A time shift parameter setting of temporal decorrelation source separation for periodic signals

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1. Introduction

Temporal Decorrelation Source SEParation (TDSEP) [1] is a blind separation scheme that uses the time structure of the sources signals. The merit of this TDSEP is that it can separate Gaussian distributed signals, as long as they have some time structural properties, such as periodicity, whereas other non-Gaussianity based methods [2] can not. The drawback is that the separation performance heavily depend upon the time shift parameters. This motivates us to investigate the optimal time shift parameters for periodic Gaussian signals. This paper proposes a method to automatically select the time shift parameters, that provides high separation performance.

2. TDSEP

The mixing model is given by the following equation.

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) \tag{1}$$

where $\mathbf{s}$ is the vector composed of J source signals $s_j(j = 1, ..., J)$, $\mathbf{x}(k)$ is the vector composed of I received signals $x_i(i = 1, ..., I)$, k is a sampling index, and $\mathbf{A}$ is the mixing matrix. Further denote $\mathbf{M}$ to be the whitening matrix and whitened received signal vector to be $\mathbf{z}(k)(= \mathbf{M}\mathbf{x}(k))$. Then, the separated signal vector $\mathbf{y}(k)$ is given as follows.

$$\mathbf{y}(k) = \mathbf{W}^H\mathbf{z}(k) = \mathbf{W}^H\mathbf{M}\mathbf{x}(k) \tag{2}$$

where $\mathbf{W}$ is the separating weighting matrix. Suppose $\mathbf{R_z}(\tau)$ to be the correlation matrix of $\mathbf{z}$ and τ is a time shift. In practice, $\mathbf{R_z}(\tau)$ itself is not symmetric, and instead, $\tilde{\mathbf{R}}_\mathbf{z}(\tau) = \{\mathbf{R_z}(\tau) + \mathbf{R_z}^H(\tau)\}/2$ is used [1]. From now on, we use $\tilde{}$ to denotes symmmetrized matrix. In TDSEP, τ is a setting parameter and can be multiple. Set τ_l to be the lth$(l = 1, ..., L)$ time shift parameter, where L is the number of τ_l. The correlation matrix $\tilde{\mathbf{R}}_\mathbf{y}(\tau_l)$ of $\mathbf{y}$, is given as follows.

$$\tilde{\mathbf{R}}_\mathbf{y}(\tau_l) = \mathbf{W}^H\tilde{\mathbf{R}}_\mathbf{z}(\tau_l)\mathbf{W} \tag{3}$$

In order to minimize the crosscorrelation of $y_j(k)$, by extended Jacobi rotation technique [1], it computes the separating matrix $\mathbf{W}$ by diagonalizing $\tilde{\mathbf{R}}_\mathbf{y}(\tau_l)(l = 1, ..., L)$ simultaneously.

$$\min_{\mathbf{W},\mathbf{W}^H\mathbf{W}=\mathbf{I}} \sum_{l=1}^{L} off(\mathbf{W}^H\tilde{\mathbf{R}}_\mathbf{z}(\tau_l)\mathbf{W}) \tag{4}$$

where $off(.)$ implies the squared sum of the off diagonal elements.

In general, there are two way of selecting time shift parameters.

(1) Select $\tau_l(> 0)$ in random manner [1]. The approach is not optimal in the sense of separation performance. However, it provides stable performance, as the number of parameters increase. The drawback is that the increasing parameters results in inevitably heavy computation.

(2) Select $\tau_l(> 0)$ where the autocorrelation of the source signals is large [3]. This approach is optimal in the sense of separation performance. However, it needs a priori information of the time structure of the source signals, namely, lacking blindness.

3. The proposed time shift parameter setting method

The proposed method takes advantage of the fact that the time shift providing large autocorrelation of source signals also attains large crosscorrelation of mixed signals. However, the problem is that autocorrelation of periodic signals has its peaks periodically. It is even worse for the crosscorrrelation of mixed signals because multiple source signals contributes to innumerable peaks. Selecting all of them as time shift parameters results in increasing of computing burden in TDSEP. Instead, we propose a method to select one single time shift parameter for each source signal. Figure 1 shows the flow of the proposed method, and we explain the method in steps.

1. Evaluate crosscorrelation index $J(\tau)$
   Define the crosscorrelation index $J(\tau)$ by the following equation.

$$J(\tau) = off(\tilde{\mathbf{R}}_{\mathbf{z}}(\tau)) = \sum_{j_1 \neq j_2} |\tilde{R}z_{j_1,j_2}(\tau)|^2 \tag{5}$$

2. Extract the time shift values at peaks
   Figure 3 shows a schematic picture of extracting the time shift values at peaks. First of all, in order for $J(\tau)$ to be called peak, the following must be satisfied.

$$J(\tau) > max(J(\tau - 1), J(\tau + 1)) \tag{6}$$

   Next, among the set of τ satisfying Equation(5), we extract the time shift values providing extraordinary peaks only, by threshold $th(\tau)$.

$$J(\tau) \geqq th(\tau) \tag{7}$$

   To compute $th(\tau)$ for a certain τ, consider mean and standard deviation of the noise level of $J(\tau)$ in neighbor of τ, denoted by $mean(J(\tau)|_{no})$,and $\sigma(J(\tau)|_{no})$. Using these terms, $th(\tau)$ is defined as follows.

$$th(\tau) = mean(J(\tau)|_{no}) + K\sigma(J(\tau)|_{no}) \tag{8}$$

   where K is a setting parameter. To compute $mean(J(\tau)|_{no})$ and $\sigma(J(\tau)|_{no})$,define N to be the number of samples. Furthermore, we define the guard samples called G, to avoid sidelobe of the peaks being included. Then, $mean(J(\tau)|_{no})$,and $\sigma(J(\tau)|_{no})$ are given by the following equations.

$$mean(J(\tau)|_{no}) = \frac{1}{2(N - G + 1)}\{\sum_{i=\tau-N}^{\tau-G-1} J(i) + \sum_{i=\tau+G+1}^{\tau+N} J(i)\} \tag{9}$$

$$\sigma(J(\tau)|_{no}) = \sqrt{\frac{1}{2(N - G + 1)}\{\sum_{i=\tau-N}^{\tau-G-1} (J(i) - mean(J(\tau)|_{no}))^2 + \sum_{i=\tau+G+1}^{\tau+N} (J(i) - mean(J(\tau)|_{no}))^2\}} \tag{10}$$

   For the refinement of $th(\tau)$, once $th(\tau)$ is calculated, we remove the τ with $J(\tau)$ larger than $th(\tau)$, from the set of τ used to calculate $th(\tau)$. Then recompute $th(\tau)$. This avoids $th(\tau)$ being too high by other peaks included in the set of τ used to calculate $th(\tau)$. Repeat it until it converges. This peak detection is conducted for all τ.

3. Period estimation
   Next, we estimate period of each source signals from the peaks extracted in Step 2. We use reminder based method. Set the first τ to be the period τ_1. Suppose a candidate of the period τ_c, and τ_l is the lth period already found. If the reminder of τ_c divided by τ_l is small enough, the τ_c is a part of periodic peak train of τ_l, and therefore abandon it. If the reminder is big enough for all τ_l, select τ_c as the $l + 1$th period τ_{l+1}. We continue this process until the last τ. In equation, the above reminder condition is given as follows.

$$\min\{mod(\tau_c, \tau_l), \tau_l - mod(\tau_c, \tau_l)\} > \epsilon,\ for\ all\ l \tag{11}$$

   where $mod(a, b)$ is a reminder of a divided by b, and $\epsilon(\geqq 0)$ is a parameter.

4. Validation of the selected τ_l
   In order to validate the τ_l obtained in Step 3, we utilize the value of $J(\tau)$ with period τ_l. Among the set of τ_l selected in Step 3, we select τ_l whose $J(\tau)$ with period of τ_l is large. To do that, first, compute M_l, which is sum of $J(\tau)$ over $\tau \in S_l$, where S_l is the set of τ with period of τ_l, and defined as follows.
$$S_l = \{\tau | min\{mod(\tau, \tau_l), \tau_l - mod(\tau, \tau_l)\} \leqq \epsilon\} \tag{12}$$
$$M_l = \frac{1}{N_{S_l}} \sum_{\tau \in S_l} J(\tau) \tag{13}$$
   where N_{S_l} is the number of elements in S_l. Then, select τ_l with the L largest M_l, where L is the necessary number of τ_l. Finally, if the number of τ_l is less than L, generate in random manner, and add to the parameter set.

4. A simulation example of the proposed method

4.1 Simulation Setting

Table1 and Figure2 show the simulation setting. We compare the proposed method with the ordinary one explained in Section2.-(a), namely, randomly selects the same number of time shift parameters from the range between 1 to 512(= $M/2$).

Next, we define the separation performance index. Let $\hat{\mathbf{A}}(m)$ to be the estimated mixing matrix at mth Monte-Carlo trial where $m = 1, ..., Monte$ and $Monte$ is the number of Monte Carlo trials, and define $\mathbf{G}(m) = \hat{\mathbf{A}}^{\dagger}(m)\mathbf{A}$. Then the separation performance index(SEP) is defined as follows.

$$SEP = \frac{\sum_{m=1}^{Monte} \sum_{i=1}^{J} max_j(|G_{i,j}(m)|^2)}{\sum_{m=1}^{Monte} \sum_{i=1}^{J} \{\sum_{j=1}^{J} |G_{i,j}(m)|^2 - max_j(|G_{i,j}(m)|^2)\}}, \ for\ i, j = 1, ..., J, \tag{14}$$

Equation(14) implies the ratio of average power of the desired and unnecessary signals components contained in separated signals, over the Monte-Carlo trial and separated signals.

4.2 Simulation result of the proposed method

Table 2 shows the SEP, Table 3 shows the estimation accuracy of τ_l over Monte Carlo trials, Figure 4 shows an example of $J(\tau)$ and selected τ_l of a certain Monte Carlo trial, and Figure 5 shows the estimated τ_l of all Monte Carlo trials. From Table 2, SEP of the proposed method is higher than the ordinary method by approximately 22dB. Also, Table 3 indicates that the estimation accuracy is 93%.

5. Conclusion

In this paper, we proposed a time shift parameter selecting method, based on crosscorrelation of the whitened received signals. The method has a merit of extracting only one single parameter for each source signal from many candidates. The simulation result showed that the proposed method increase the separation performance index by 22dB, compared with the ordinary method.

References

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[3] Shizuo Akiyama, "2-D DOA Estimation Method in Closely Spaced Coherent Multipath Environment : Geometric Analysis of the Superimposed Modevector ," The transactions of IEICE B, J92-B(3), pp.555-566, 2009(in Japanese).

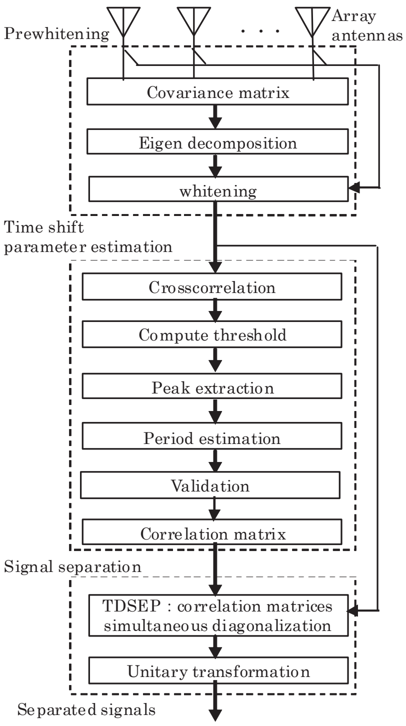

Figure 1: Flow of the proposed time shift parameter selection method.

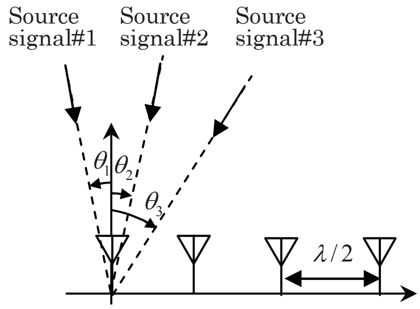

Figure 2: Sensors and source signals geometry

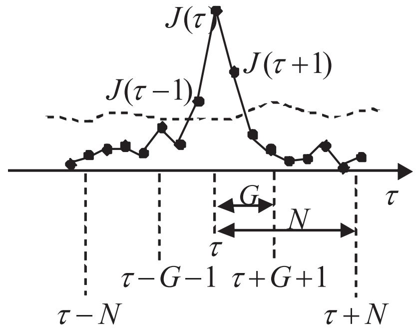

Figure 3: Schematic picture of the peak detection by threshold.

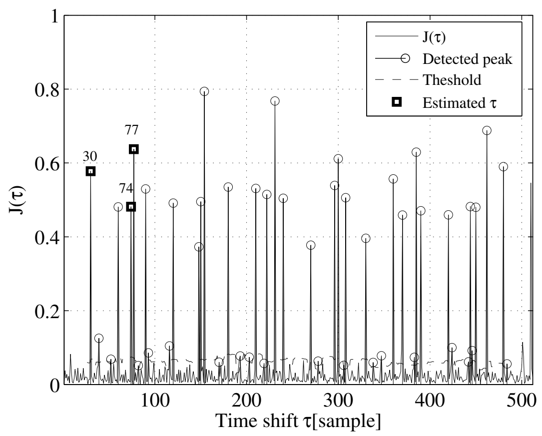

Figure 4: An example of $J(\tau)$ and selected τ_l.

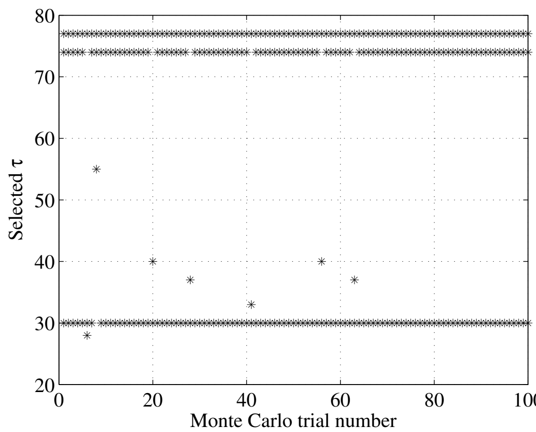

Figure 5: Selected τ vs. Monte Carlo trial.

Table 1: Simulation parameter setting

Parameter	Value
Period of signal# 1,# 2,# 3	Gaussian, period of 30,74,77 samples
DOA of signal# 1,# 2,# 3	-10deg,+10deg,+30deg
S/N # 1,# 2,# 3	10dB,10dB,10dB
Antenna configuration	4 elements linear array
Aperture length	Half of wave length
Number of samples M	1024 samples
Mod parameter ϵ	0.1
The number of guard samples G	3 samples
The number of samples to compute the threshold N	40 samples
Coefficient on threshold deviation K	3
Number of setting parameter L	3
The number of Monte Carlo trials	100 trials

Table 2: SEP of the proposed and ordinary method.

	Ordinary	The proposed
Separation Performance Index	11.75dB	33.7dB

Table 3: Estimation accuracy of τ.

Signal No.	Overall	Signal#1	Signal#2	Signal#3
Percentage	93%	99%	94%	100%