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SYMMETRIZED ENERGY-MOMENTUM TENSOR OF THE ELECTROMAGNETIC WAVE IN A MOVING DISPERSIVE MEDIUM

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In the Minkowski formulation of electromagnetism the electric field E, the electric displacement D, the magnetic induction B are introduced as field quantities. In the usual linear treatments D and B are assumed proportional to E and H, respectively. In a dynamical system composed of such linear responses, which we call the linear Minkowski subsystem(LMS), the power flow density S, the energy density W, the stress tensor T, the momentum density G, and the time rate of wave energy dissipation Q in a unit volume and time are defined for quasi-monochromatic waves with the real wave number k and the real frequency ω as the quadratic quantities in LMS. They satisfy the conservation relations[1]:

$$\nabla\cdot S + \partial W/\partial t + Q = 0 \quad (1)$$

$$\nabla\cdot T + \partial G/\partial t + \frac{k}{\omega} Q = 0 \quad (2)$$

where

$$S=\mathrm{Re}(\bar{E}^*\times\bar{H}) - \frac{1}{2}\frac{\partial}{\partial k}\omega(\varepsilon^h:\bar{E}^*\bar{E}+\mu^h:\bar{H}^*\bar{H}) \quad (3)$$

$$W=\frac{1}{4}\frac{\partial}{\partial\omega}\omega(\varepsilon^h:\bar{E}^*\bar{E} + \mu^h:\bar{H}^*\bar{H}) \quad (4)$$

$$T=-\frac{1}{2}\mathrm{Re}(\bar{E}^*\bar{D}+\bar{H}^*\bar{B}) + \frac{1}{4}(I-k\frac{\partial}{\partial k})(\varepsilon^h:\bar{E}^*\bar{E} + \mu^h:\bar{H}^*\bar{H}) \quad (5)$$

$$G=\frac{1}{2}\mathrm{Re}(\bar{D}\times\bar{B}^*)+\frac{1}{4}k\frac{\partial}{\partial\omega}(\varepsilon^h:\bar{E}^*\bar{E}+\mu^h:\bar{H}^*\bar{H}) \quad (6)$$

$$Q=\frac{1}{2}\,\mathrm{Im}\,\omega(\varepsilon^a:\bar{E}^*\bar{E} + \mu^a:\bar{H}^*\bar{H}). \quad (7)$$

Here h and a denote hermitian and antihermitian parts of ε and μ. The above relations are valid to the first order in ε^a and μ^a even for a medium in motion.

The energy-momentum tensor T^{LMS} for LMS is then given by

$$T^{LMS}=\begin{pmatrix} \frac{d\omega}{dk}\frac{k}{\omega}W & \frac{i}{c}\frac{d\omega}{dk}W \\ ic\frac{k}{\omega}W & -W \end{pmatrix} \quad (8)$$

where the relations

$$T = S\frac{k}{\omega} \quad (9)$$

$$G = W\frac{k}{\omega} \quad (10)$$

have been used. This tensor is not symmetric except for the case where waves obey the following Klein-Gordon type dispersion

$$\omega^2 = c^2 k^2 + \Omega^2 \quad (11)$$

where Ω is a constant. Polarizability of this medium must be $-\varepsilon_0\Omega^2/\omega^2$, which would be the case when the polarizations are free from restoring forces.

In an open system the power flow density and the energy density should be understood merely as potential functions which altogether give amount of work that would be done upon the externally introduced currents. On the other hand the stress tensor and the momentum density form potential functions for the force upon the external currents.

It has been proved[2] that the time rate of change in the momentum density defined by (6), for instance, is composed of the time rate of change in the momentum density of electromagnetic fields $(1/c^2)E\times H$ and latent forces which would act on the external currents at the expense of kinetic momentum of a medium[3] (Here 'external' means the currents not included in ε) All the other elements of (8) include contribution from kinetic motion of a medium.

The interaction between the kinetic motion of a medium and the fields in the linear response subsystem takes place via deviation of a center of oscillation of each harmonic oscillator which constitutes polarization from a center of restoring force and via polarization charges which would appear upon introduction of the external currents.

Quantities defined by (3) through (7) acquire different physical meanings if the medium is in motion. The transformation laws for S, W, and Q have already beenderived by Musha and Agu[1]. In reference to these transformation laws and (9) and (10) we can know how T and G vary under a Lorentz transformation. Q, for example, represents amount of heat evolved per unit volume and time when the medium is at rest. However, if the medium is moving at a velocity v, Q defined in the laboratory frame of reference is related to Q^o defined in the medium frame of reference as

$$Q=\frac{Q^o}{\gamma}+\gamma v\cdot\frac{k^o}{\omega^o}\,Q^o+\gamma\,\frac{v^2}{c^2}\,Q^o \qquad (12)$$

where $\gamma=(1-v^2/c^2)^{-1/2}$ .and k^o and ω^o are wave number and frequency in the medium frame. The first term represents transformation of heat evolved[4]. During heat evolution the dissipation subsystem is accelerated as a result of decrease in the wave momentum and this rate of increase in the kinetic energy of the dissipation subsystem, which is eventually transferred to the kinetic motion of the medium itself, is given by the second term of (12). The last term is purely relativistic because it represents energy increse of the dissipation subsystem associated with mass of the heat evolved.

Through the analysis on what S, W, T, G, and Q stand for in LMS of spatial and temporal dispersion it has been found: The kinetic subsystem of the medium including translational as well as rotational(for anisotropic material) motion, the dissipation subsystem, subsystems of nonlinear polarization and magnetization (and fields), and LMS altogether constitue an entire closed system. Sum of the energy-momentum tensors for these subsystems forms a symmetric tensor.

References

1) T.Musha and M.Agu, J.Phys.Soc. Japan vol.26, no.2, pp.541-549 (1969).
2) T.Musha(to be published).
3) Similar discussions about contribution of kinetic momentum of the medium to the wave momentum for non-dispersive media (not LMS) are made by H.A.Haus, Physica vol.43, pp.77-91 (1969).
4) $Q^o dV^o d\tau^o$ is heat developed in a volume dV^o and a time $d\tau^o$. Since $dVd\tau$ is invariant under a Lorentz transformation, Q^o transforms like heat itself. Concerning the transformation of heat see, for instance, C. Møller, The Theory of Relativity (Oxford Univ. Press, Oxford, England, 1952) Eq.(121).