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PROPAGATION CHARACTERISTICS IN THE WAVEGUIDE LOADED WITH A CYLINDRICAL ROD OF MAGNETIZED FERRITE

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1. Introduction

When a cylindrical ferrite rod magnetized axially in static magnetic field is inserted parallel to the E-plane of a rectangular waveguide where TE_{10} mode is propagating (see Fig.1), we exactly solve the electromagnetic fields at the in- and outsides of ferrite rod and also obtain the transmission and reflection coefficients[1].

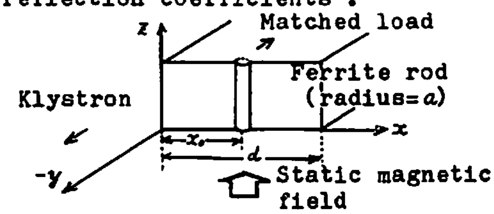

Fig.1 Waveguide inserted with a ferrite rod

Though this problem was solved by Epstein and Berk[2], their solution is an approximate solution that is available only when the radius of the rod is enough small.

2. Theory

The electromagnetic field in the waveguide is given as the sum of an incident wave and scattered wave. The field in the waveguide is decided by the boundary conditions at the surface of ferrite rod.

The incident electric field E_z^i is expressed as follows:

$$E_z^i = E_0 \sin\frac{\pi x}{d}\, e^{-j\sqrt{k_0^2-(\pi/d)^2}\,y} \qquad (1)$$

Expanding this expression by using the polar coordinates, we obtain

$$E_z^i = E_0 \sum_{m=-\infty}^{\infty} S_m J_m(k_0 r_0) e^{-jm\theta_0} \qquad (2)$$

,where $S_m = \sin\left(\frac{\pi}{d}x_0 + m\tan^{-1}\frac{\pi}{d\sqrt{k_0^2-(\pi/d)^2}}\right)$

From Helmholz equation, the scattered electric field can be expressed as follows.(see Fig.2)

$$E_z^s = E_0 \sum_{m=-\infty}^{\infty} A_m \sum_{l=-\infty}^{\infty} \{ H_m^{(2)}(k_0 r_{2l}) e^{-jm\theta_{2l}} - (-1)^m H_m^{(2)}(k_0 r_{2l-1}) e^{jm\theta_{2l-1}} \} \qquad (3)$$

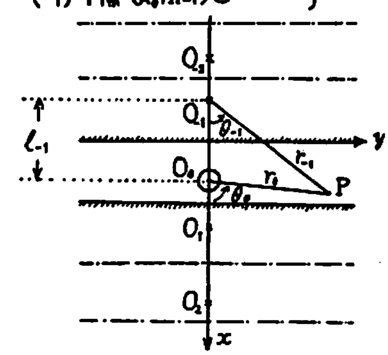

Fig.2 Image points

Using the addition theorem for Bessel function, Eq.(3) becomes

$$E_z^s = E_0 \sum_{n=-\infty}^{\infty} J_n(k_0 r_0) e^{-jn\theta_0} \sum_{m=-\infty}^{\infty} A_m C_{mn} + E_0 \sum_{m=-\infty}^{\infty} A_n H_n^{(2)}(k_0 r_0) e^{-jn\theta_0} \qquad (4)$$

,where

$$C_{mn} = \sum_{l=-\infty}^{-1} H_{m-n}^{(2)}(k_0 l_{2l}) + \sum_{l=1}^{\infty} (-1)^{m-n} H_{m-n}^{(2)}(k_0 l_{2l}) - \sum_{l=-\infty}^{-1} (-1)^{m+n} H_{m+n}^{(2)}(k_0 l_{2l+1}) - \sum_{l=1}^{\infty} H_{m+n}^{(2)}(k_0 l_{2l-1})$$

From Maxwell's equation, the electric field E_z^f in ferrite rod is expressed by

$$E_z^f = E_0 \sum_{m=-\infty}^{\infty} B_m J_m(\eta r_0) e^{-jm\theta_0} \qquad (5)$$

,where $\eta^2 = k_0^2 \dfrac{\varepsilon\{\mu^2+(j\kappa)^2\}}{\mu}$

To decide the unknown A_m and B_m, we consider the continuous conditions on the tangential components of the electric and magnetic fields.

$$(S_n + \sum_{m=-\infty}^{\infty} A_m C_{mn}) J_n(k_0 a) + A_n H_n^{(2)}(k_0 a) = B_n J_n(\eta a) \qquad (6)$$

$$k_0\{(S_n + \sum_{m=-\infty}^{\infty} A_m C_{mn}) J_n'(k_0 a) + A_n H_n^{(2)}(k_0 a)\} = B_n\left[\frac{1}{\mu^2-\kappa^2}\{\mu\eta J_n'(\eta a) + \kappa n J_n(\eta a)\}\right] \qquad (7)$$

From Eqs.(6) and (7), we can obtain each A_m and B_m by taking the m to a proper order, and so decide all the field.

Next, by transforming Eq.(4)

to the rectangular coordinates (x, y), we obtain

$$E_z^s = E_0 \frac{4}{d} \sum_{n=1}^{\infty} \frac{e^{-(\pm j) f_n \sqrt{k_0^2-(n\pi/d)^2}\, y}}{f_n \sqrt{k_0^2-(n\pi/d)^2}} \sin\frac{n\pi}{d}x \sum_{m=-\infty}^{\infty} (\pm 1)^m A_m \sin\left(\frac{n\pi}{d}x_0 \pm m \sin^{-1}\frac{n\pi}{k_0 d}\right) \quad (8)$$

, where

$$f_n = \begin{cases} 1 \cdots\cdots & n\pi/d < k_0 \\ -1 \cdots\cdots & n\pi/d > k_0 \end{cases}$$

and, + and - signs correspond to the transmitted and reflected waves respectively.

The transmission coefficient T and the reflection one R are given by

$$T = 1 + \frac{4}{d\sqrt{k_0^2-(\pi/d)^2}} \sum_{m=-\infty}^{\infty} A_m \sin\left(\frac{\pi}{d}x_0 + m \sin^{-1}\frac{\pi}{k_0 d}\right) \quad (9)$$

$$R = \frac{4}{d\sqrt{k_0^2-(\pi/d)^2}} \sum_{m=-\infty}^{\infty} (-1)^m A_m \sin\left(\frac{\pi}{d}x_0 - m \sin^{-1}\frac{\pi}{k_0 d}\right) \quad (10)$$

3. Calculated Results

We obtained the numerical values of the above expression by a computer for a lossless ferrite rod, when $x_0 = d/2$. Fig.3 shows the absolute values of the transmission and reflection coefficients.

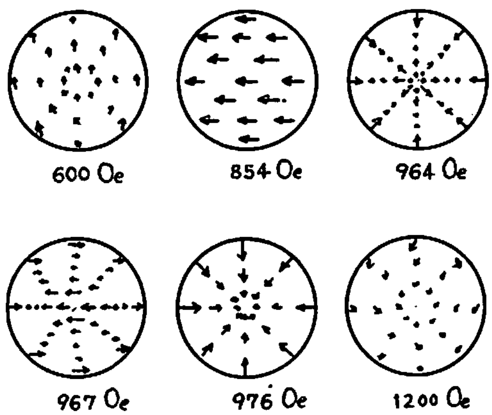

Fig.3 Magnetization distributions on cross section of the rod

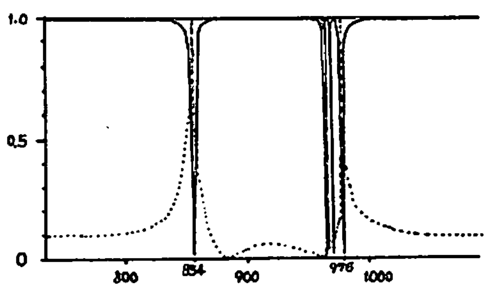

Fig.4 Transmission coefficient(——) reflection coefficient(·······)

Fig.4 shows the magnetization distributions on the cross section of ferrite rod, and Fig.5 shows the electric field on $x = x_0$ in the rod (E_0 is normalized to unity).

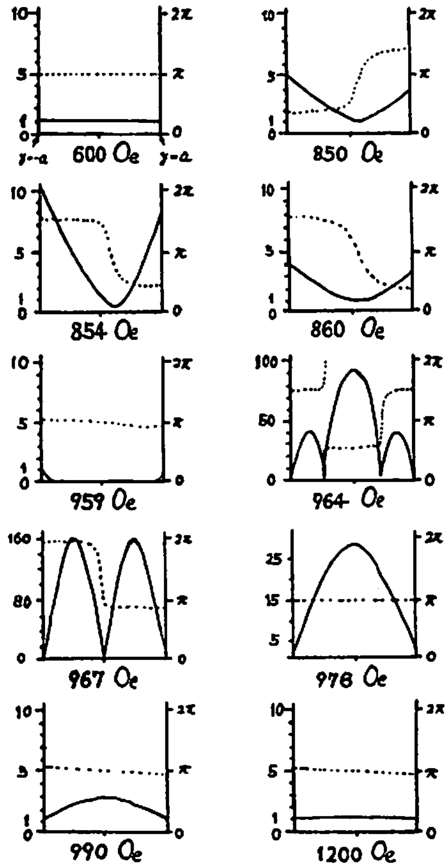

Fig.5 Electric field distributions along $x = x_0$ of cross section of the rod.
Absolute value(——), phase(·······)

4. Conclusion

The transmission and reflection coefficients and the field distributions in ferrite rod were obtained.

The experimental results for a lossy ferrite rod were in good agreement with the theoretical results.

References

1) T.Yoshida, M.Umeno and S.Miki: Inst.Electronics & Communication Engineers of Japan, MW70-46.

2) P.S.Epstein and A.D.Berk: J.Appl. Phys, 27, 11, p.1452 (Nov.1956)