OPTIMUM PARAMETERS OF A TWO DIMENSIONAL SMITH-PURCELL RADIATING SYSTEM

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1. INTRODUCTION

There has been recently considerable theoretical and experimental interest in the application of relativistic electron beam to new electromagnetic wave sources [1],[2]. One of the important research areas is a high-power microwave radiation by a relativistic electron beam using slow wave effects in open periodic structures, which is referred generally to as the Smith-Purcell radiation. Two types of emission mechanism, i.e., a spontaneously emitted radiation [3],[4] and a stimulated emission [5], can be the basis of the coherent radiation source. From the wave-theoretical viewpoint [6], the spontaneous emission can be interpreted as a radiative leakage of slow space-charge waves and the stimulated emission can be as an instabilities of electromagnetic surface waves coupled to the electron beam. Then the analysis of spontaneous emission is reduced to determine the complex eigen wavenumber of the space-charge waves under a specified periodic structure, whereas for the case of stimulated emission, the analysis is reduced to determine the complex eigen wave-frequency of the unstable electromagnetic surface waves.

In this paper, we shall investigate a two-dimensional Smith-Purcell radiation by a relativistic sheet electron beam propagating parallel to a reflection grating. A self-consistent analysis of the eigen wavenumber and eigen wave-frequency of the radiation is developed using the mode-matching method [7]. For the case of sinusoidal grating, the precise numerical results are presented for the leakage coefficient of spontaneous emission and for the growth rate of stimulated emission. Their maximum values that depend on the grating dimensions and the beam configuration are discussed at length. Based on the results, some of the optimum parameters to achieve an efficient radiating system are brought out.

2. FORMULATION OF THE PROBLEM

The geometry considered here is shown in Fig. 1. A sheet electron beam having a characteristic thickness $2b$ about the midplane $x = a$ is located in free space above a reflection grating. The electron beam propagates at a constant velocity v_0 in the z direction along an externally applied magnetic field. The grating consists of a perfectly conducting surface that is uniform in the y direction and is corrugated in the z direction according to a periodic function $x = f(z) = f(z + D)$, where D is the period of the corrugation. For the sake of simplicity, we assume that (a) the externally applied magnetic field is so strong that electron motions are restricted only in the z direction, (b) the effects of beam self-electric and self-magnetic fields can be neglected, and (c) perturbations are uniform in the y direction. We describe this system using the Maxwell equations and relativistic hydrodynamic equations

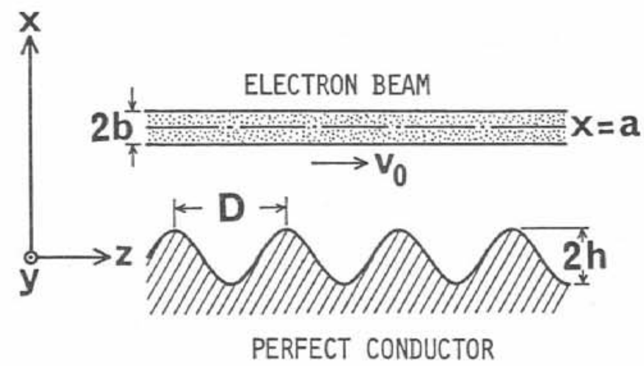

Fig. 1. Geometry of the problem.

for cold electrons. Assuming the time dependence in the form $exp(-i\omega t)$ and denoting the H_y field as $\Psi_\ell(x,z)$, the two dimensional TM fields coupled to the electron beam are governed by the following equation:

$$[\frac{\partial^2}{\partial x^2} + \chi_\ell(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2})]\,\Psi_\ell(x,z) = 0 \quad (\ell = 1,\ 2,\ 3) \tag{1}$$

$$E_{\ell,z} = \frac{i}{\omega\varepsilon_0\chi_\ell}\frac{\partial}{\partial x}\Psi_\ell(x,z), \quad E_{\ell,x} = -\frac{i}{\omega\varepsilon_0}\frac{\partial}{\partial z}\Psi_\ell(x,z) \tag{2}$$

with

$$\chi_2 = 1 - \frac{\omega_p^2}{\gamma^2(\omega + iv_0\partial/\partial z)^2}, \quad \chi_1 = \chi_3 = 1, \tag{3}$$

where $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v_0/c$, c is the velocity of light in free space, ω_p is the relativistic plasma frequency of the electrons, and the subscript ℓ indicates the three regions; region 1 $(a+b \leq x)$, region 2 $(|x-a| < b)$, and region 3 $(f(z) \leq x \leq a-b)$. The H_y and E_z fields must be continuous across the boundary surfaces $x = a \pm b$ and the tangential electric field must vanish on the grating surface $x = f(z)$.

3. NUMERICAL ALGORITHM

We assume that the TM fields vary in the form $exp(ikz)$ where k is a propagation constant, and introduce a set of modal functions $\phi_{\ell,n}(x,z)$ $(\ell = 1,2,3;\ n = 0,\ \pm1,\ \pm2,\cdots)$ which satisfy the wave equations (1), the periodic condition, and the radiation condition. These functions are given by the plane wave solutions [6] to Eqs.(1) having the z dependence as $exp[i(k+2n\pi/D)z]$. Referring to the mode-matching method[7], we approximate $\Psi_\ell(x,z)$ by the following finite series of the modal functions:

$$\Psi_{\ell,N}(x,z) = \sum_{n=-N}^{N} a_{\ell,n}(N)\,\phi_{\ell,n}(x,z) \tag{4}$$

where $\{a_{\ell,n}(N)\}$ are unknown modal amplitudes that depend on the truncation number N. Applying the boundary conditions at $x = a \pm b$ to the approximate fields $\Psi_{\ell,N}(x,z)$ $(\ell = 1,2,3)$, we have the relations to express $\{a_{1,n}(N)\}$ and $\{a_{2,n}(N)\}$ in terms of $\{a_{3,n}(N)\}$. The remaining boundary condition on the grating surface for the approximate field $\Psi_{3,n}(x,z)$ is matched in the sense of least squares. For this purpose, we define the boundary residuals Ω_N on $x = f(z)$ as follows:

$$\Omega_N = \int_0^D |\frac{\partial}{\partial\nu}\tilde{\Psi}_{3,N}(z)|^2 dz \tag{5}$$

with

$$\frac{\partial}{\partial\nu}\tilde{\Psi}_{3,N}(z) = exp(-ikz)\,\frac{\partial}{\partial\nu}\Psi_{3,N}(x,z)\,|_{x=f(z)}, \tag{6}$$

where $\partial/\partial\nu$ means the normal derivative on $x = f(z)$. The value of Ω_N should be minimized with respect to $\{a_{3,n}(N)\}$ under the constraint $a_{3,0}(N) = 1$. Therefore, introducing a Lagrange's multiplier λ_N, we consider the quantity:

$$\bar{\Omega}_N = \Omega_N + \lambda_N^*[a_{3,0}(N) - 1] + \lambda_N[a_{3,0}(N) - 1]^* \tag{7}$$

where the asterisk denotes the complex conjugate. Letting the first derivative of $\bar{\Omega}_N$ with respect to $\{a_{3,n}^*(N)\}$ and λ_N^* be zero, we have a set of simultaneous linear equations of $2N+2$ unknowns for $\{a_{3,n}(N)\}$ and λ_N. The solutions are obtained by solving numerically the linear equations for properly fixed N, ω, and k. Letting $\Omega_{N,min}$ be the minimum value of Ω_N for such solutions, we have the relation; $\lambda_N = -\Omega_{N,min}$. Then if we calculate λ_N as a function of k for the fixed N and ω, the approximate propagation constant $k_N(\omega)$ can be obtained by the value of k which minimizes $-\lambda_N$. These steps can be readily carried out by numerical computations using the well-known Newton method. If we interchange the role of k and ω, we can calculate the frequency ω as a function of the propagation constant k.

4. NUMERICAL RESULTS AND DISCUSSIONS

For the case of sinusoidal grating given by $f(z) = h\cos(2\pi z/D)$, we have carried out numerical computations on the dispersion characteristics of the lowest eigenmodes supported by the two-dimensional structure as shown in Fig. 1. For convenience in computations, we have introduced a dimensionless parameter $\alpha = (\omega_p D/c)^2$ signifying the electron-beam strength. The truncation size of the modal expansion was chosen as $2N+1 = 15$ after confirming the convergence of the numerical solutions.

The spontaneous emission takes place outside the radiation triangle in the Brillouin diagram. The leakage coefficient of the emission is given by the imaginary part k_i of the complex wavenumber k. The sign of k_i is negative and $|k_i|$ has a maximum at some real wave-frequency ω just outside the radiation triangle. Figures 2 (a)-(c) show the maximum leakage coefficient k_i , the corresponding real part k_r of the wavenumber, and the wave-frequency ω giving the maximum k_i as a function of (a) the electron velocity $\beta = v_0/c$, (b) the grating amplitude h/D, and (c) the electron-beam strength parameter α.

The stimulated emission takes place inside the radiation triangle. The growth rate of the emission is given by the imaginary part ω_i of the complex eigen wave-frequency ω and has a maximum at some real wavenumber k just inside the radiation triangle. Figures 3 (a)-(c) show the maximum growth rate ω_i, the corresponding wave-frequency ω_r, and the real wavenumber k giving the maximum ω_i as a function of (a) the electron velocity β , (b) the grating amplitude h/D, and (c) the electron-beam strength parameter α.

Figures 2 and 3 are instructive to know what amount of the leakage coefficient and the growth rate can be attainable for a given grating structure coupled to the electron beam. It is of particular importance to note that both of the maximum leakage coefficient and the maximum growth rate are much sensitive to the electron velocity as well as to the grating amplitude.

ACKNOWLEDGMENT: This work was supported in part by a scientific Research grant-in-aid from the Ministry of Education, Science, and Culture, Japan.

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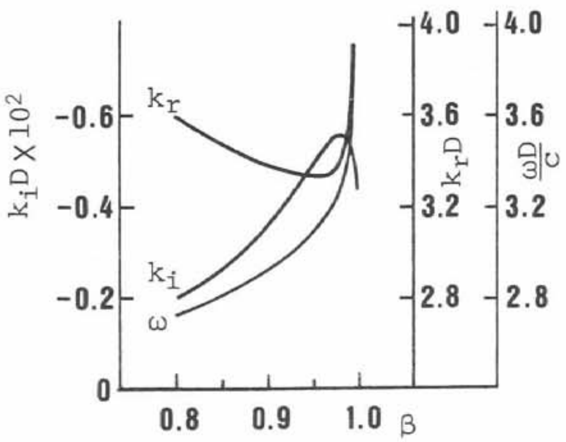
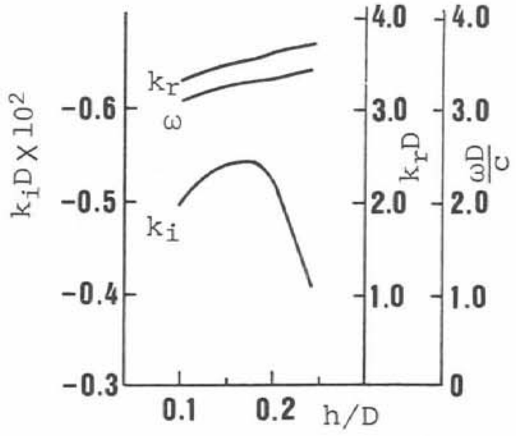
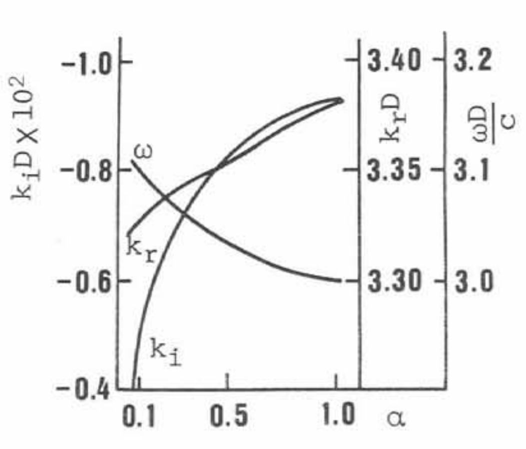


(a) h/D = 0.1 and α = 0.1

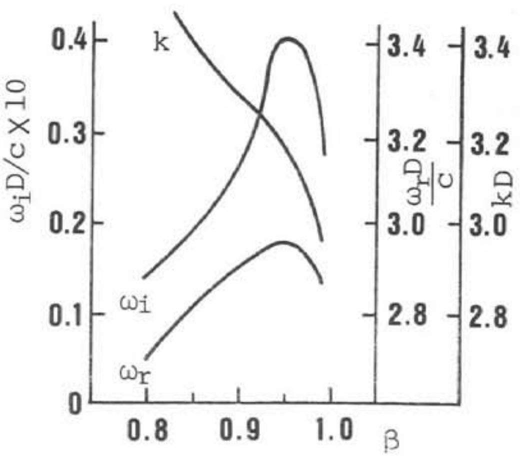


(b) β = 0.95 and α = 0.1

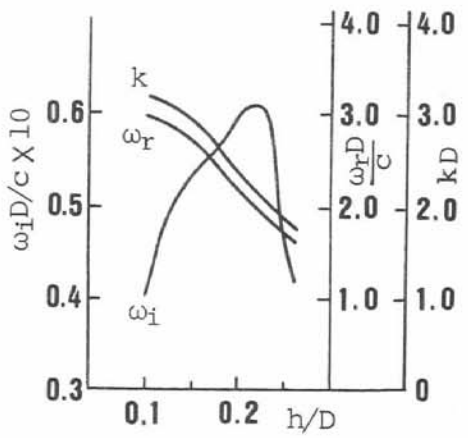


(c) h/D = 0.1 and β = 0.95

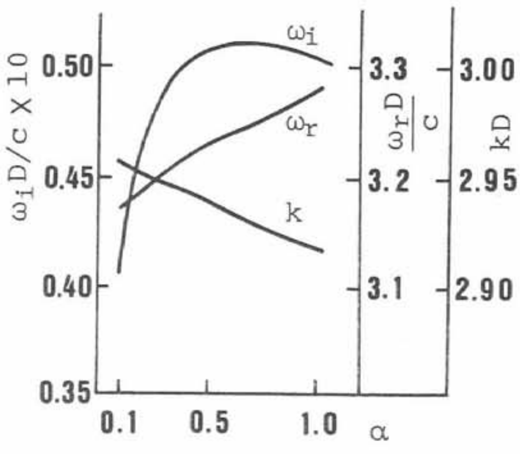
Fig. 2. The maximum leakage coefficient k_i, the corresponding real part k_r of the wavenumber, and the wave frequency ω giving the maximum k_i as a function of (a) the electron velocity β, (b) the grating amplitude h/D, and (c) the electron-beam strength parameter α for a/D = 1.0 and b/D = 0.5.



(a) h/D = 0.1 and α = 0.1



(b) β = 0.95 and α = 0.1



(c) h/D = 0.1 and β = 0.95

Fig. 3. The maximum growth rate ω_i, the corresponding wave-frequency ω_r, and the real wavenumber k giving the maximum ω_i as a function of (a) the electron velocity β, (b) the grating amplitude h/D, and (c) the electron-beam strength parameter α for a/D = 1.0 and b/D = 0.5.