects of perturbations on the performance of optimum adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. AES-15, No. 1, pp 76-87, 1979.

[3] S.P. Applebaum, and D.J. Chapman, "Adaptive arrays with main beam constraints," *IEEE Trans. Antennas & Propagation*, AP-24, No. 5, pp 650-662, 1976.

[4] M.H. Er, "Adaptive antenna array under directional and spatial derivative constraints," *Proc. IEE*, Pt H, Vol. 135, No. 6, pp 414-419, 1988.

[5] M.H. Er and A. Cantoni, "An alternative formulation for an optimum beamformer with robustness capability," *Proc. IEE*, Pt H, Vol. 132, pp 447-460, 1985.

[6] K. Takao and N. Kikuma, "Tamed adaptive antenna array," *IEEE Trans. Antennas & Propagation*, AP-34, No. 6, pp 388-394, 1986.

[7] Y.J. Hong, D.R. Ucci and C.C. Yeh, "The performance of an adaptive array combining reference signal and steering vector," *IEEE Trans. Antennas & Propagation*, AP-35, pp 763-770, July 1987.

[8] J.W. Kim and C.K. Un, "Signal subspace method for beam-steered adaptive array," *Electron, Lett.*, Vol. 25, No. 16, pp 1076-1077, Aug 1989.

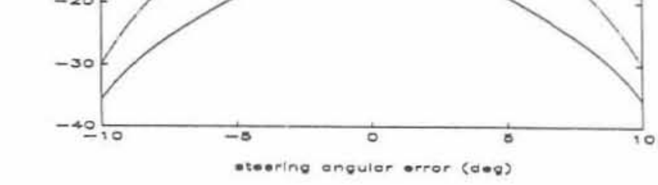

Figure 1

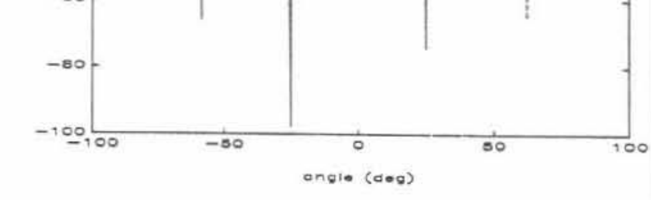

Figure 2