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ON EXTRAPOLATED ABSORBING BOUNDARY CONDITION FOR FVTD METHOD

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Abstract: In this paper we propose an extrapolated absorbing boundary conditions (EABCs) for solving electromagnetic problems based on finite volume time domain (FVTD) method. It is sown that the present method exhibits a much better accuracy than the Mur's ABC and it requires much smaller computer memories than the perfectly matched layers (PMLs). Numerical examples are given for field distributions in case of 1D, 2D and 3D boundary value problems including a semi-infinite dielectric and magnetic material.
Key words: ABC, Extrapolation, FVTD, FDTD.

1. Introduction

With the rapid development of high speed and large memory computers, the finite difference time domain (FDTD) method have been widely used, since Yee [1] first proposed this numerical method. Because of the finite memory sizes of computers, we have to realize virtual computational spaces by introducing absorbing boundary conditions (ABCs). In the earlier times, Mur's ABCs [2] were used by many researchers, but fictitious reflections were always observed. It is well known that PMLs were successfully introduced by Berenger [3] to overcome this difficult situations.

ABCs based on PMLs are excellent, but a lot of computer memories are needed to implement them on computers. In this paper, we propose the EABCs in a very compact form. It is demonstrated that the accuracy of the EABCs is much better than the Mur's ABC, and it requires much less memories than the PMLs.

2. FVTD equations.

For the computational reason, we normalize the magnetic field by the intrinsic impedance as $\tilde{\boldsymbol{H}} = Z_0\boldsymbol{H} = \sqrt{\mu_0/\epsilon_0}\boldsymbol{H}$. In the time domain, we discretize the electric fields at $t = n\Delta t$ and magnetic fields at $t = n'\Delta t = (n-1/2)\Delta t$ as Yee proposed [1] , and in the space domain we discretize electromagnetic fields at $(x, y, z) = (i\Delta x, j\Delta y, k\Delta z)$. Then we have the following FVTD equations [4]:

$$
\begin{aligned}
&\tilde{H}_x^{n'+1}(i,j,k) = \tilde{H}_x^{n'}(i,j,k) \\
&\qquad + \Lambda_z[E_y^n(i,j,k+1) - E_y^n(i,j,k-1)] - \Lambda_y[E_z^n(i,j+1,k) - E_z^n(i,j-1,k)] \\
&\tilde{H}_y^{n'+1}(i,j,k) = \tilde{H}_y^{n'}(i,j,k) \\
&\qquad + \Lambda_x[E_z^n(i+1,j,k) - E_z^n(i-1,j,k)] - \Lambda_z[E_x^n(i,j,k+1) - E_x^n(i,j,k-1)] \\
&\tilde{H}_z^{n'+1}(i,j,k) = \tilde{H}_z^{n'}(i,j,k) \\
&\qquad + \Lambda_y[E_x^n(i,j+1,k) - E_x^n(i,j+1,k)] - \Lambda_x[E_y^n(i+1,j,k) - E_y^n(i-1,j,k)]
\end{aligned}
\tag{1}
$$

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$$
\begin{aligned}
E_x^{n+1}(i,j,k) &= E_x^n(i,j,k) \\
&- \Gamma_z[\tilde{H}_y^{n'+1}(i,j,k+1) - \tilde{H}_y^{n'+1}(i,j,k-1)] + \Gamma_y[\tilde{H}_z^{n'+1}(i,j+1,k) - \tilde{H}_z^{n'+1}(i,j-1,k)] \\
E_y^{n+1}(i,j,k) &= E_y^n(i,j,k) \\
&- \Gamma_x[\tilde{H}_z^{n'+1}(i+1,j,k) - \tilde{H}_z^{n'+1}(i-1,j,k)] + \Gamma_z[\tilde{H}_x^{n'+1}(i,j,k+1) - \tilde{H}_x^{n'+1}(i,j,k-1)] \\
E_z^{n+1}(i,j,k) &= E_z^n(i,j,k) \\
&- \Gamma_y[\tilde{H}_x^{n'+1}(i,j+1,k) - \tilde{H}_x^{n'+1}(i,j+1,k)] + \Gamma_x[\tilde{H}_y^{n'+1}(i+1,j,k) - \tilde{H}_y^{n'+1}(i-1,j,k)]
\end{aligned}
\tag{2}
$$

where the step parameters are defined by

$$\Lambda_{x,y,z} = (c\Delta t)/(2\mu_r \Delta_{x,y,z}) \quad \Gamma_{x,y,z} = (c\Delta t)/(2\epsilon_r \Delta_{x,y,z}) \quad . \tag{3}$$

It is worth noting that FVTD is useful for treating electromagnetic problems with inhomogeneous dielectric and magnetic materials.

3. EABC in case of 1D.

We consider the plane wave $\tilde{\boldsymbol{H}} = (0, \tilde{H}_y, 0)$ and $\boldsymbol{E} = (0,0,E_z)$ traveling in x-direction. The FVTD equations are given by

$$
\begin{aligned}
\tilde{H}_y^{n'+1}(i) &= \tilde{H}_y^{n'}(i) + \Lambda_x[E_z^n(i+1) - E_z^n(i-1)] \\
E_z^{n+1}(i) &= E_z^n(i) + \Gamma_x[\tilde{H}_y^{n'+1}(i+1) - \tilde{H}_y^{n'+1}(i-1)] \quad .
\end{aligned}
\tag{4}
$$

We assume that there exists an absorbing boundary at $i = N_x$ which absorbs the plane wave traveling in the positive x-direction. To realize this assumption we extrapolate the value of magnetic field at $i = N_x$ by taking into account the time and space intervals together with the wave velocity in the a dielectric and magnetic material as follows:

$$
\begin{aligned}
\text{Type1}: \ \tilde{H}_y^{n'+1}(N_x) &= -W_1 Y E_z^n(N_x - 1) + W_2 Y E_z^n(N_x - 3) \\
\text{Type2}: \ \tilde{H}_y^{n'+1}(N_x) &= -W_3 Y E_z^n(N_x - 1) - W_4 \tilde{H}_y^{n'+1}(N_x - 2)
\end{aligned}
\tag{5}
$$

where $Y = \sqrt{\epsilon_r/\mu_r}$ and the weights are given by

$$
\begin{aligned}
W_1 &= (6\sqrt{\epsilon_r\mu_r} - 1)/(4\sqrt{\epsilon_r\mu_r}) \quad W_2 = (2\sqrt{\epsilon_r\mu_r} - 1)/(4\sqrt{\epsilon_r\mu_r}) \\
W_3 &= (4\sqrt{\epsilon_r\mu_r})/(2\sqrt{\epsilon_r\mu_r} + 1) \quad W_4 = (2\sqrt{\epsilon_r\mu_r} - 1)/(2\sqrt{\epsilon_r\mu_r} + 1) \ .
\end{aligned}
\tag{6}
$$

It is evident that the above weights become in the free space as $W_1 = 1.25$, $W_2 = 0.25$, $W_3 = 1.5$ and $W_4 = 0.5$. The above 1D relations are the essence of the present algorithm and modifications to 2D and 3D are discussed in the subsequent sections.

Fig.1 shows reflection errors caused by ABCs with the accuracy in dB such as PML(10) < Type 1< PML(9) < Type 2. Thus EABC of Type 2 is better than that of Type 1 or better than even that of PML(9). Fig.2 well simulates that both the reflection and transmission coefficients are 0.5. Fig.3 shows impedance matching at the semi-infinite dielectric and magnetic material, and almost no fictitious reflections are observed.

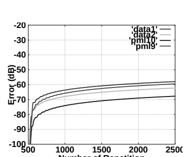

Fig.1 Reflection (dB).

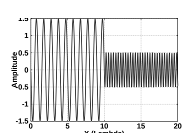

Fig.2 Electric field (ϵ_r=9.0 μ_r=1.0).

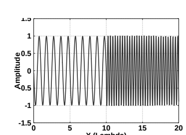

Fig.3 Electric field (ϵ_r=3.0 μ_r=3.0).

4. EABC in case of 2D.

For the TM-wave with $\tilde{\boldsymbol{H}} = (\tilde{H}_x, \tilde{H}_y, 0)$ and $\boldsymbol{E} = (0, 0, E_z)$, the FVTD equations for electromagnetic fields are derived from Eqs.(1) and (2) as follows:

$$
\begin{aligned}
\tilde{H}_x^{n'+1}(i,j) &= \tilde{H}_x^{n'}(i,j) - \Lambda_y[E_z^n(i,j+1) - E_z^n(i,j-1)] \\
\tilde{H}_y^{n'+1}(i,j) &= \tilde{H}_y^{n'}(i,j) + \Lambda_x[E_z^n(i+1,j) - E_z^n(i-1,j)] \\
E_z^{n+1}(i,j) &= E_z^n(i,j) + \Gamma_x[\tilde{H}_y^{n'+1}(i+1,j) - \tilde{H}_y^{n'+1}(i-1,j)] \\
&\qquad - \Gamma_y[\tilde{H}_x^{n'+1}(i,j+1) - \tilde{H}_x^{n'+1}(i,j+1)] \ .
\end{aligned} \tag{7}
$$

Now we split the electric field as $E_z = E_{zx} + E_{zy}$ [3], then, analogous to 1D case, we can extrapolate the magnetic fields in the boundary cells $i = N_x$ or $j = N_y$ in terms of the electric fields in the inner cells as follows:

$$
\begin{aligned}
\tilde{H}_y^{n'+1}(N_x,j) &= -W_1 Y E_{zx}^n(N_x-1,j) + W_2 Y E_{zx}^n(N_x-3,j) \\
\tilde{H}_x^{n'+1}(i,N_y) &= +W_1 Y E_{zy}^n(i,N_y-1) - W_2 Y E_{zy}^n(i,N_y-3)
\end{aligned} \tag{8}
$$

EABCs of Type2 are not shown here for simplicity.

Next we consider the TE-wave with $\tilde{\boldsymbol{H}} = (0, 0, \tilde{H}_z)$ and $\boldsymbol{E} = (E_x, E_y, 0)$. The FVTD equation for the electromagnetic fields are derived from Eqs.(1) and (2) as follows:

$$
\begin{aligned}
\tilde{H}_z^{n'+1}(i,j) &= \tilde{H}_z^{n'}(i,j) + \Lambda_y[E_x^n(i,j+1) - E_x^n(i,j+1)] \\
&\qquad - \Lambda_x[E_y^n(i+1,j) - E_y^n(i-1,j)] \\
E_x^{n+1}(i,j) &= E_x^n(i,j) + \Gamma_y[\tilde{H}_z^{n'+1}(i,j+1) - \tilde{H}_z^{n'+1}(i,j-1)] \\
E_y^{n+1}(i,j) &= E_y^n(i,j) - \Gamma_x[\tilde{H}_z^{n'+1}(i+1,j) - \tilde{H}_z^{n'+1}(i-1,j)] \ .
\end{aligned} \tag{9}
$$

Now we split the electric field as $H_z = H_{zx} + H_{zy}$ [3], then the EABCs in case of TE-wave are summarized as follows:

$$
\begin{aligned}
\tilde{H}_{zx}^{n'+1}(N_x,j) &= +W_1 Y E_y^n(N_x-1,j) - W_2 Y E_y^n(N_x-3,j) \\
\tilde{H}_{zy}^{n'+1}(i,N_y) &= -W_1 Y E_x^n(i,N_y-1) + W_2 Y E_x^n(i,N_y-3) \quad .
\end{aligned} \tag{10}
$$

Expressions for Type 2 are also omitted here.

Figs.4 and 5 show numerical examples for radiation from an electric or magnetic line source near a semi-infinite dielectric ($\epsilon_r = 4.0, \mu_r = 1.0$).

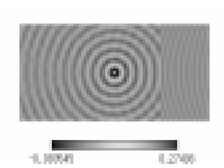

Fig.4 TM-wave. ($\Delta_{x,y} = \lambda/20$).

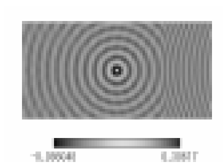

Fig.5 TE-wave. ($\Delta_{x,y} = \lambda/20$).

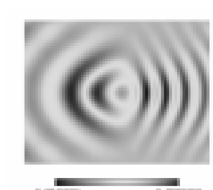

Fig.6 Electric field in case of 3D. ($\Delta_{x,y} = \lambda/20$).

5. ABC in case of 3D.

In case of 3D, we split the electromagnetic fields into two parts in the cell layers just inside the absorbing boundaries as follows:

$$
\begin{aligned}
&E_x = E_{xy} + E_{xz} \quad E_y = E_{yz} + E_{yx} \quad E_z = E_{zx} + E_{zy} \\
&H_x = H_{xy} + H_{xz} \quad H_y = H_{yz} + H_{yx} \quad H_z = H_{zx} + H_{zy} \ .
\end{aligned} \tag{11}
$$

Now we assume that the absorbing boundaries are at $i = N_x$, $j = N_y$ or $k = N_z$, then we can extrapolate the magnetic fields at the boundaries just outside the computational regions as follows:

$$\begin{aligned}\tilde{H}_{zx}^{n'+1}(N_x, j, k) &= +W_1 Y E_{yx}^{n}(N_x - 1, j, k) - W_2 Y E_{yx}^{n}(N_x - 3, j, k)\\ \tilde{H}_{yx}^{n'+1}(N_x, j, k) &= -W_1 Y E_{zx}^{n}(N_x - 1, j, k) + W_2 Y E_{zx}^{n}(N_x - 3, j, k)\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{H}_{xy}^{n'+1}(i, N_y, k) &= +W_1 Y E_{zy}^{n}(i, N_y - 1, k) - W_2 Y E_{zy}^{n}(i, N_y - 3, k)\\ \tilde{H}_{zy}^{n'+1}(i, N_y, k) &= -W_1 Y E_{xy}^{n}(i, N_y - 1, k) + W_2 Y E_{xy}^{n}(i, N_y - 3, k)\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{H}_{yz}^{n'+1}(i, j, N_z) &= +W_1 Y E_{xz}^{n}(i, j, N_z - 1) - W_2 Y E_{xz}^{n}(i, j, N_x - 3)\\ \tilde{H}_{xz}^{n'+1}(i, j, N_z) &= -W_1 Y E_{yz}^{n}(i, j, N_z - 1) + W_2 Y E_{yz}^{n}(i, j, N_z - 3)\ .\end{aligned} \tag{14}$$

Expressions for Type 2 are also omitted here.

Fig.6 shows radiation from a small dipole antenna above a semi-infinite dielectric ($\epsilon_r = 4.0, \mu_r = 1.0$). Almost no fictitious reflections are observed, and thus, we can deal with a lot of electromagnetic problems with inhomogeneous dielectric and magnetic materials by applying the FVTD method combined with the proposed EABCs.

6. Conclusion.

In this paper, we have proposed the EABCs for FVTD method by extrapolating the magnetic fields just outside the computational region in terms of the electromagnetic fields inside it. It should be noted that the extrapolation of the electric fields is also possible and that the present method can also be applied to FDTD in a similar fashion. Numerical calculations were carried out for field distributions in case of 1D, 2D and 3D problems including a semi-infinite dielectric and magnetic material. The present method exhibits better accuracy than the Mur's ABCs and it requires smaller computer memories than the PMLs.

The proposed EABCs can be extended to lossy materials. It deserves as a future problem together with the application of the present method to various electromagnetic boundary value problems.

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