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APPLICATION OF THE RADON TRANSFORM THEORY TO ELECTROMAGNETIC INVERSE SCATTERING

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The determination of a target's shape and size from its electromagnetic ramp response signature can be reduced to the geometrical problem of determining a three-dimensional body from its cross-sectional areas. In this paper, the theory of Radon transforms is used to discuss the solution to this problem and to demonstrate that certain aspects of this problem are equivalent to the problem of image reconstruction from projections, which have been solved in various other disciplines. Two of the data inversion algorithms developed to solve the latter problem have been used to determine radar target shapes from their ramp responses.

It is known [1,2] that the cross-sectional area of a target as a function of the distance along the line of sight can be estimated from its backscattered electromagnetic ramp response, which in turn can be approximately synthesized by using a 10:1 frequency bandwidth in the target's low resonance range. Thus the determination of the target shape and size using the ramp response signature is reduced to the geometrical problem of reconstructing a body from its cross-sectional areas. This problem is most naturally tackled by employing the theory of Radon transforms [3,4], because it is known that for the characteristic function, γ of a three-dimensional body, the values of its Radon transform are given by the appropriate cross-sectional areas of the body. Well-known results in the theory of Radon transforms are used to obtain the following results for a three-dimensional body, some of which have been reported previously [5,6].

(1) Let us assume that the cross-sectional areas $A(\eta,\psi,p)$ of a three-dimensional body (Fig.1a) are known for all directions of viewing (η,ψ) and for all p, the distances from the origin. It is shown that the characteristic function of the body $\gamma(r,\theta,\phi)$ is related to the areas by

$$\gamma(r,\theta,\phi) = -\frac{1}{8\pi^2}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \left.\frac{\partial^2 A(\eta,\psi,p)}{\partial p^2}\right|_{p=r[\cos\theta\cos\eta\cos(\phi-\psi)+\sin\theta\sin\eta]} \cos\eta\, d\eta\, d\psi \qquad (1)$$

where the notations are explained in Fig. 1.

(2) If only $A(\psi,p)$, the areas corresponding to aspects (ψ) lying on the equatorial plane $(\eta=0)$ are known, then the width function $W(x_1,x_2)$ normal to the $x_3=0$ plane is related to $A(\psi,0)$ as follows

$$A(\psi,p) = \int_{-\infty}^{\infty}\int W(x_1,x_2)\,\delta[p-\{x_1\cos\psi + x_2\sin\psi\}]\,dx_1\,dx_2 \qquad (2)$$

where δ denotes the dirac delta function.
The solution to (2) is

$$W(r_0,\phi) = S_1 - S_2 = -\frac{1}{2\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty}\frac{\partial A(\psi,p)}{\partial p}\;\frac{dp\,d\psi}{p-r_0\cos(\psi-\phi)} \qquad (3)$$

where $x_1 = r_0\cos\phi$, $x_2 = r_0\sin\phi$. $x_3 = S_1(r_0,\phi)$ and $x_3 = S_2(r_0,\phi)$ are respectively the upper and lower surfaces of the body as illustrated in Fig. 1b.

(3) If in addition to $A(\psi,p)$, also the derivatives $\frac{\partial A}{\partial \eta}$ are known at the $\eta=0$ plane, then

$$\beta(r_0,\phi) = \frac{S_1^2-S_2^2}{2} = \frac{1}{2\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty}\left.\frac{\partial A(\eta,\psi,p)}{\partial \eta}\right|_{\eta=0}\frac{dp\,d\psi}{p-r_0\cos(\psi-\phi)} \qquad (4)$$

from (3) and (4), one can determine the body through

$$x_3 = S_1 = \frac{W}{2}+\frac{\beta}{W} \quad \text{and} \quad x_3 = S_2 = -\frac{W}{2}+\frac{\beta}{W} \qquad (5)$$

where W and β are given by (3) and (4) respectively.

Equations (1) to (5) represent formal analytical solution to the problem of determining different features of a target from its cross-sectional areas. However, the emphasis here is put on equation (2), because it demonstrates the equivalence of the target identification problem to the problem of image reconstruction from a finite number of projections [7,8], which have been solved in many different fields ranging from computerized tomography to radio astronomy. Two of the reconstruction algorithms (e.g. the convolution [9] and the simultaneous iterative reconstruction technique [10] have been applied to solve for W in Eq.(2) using cross-sectional areas, which are either geometrically calculated or derived from synthesized ramp responses. Numerical results for a sphere and a spheroid are discussed [11]. In [11] and [12] it is shown how the transient ramp response method [1,2] is related to the physical optics inverse transform method [13], which was derived from Bojarski's identity [14], via the Radon transform approach of [6].

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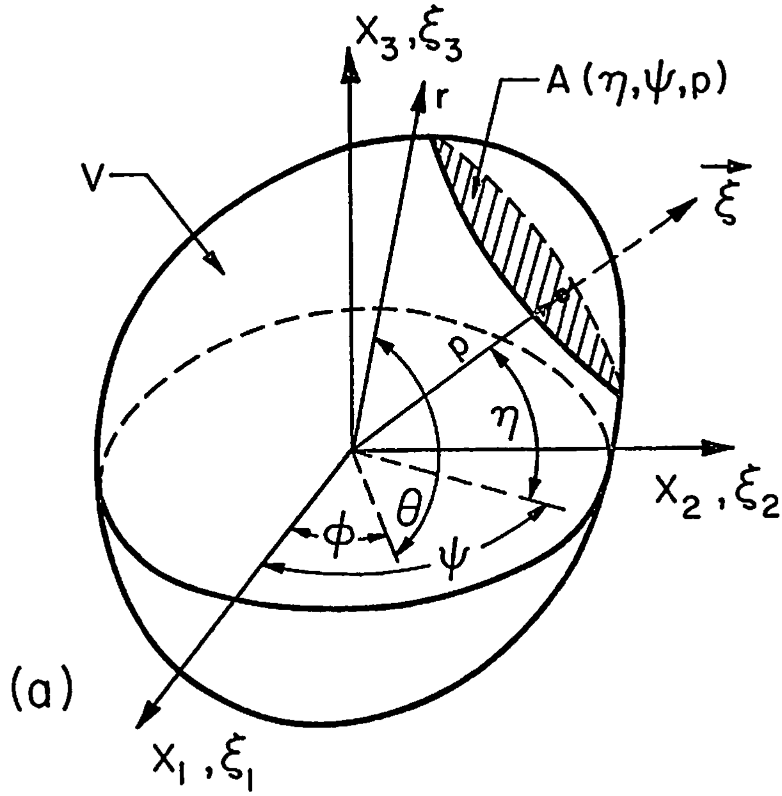

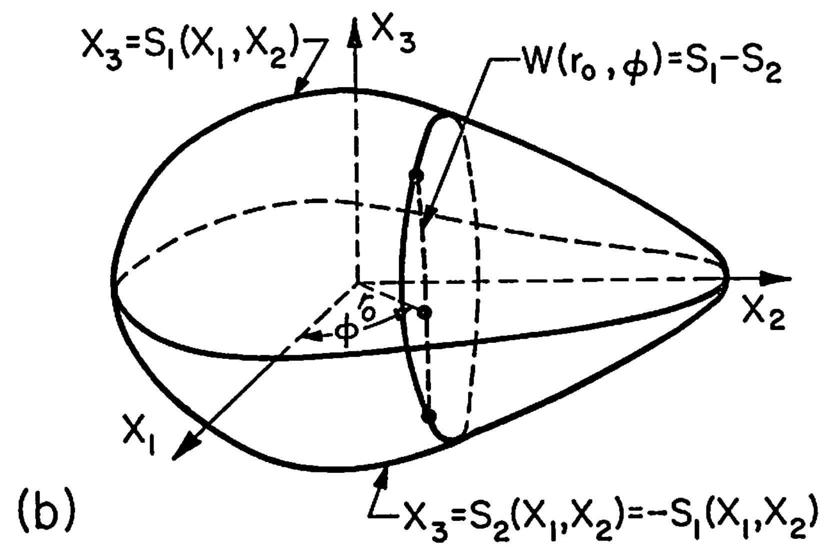

Fig.1 Geometry of the Problem