1-III D2

THE MEAN GREEN'S DYADIC FOR A RANDOM UNIAXIAL ANISOTROPIC MEDIUM WITH SMALL SCALE FLUCTUATIONS

D. Dence
Communications/Automatic Data Processing Laboratory
Fort Monmouth, N. J. 07703

John E. Spence
Department of Electrical Engineering
University of Rhode Island
Kingston, Rhode Island 02881
UNITED STATES

Under investigation is the mean Green's dyadic for an unbounded medium characterized by a permittivity tensor whose elements can be expressed as the sum of two terms, one deterministic and the other random. The deterministic and random parts of the permittivity tensor are considered to be uniaxial anisotropic; thus it is a diagonal tensor where two of the three elements are equal. Our main concern is with determining the effect of small scale fluctuations defined by (free space wavenumber) x (dielectric tensor element)$^{1/2}$ x (correlation length) $\ll 1$. An effective permittivity tensor is defined to provide for this case an equivalent deterministic medium description. It is shown that this "effective" permittivity tensor is also uniaxial anisotropic. Utilizing this medium description the mean Green's dyadic is obtained thereby revealing the nature of the mean ordinary and mean extraordinary waves. They are shown to have two important properties 1) an effective conductivity term appears which is different from that due to absorption and is caused by the scattering of the wave as it propagates and 2) there is an increase in the real part of the dielectric constant which causes both waves to travel at speeds slower than had occurred when the randomness was zero.

A complete description of long range propagation through a deterministic uniaxial medium or a knowledge of the properties of antennas immersed in such media is often times hampered by the assumption of homogeneity. The introduction of the random anisotropic permittivity concept, which accounts for the random inhomogeneities, lends credence to the usefulness of such models for describing known anisotropic media such as the ionosphere (1) or forest (2,3) among others. The results of our analysis represent an essential step in the investigation of layered media and can be used as presented for situations where boundary effects are unimportant.

Previous investigators (4-7), utilizing different procedures, have studied the coherent wave in a medium where the permittivity tensor takes a variety of forms. One of these procedures involves casting the governing equation for the wave process into an integral equation involving a centered (zero mean) random function ξ and a Green's dyadic $\underset{\approx}{G}_s$ which by continued iteration yields the Neumann series. ξ describes the random character of the permittivity (random inhomogeneities) whereas $\underset{\approx}{G}_s$ represents the solution for the wave when ξ is zero (called the background solution). Utilizing established techniques employing Feynmann diagrams leads to Dyson's equation for the mean Green's dyadic. These procedures can be utilized quite readily for those situations where the random fluctuations are isotropic and the background is either isotropic or anisotropic, but must be modified when the fluctuations are anisotropic. These modifications, which have been set forth in a recent study (8), cause the Feynmann diagrammatical representation of Dyson's equation to remain unaltered except that the "mass operator" contains additional terms. The analysis starts with the bilocal approximation to Dyson's equation in this form. Some results of our study are presented in Figure No. 1.

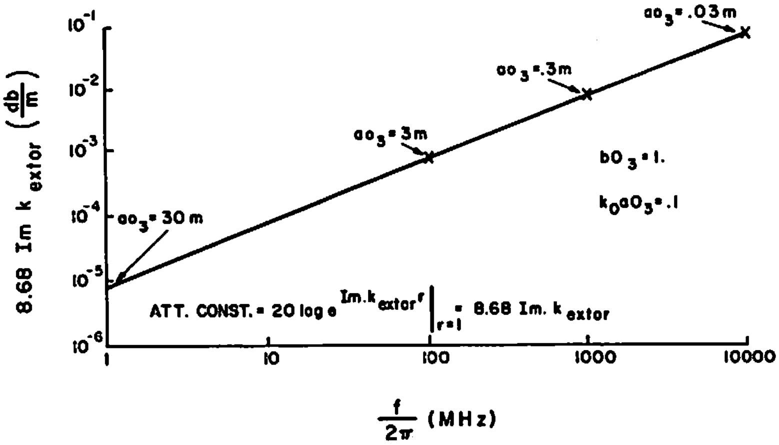

Fig. 1 Attenuation constant in $\frac{db}{m}$ for the situation $K = K_3 = 1$ resulting from a plane wave traveling at $\theta = \pi/2$ and possessing the extraordinary wave number.

Plotted in Fig. No. 1 is the attenuation constant in db/m for a plane wave travelling with the extraordinary wave number where k_{exord} is defined by (k_o = freespace wavenumber)

$$k_{exord} = k_o\sqrt{(K_3+R_{33})\sin^2\theta+(K+R)\cos^2\theta}$$

The functional variation of the attenuation constant reveals the significance of the scattering mechanism. $K + R$, $K_3 + R_{33}$ represent the "effective" dielectric constant in the horizontal and vertical direction respectively. K, K_3 represent the dielectric constant of the background whereas R, R_{33} represent the dielectric constant caused by the random fluctuations. The correlation function describing the two point correlation statistic for the three dimensional fluctuations in the vertical direction was chosen to be an exponential. The amplitude of the correlation function was bO_3 and the correlation length was aO_3. θ represents the angle of the incident wave as measured from the vertical direction.

REFERENCES

(1) K. G. Budden, Radio Waves in the Ionosphere, Cambridge University Press, 1961.

(2) T. Tamir, "On Radio Wave Propagation in Forest Environments," IEEE Trans. Antennas Propagat., Vol. AP-15, pp. 806-817, November, 1967.

(3) D. Dence and T. Tamir, "Radio Loss of Lateral Waves in Forest Environments," Radio Sci., Vol. 4, No. 4, pp. 307-318, 1969.

(4) J. B. Keller, "Stochastic Equations and Wave Propagation in Random Media, "Proc. Symp, Appl. Math., Vol. 16, pp. 145-170, 1960.

(5) V. Í. Tartarskii and M. E. Gertsenshtein, "Propagation of Waves in a Medium with Strong Fluctuations of the Refractive Index," Soviet Phys. JETP, 17, pp. 458-463, August, 1963.

(6) W. P. Brown, "Propagation in Random Medium--Cumulative Effect of Weak Inhomogeneities," IEEE Trans. on Antennas and Propagation, Vol. AP-15, pp. 81-89, January, 1967.

(7) S. Rosenbaum, "On Energy-conserving Formulations in a Randomly Fluctuating Medium" Proc. of the Symp. on Turbulence of Fluids and Plasmas, Polytechnic Institute of Brooklyn, pp. 163-185, April, 1968.

(8) D. Dence and John E. Spence, "The Coherent Wave in a Random Uniaxial Anisotropic Medium," IEEE Trans. on Antennas and Propagation, Vol. AP-19, pp. 302-305, March, 1971.