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A COUPLING MODEL FOR A PAIR OF SKEWED TRANSMISSION LINES

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There has been a renewed interest in the subject of multiconductor transmission lines over the past few years, partially because of a direct application in the area of nuclear electromagnetic pulse (NEMP). Once the NEMP energy penetrates into the interior of a system (such as an aircraft or missile) it can couple to and propagate along the cables inside the structure. In paractice, the cables consist of multiconductors that involve complicated branching. In this context, several canonical problems have been identified and are being investigated by various researchers.

In this paper, we address one such canonical problem involving the coupling of a pair of skewed transmission lines formed by two wires parallel to an image plane. The two wires shown in figure 1 are at heights h_1 and h_2 and assumed to be of the same radius, although this is not essential. The objective is to compute the elements of a coupling model in the form of a junction equivalent circuit. In general, for arbitrary values of θ in the range of $[0 < \theta \leq (\pi/2)]$, one would expect both inductive and capacitive coupling between the transmission lines. The special case of $\theta = 0$ is precluded here because of the distributed nature of the coupling which cannot be treated with localized lumped elements, and this problem has been reported elsewhere, e.g., [1]. For values of θ in the range of $[0 < \theta \leq (\pi/2)]$, one has L_m, C_1, C_2 and C_{12} to evaluate and they respectively are the mutual inductance, excess self-capacitance in lines 1 and 2 and the mutual capacitance between the two lines. Since the wires are assumed to be perfect electrical conductors and hence nonmagnetic, there is no change in self-inductance of each wire because of the proximity of the other wire resulting in only inductive coupling via L_m.

It is believed that more insight into the problem is gained by looking at the skewed transmission lines of finite length 2L (see figure 2) and later let L tend to infinity. If the wires are sufficiently far apart $[a \ll |b|]$, we may assume rotational symmetry and write the mutual inductance as [2]

$$\left[L_m = \frac{\mu_o}{\pi} \oint_{C_1} \vec{ds}_1 \cdot \oint_{C_2} \frac{1}{R_{12}} \vec{ds}_2 \right] \qquad (1)$$

where μ_0 = free space permeability = $4\pi \times 10^{-7}$ (H/m) and C_1 and C_2 represent the two closed (assumed) loops formed by the two transmission lines, as illustrated in figure 2. Taking advantage of symmetry, we analytically perform the above integrals [3] and finally the normalized mutual inductance $[L_m^{(norm)} = L_m/(h_1 L_1')]$ is computed and parametrically shown plotted in figure 3. The normalization scheme uses the height h_1 and the inductance-per-unit length L_1' of line 1. As a function of the skew angle θ, $L_m^{(norm)}$ goes to zero when the transmission lines are at right angles. The detailed behavior of $L_m^{(norm)}$ as a function of lengths and heights will be discussed in the presentation.

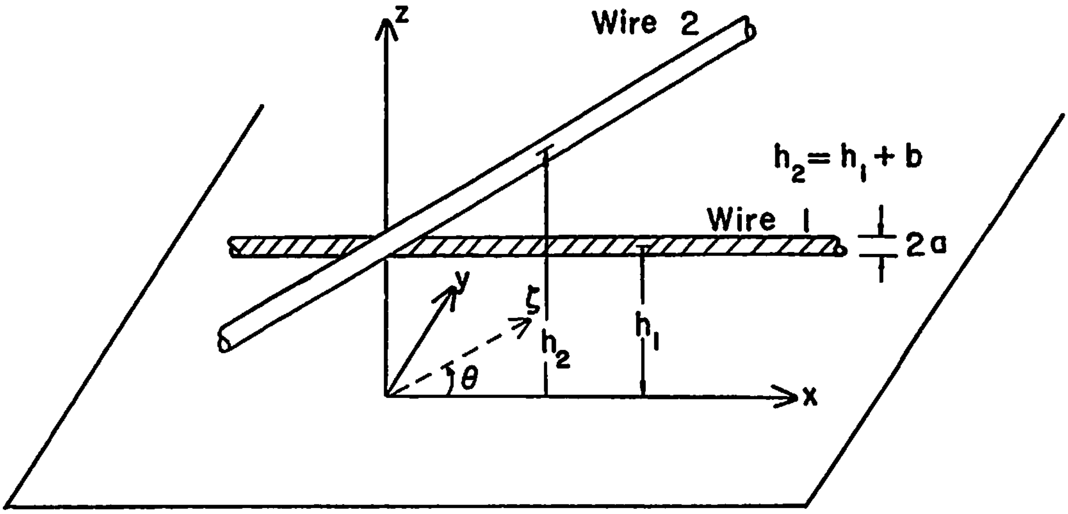

Figure 1. Two skewed wires of the same radius above an image plane.

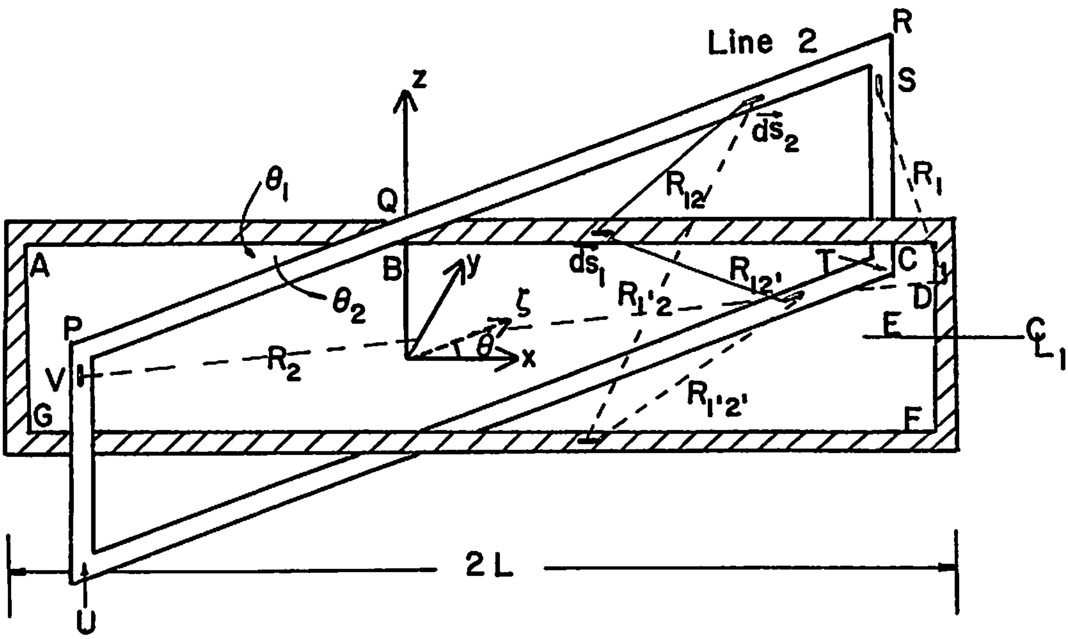

Figure 2. Equivalent pair of skewed two-wire transmission lines of length 2L.

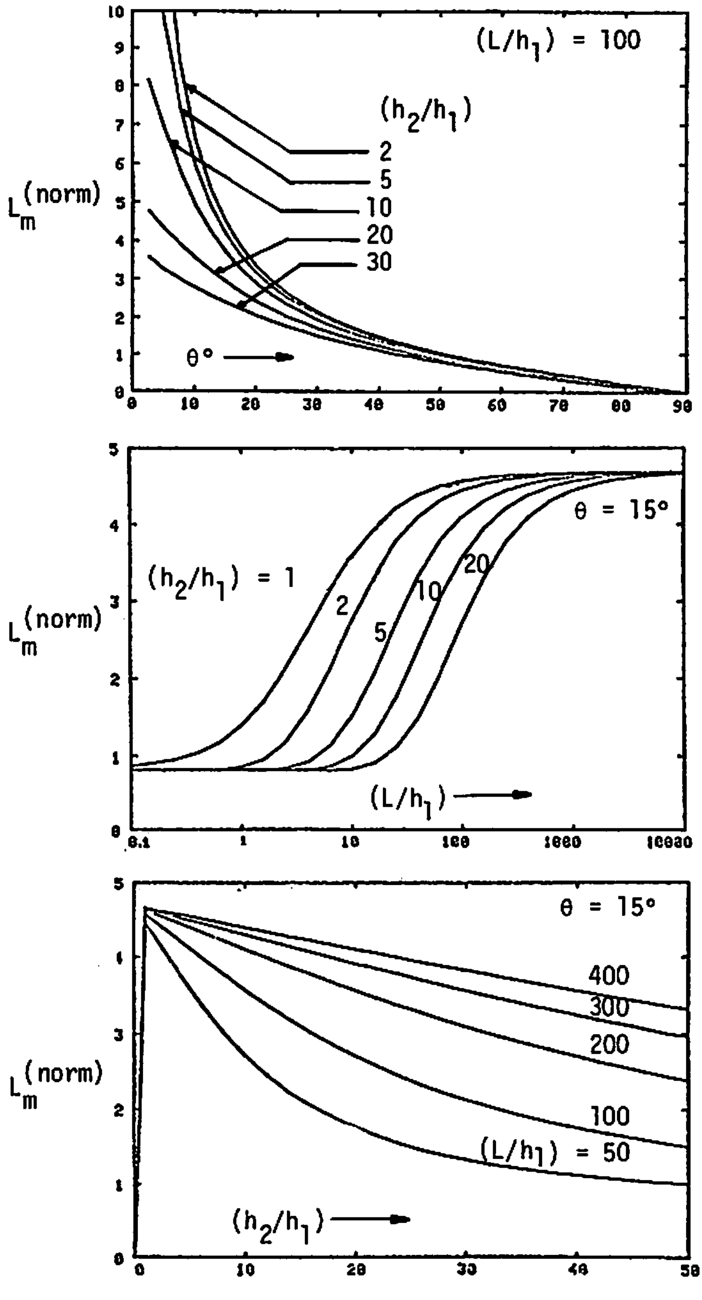

Figure 3. Parametric variation of the normalized mutual inductance.

In order to determine the capacitive coupling elements, we have formulated a coupled pair of integral equations

$$\int_{-\infty}^{\infty} q_1'(x') \, K_{11}(x,x') \, dx' + \int_{-\infty}^{\infty} q_2'(\zeta') \, K_{12}(x,\zeta') \, d\zeta' = V_2 \, C_2' \, p(x) \qquad (2)$$

$$\int_{-\infty}^{\infty} q_1'(x') \, K_{21}(\zeta,x') \, dx' + \int_{-\infty}^{\infty} q_2'(\zeta') \, K_{22}(\zeta,\zeta') \, d\zeta' = V_1 \, C_1' \, g(\zeta) \qquad (3)$$

where the various kernel and the forcing functions are known. V_1 and V_2 are the potentials on wires 1 and 2. C_1' and C_2' are the line capacitances. We solve for the excess charges $q_1'(x)$ and $q_2'(\zeta)$ due to the capacitive coupling. From the known excess charges, the elements of capacitive coupling circuit can then be evaluated. Time permitting, these results will also be presented.

References

[1] C.R. Paul and A.E. Feather, "Computation of the Transmission Line Inductance and Capacitance Matrices from the Generalized Capacitance Matrix", IEEE Trans., Vol. EMC-18, No. 4, November 1976.

[2] R.W.P. King, Fundamental Electromagnetic Theory, Chapter VI, Dover Publications, New York, 1963.

[3] I.S. Gradshteyn and I.W. Ryzhik, Table of Integrals Series and Products, Academic Press, New York, 1965.