NEW BOUNDARY INTEGRAL EQUATIONS FOR CAD OF DIELECTRIC OPTICAL CIRCUITS

KAZUO TANAKA
Department of Electronics and Computer Engineering,
Gifu University, Yanagido 1-1, 501-11, Japan

1. Introduction

Since complicated optical waveguide circuits will be used in the future, it is necessary to construct computer-aided design (CAD) software for optical waveguide circuits. In this paper, we present new boundary integral equations (BIE's) that are suitable for CAD software for optical waveguide circuits. Since general forms of the new BIE's have very complicated expressions, we derive new BIE's for a simple three-ports optical waveguide circuits i.e., the optical branching circuit. The new BIE's can be solved numerically by the conventional boundary element method (BEM). Since new BIE's are exact, the exact solution can be obtained if sufficiently large computer memory and computational time can be employed [1]-[3].

2. Branching Waveguide Circuit

The geometry of the problem to treat is shown in Fig. 1. For the mathematical simplicity, we consider the two-dimensional problem for the case of TE mode. The dielectric waveguide 1, 2 and 3 whose indices of refraction are given by n_2 are joined together at an angles θ_1, θ_2 and θ_3 with positive x axis as shown in Fig. 1. The index of refraction in the surrounding space is given by n_1. It is assumed that all waveguides satisfy the single-mode condition and only a TE-even mode propagates in waveguides.

We denote the outer surrounding space the region I and the inner space in the waveguides the region II as shown in Fig. 1. We next denote the boundary between the surrounding space and waveguides C_j (j=1,2,3) and also denote virtual boundaries between waveguides C_{ji} (i=1,2,j=1,2,3) as shown in Fig. 2.

A dominant TE even-mode is incident from waveguide 2 to the branch. The incident wave denoted by $E^{-(2)}$ is reflected, transmitted and scattered by

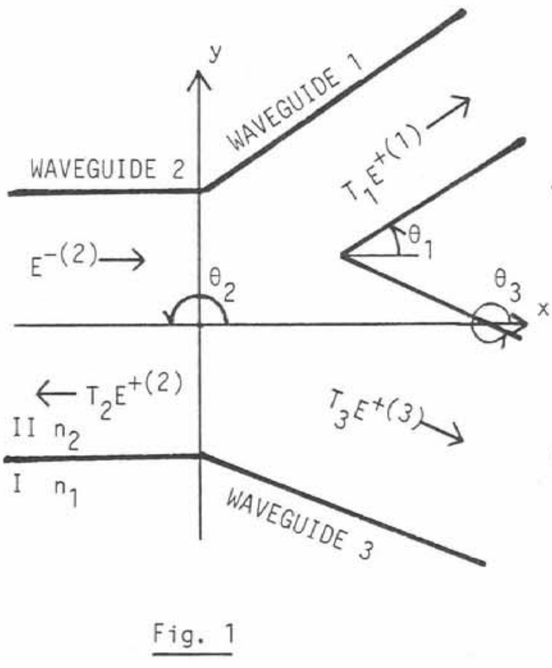

Fig. 1

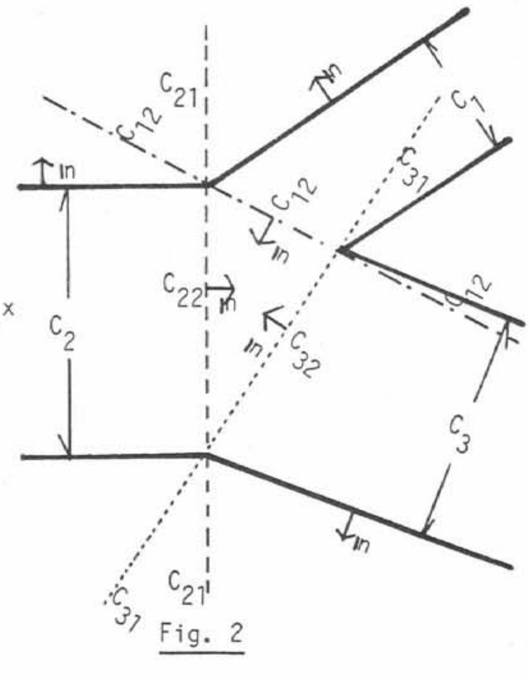

Fig. 2

the branch. The total electric fields denoted by $E_z(x,y)=E(\mathbf{x})$ near the branch that are created by the incident wave are very complicated. However, only reflected or transmitted wave can survive at points far away from the branch in each waveguide. Hence, we decompose total fields on the boundaries C_1-C_3 of waveguides 1,2,3 into field components as

$$E(\mathbf{x})=E^c(\mathbf{x}) + T_j E^{+(j)}(\mathbf{x}) \qquad \text{on } C_j \quad (j=1,3) \tag{1}$$
$$E(\mathbf{x})=E^c(\mathbf{x}) + T_2 E^{+(2)}(\mathbf{x}) + E^{-(2)}(\mathbf{x}) \qquad \text{on } C_2 \tag{2}$$

respectively, as shown in Fig. 1. In (1) and (2), $T_j(j=1,2,3)$ means the transmission or reflection coefficient of the surface wave denoted by $E^{+(j)}$ in the j-th waveguide. Therefore, T_2 is the reflection coefficient in waveguide 2 and T_1 and T_3 are the transmission coefficient in waveguide 1 and 3, respectively. In (1) and (2), $E^c(\mathbf{x})$ represents the field that is a result of subtraction of the transmitted waves or the incident plus the reflected wave from the total fields. We call the field $E^c(\mathbf{x})$ the disturbed field.

3. Transmission and Reflection Coefficients

We first consider the case in which an observation point $\mathbf{x}$ approaches to boundaries $C=C_1+C_2+C_3$ from the region II in Fig. 1. From Maxwell's equations and Green's theorem, the well-known boundary integral equation for the total electric field $E(\mathbf{x})$ is:

$$1/2\cdot E=\int_C (G_2 \partial E/\partial n' - E \partial G_2/\partial n')dl'. \tag{3}$$

The integral means the Cauchy principal value integral with singularities removed. The notation $\partial/\partial n$ stands for the derivative with respect to the outward unit normal vector $\mathbf{n}$ to C as shown in Fig. 2. The expressions G_i $(i=1,2)$ represent Green's functions in the free space whose indices of refraction are given by $n_i(i=1,2)$ and they are expressed as

$$G_i=G_i(\mathbf{x}|\mathbf{x}') = -j/4\cdot H_0^{(2)}(n_i k_0|\mathbf{x}-\mathbf{x}'|) \quad (i=1,2) \tag{4}$$

where $k_0=\omega/c$ and $H_0^{(2)}(x)$ denotes the zero-th order Hankel function of the second kind. It can be seen that the disturbed field $E^c(\mathbf{x})$ will be confined to the vicinity of the branch, i.e., it will satisfy the following conditions:

$$E^c(\mathbf{x}) \text{ and } \partial E^c(\mathbf{x})/\partial n \to 0. \qquad (r=\infty, \theta=\theta_1, \theta_2 \text{ and } \theta_3) \tag{5}$$

We first consider a condition which must hold at points on the boundary C_1 far away from the bend i.e., the condition at $r=\infty$, $\theta=\theta_1$. When we substitute (1) and (2) into the integral equation (3), condition (5) shows that following relation must hold at points on C_1 far away from the branch:

$$1/2\cdot T_1 E^{+(1)}=\int_C (G_2 \partial E^c/\partial n' - E^c \partial G_2/\partial n')dl'$$
$$+\sum_{j=1}^{3} T_j \int_{C_j} (G_2 \partial E^{+(j)}/\partial n' - E^{+(j)} \partial G_2/\partial n')dl' + \int_{C_2} (G_2 \partial E^{-(2)}/\partial n' - E^{-(2)} \partial G_2/\partial n')dl' \tag{6}$$

for $r=\infty$ and $\theta=\theta_1$. Semi-infinite line integrals of the surface waves along C_j $(j=1,2,3)$ in (6) can be rewritten with line integrals along C_{j2} $(j=1,2,3)$ by Green's theorem as

$$1/2\cdot E^{+(j)} \quad -\int_{C_j} (G_2 \partial E^{+(j)}/\partial n' - E^{+(j)} \partial G_2/\partial n')dl'$$

$$=\int_{C_{j2}} (G_2\partial E^{+(j)}/\partial n' - E^{+(j)}\partial G_2/\partial n')dl' \quad (j=1,2,3) \tag{7}$$

where vector **n** normal to C_{j2} are shown in Fig. 2. By using (7), we can rewrite (6) as

$$\sum_{j=1}^{3} T_j\int_{C_{j2}} (G_2\partial E^{+(j)}/\partial n' - E^{+(j)}\partial G_2/\partial n')dl'$$

$$=\int_C (G_2\partial E^c/\partial n' - E^c\partial G_2/\partial n')dl' - \int_{C_{22}} (G_2\partial E^{-(2)}/\partial n' - E^{-(2)}\partial G_2/\partial n')dl' \tag{8}$$

for $r=\infty$ and $\theta=\theta_1$. Since the observation point **x** considered here is far away from the boundaries C_{j2}, and the disturbed field $E^c(\mathbf{x})$ is assumed to be nonzero in the vicinity of the branch, we can use the asymptotic form of the Hankel function as

$$G_i(\mathbf{x}|\mathbf{x}') = A(r)g_i(\theta|\mathbf{x}') + O[(n_i k_0 r)^{-3/2}] \quad (i=1,2) \tag{9}$$

where

$$A(r) = -j/4[2j/(\pi n_i k_0 r)]^{1/2}\exp(-jn_i k_0 r) \tag{10}$$

$$g_i(\theta|\mathbf{x}') = \exp(jn_i k_0 x'\cos\theta + jn_i k_0 y'\sin\theta). \quad (i=1,2) \tag{11}$$

After substituting equation (9) into (8), we divide both sides of the equation by the common function $A(r)$. Setting $r\to\infty$ in the resulting equation, we can obtain a linear equation. We next consider conditions that must hold at points on boundaries C_2 and C_3 far away from the branch. By the same procedure as that which is used for the case of $\theta=\theta_1$, we can also obtain the similar equations for $r=\infty$ and $\theta=\theta_2,\theta_3$. Finally, we can obtain a system of three linear equations for unknown coefficients T_j as

$$\sum_{j=1}^{3} A_{ij}T_j = B_i \quad (i=1,2,3) \tag{12}$$

where

$$A_{ij}=\int_{C_{j2}} [g_2(\theta_i|\mathbf{x}')\partial E^{+(j)}/\partial n' - E^{+(j)}\partial g_2(\theta_i|\mathbf{x}')/\partial n']dl' \tag{13}$$

$$B_i=\int_C [g_2(\theta_i|\mathbf{x}')\partial E^c/\partial n' - E^c\partial g_2(\theta_i|\mathbf{x}')/\partial n']dl' - \int_{C_{22}} [g_2(\theta_i|\mathbf{x}')\partial E^{-(2)}/\partial n' - E^{-(2)}\partial g_2(\theta_i|\mathbf{x}')/\partial n']dl'. \quad (i=1,2,3) \tag{14}$$

If we solve the system of linear equations (12) for unknown coefficients T_j, the transmission coefficient and the reflection coefficient T_j can be expressed in terms of the disturbed field $E^c(\mathbf{x})$ as

$$T_j=\int_C [W_j(\mathbf{x}')\partial E^c/\partial n' - E^c\partial W_j(\mathbf{x}')/\partial n']dl' - M_j \quad (j=1,2,3) \tag{15}$$

where

$$W_j(\mathbf{x}') = \sum_{i=1}^{3} \Delta_{ij} g_2(\theta_i|\mathbf{x}')/\Delta_A \tag{16}$$

$$M_j = \int_{C_{22}} [g_2(\theta_j|\mathbf{x}')\partial E^{-(2)}/\partial n' - E^{-(2)}\partial g_2(\theta_j|\mathbf{x}')/\partial n']dl'/\Delta_A \quad (j=1,2,3) \tag{17}$$

and $\Delta_A=\det|A_{ij}|$ and Δ_{ij} is determinant of cofactor of element A_{ij}.

4. New Boundary Integral Equations

We first substitute expressions (15) into (1) and (2), and then substitute resulting expressions of total fields into the original integral equation (3). Using relations (7), we can finally derive the following boundary integral equation for the disturbed field $E^c(\mathbf{x})$ for the case in which the observation point $\mathbf{x}$ approaches the boundaries C from (inner) region II:

$$1/2\cdot E^c=\int_C (P_2\partial E^c/\partial n'-E^c\partial P_2/\partial n')dl' -S_2(\mathbf{x}), \tag{18}$$

where

$$P_2=G_2(\mathbf{x}|\mathbf{x}') - \sum_{j=1}^{3} U_2{}^{+(j)}(\mathbf{x})W_j(\mathbf{x}'), \quad S_2(\mathbf{x})=U_2{}^{-(2)}(\mathbf{x})- \sum_{j=1}^{3} M_jU_2{}^{+(j)}(\mathbf{x})/\Delta_A \tag{19}$$

and

$$U_i{}^{+(j)}(\mathbf{x})=\int_{C_{ji}} (G_i\partial E^{+(j)}/\partial n'-E^{+(j)}\partial G_i/\partial n')dl'.(i=1,2,j=1,2,3) \tag{20}$$

So far, we have considered the case in which an observation point $\mathbf{x}$ approaches the boundaries C from the (interior) region II. We next consider the case in which the observation point approaches to the boundaries C from the (exterior) region I. From Maxwell's equations and Green's theorem, the integral equation for the disturbed electric field $E^c(\mathbf{x})$ is

$$1/2\cdot E^c=\int_C (P_1\partial E^c/\partial n'-E^c\partial P_1/\partial n')dl'-S_1(\mathbf{x}), \tag{21}$$

where

$$P_1=-G_1(\mathbf{x}|\mathbf{x}')- \sum_{j=1}^{3} U_1{}^{+(j)}(\mathbf{x})W_j(\mathbf{x}'), \quad S_1(\mathbf{x})=U_1{}^{-(2)}(\mathbf{x})- \sum_{j=1}^{3} M_jU_1{}^{+j}(\mathbf{x})/\Delta_A. \tag{22}$$

The original BIE (4) cannot be applied to the problem containing dielectric waveguides of infinite length such as branching waveguides, because they have infinite-sized boundaries. In contrast, since the disturbed field vanishes far from the branch, we can solve the new BIE's (18) and (21) numerically by the conventional BEM (or moment method) as if the waveguide had finite-sized boundaries as in the scattering problem for a finite-sized object. Moreover, the structure of the new BIE's is similar to that of the original BIE's. Hence, we can use various techniques that were developed for solving original BEI's to solve BIE's (18) and (21).

5. Conclusion

New BIE's have been presented that are suitable for the basic theory of CAD software for optical waveguide circuits. The new BIE's are exact and can be solved numerically by the conventional BEM. The concrete expressions of the new BIE's are derived for a typical dielectric optical circuit, i.e., the two-dimensional branching waveguide. It is easy to extend new BIE's to other complicated dielectric optical waveguide circuits. For example, new BIE's and numerical examples for the corner bend of dielectric waveguide are shown in reference [3]. Since the theory is based on the exact theory, the solution is exact if sufficiently large computer memory and computational time can be employed.

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