

**Modification of FDTD Method For Highly Accurate Analysis of Microstrip Lines**

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**1 Introduction**

The FDTD method[1][2] is widely used for electromagnetic problems including the EM scattering, antennas and so on. Its reason is considered that the FDTD method has a simple algorithm and a high capability of modeling the complicated structures, and can easily obtain the practical level of the accuracy. However, the original FDTD method needs smaller cell and requires great amount of computer resources when requiring the extremely accuracy result. This is one of the disadvantages of the FDTD method. On the other hand, we have proposed the highly accurate FDTD[3][4] technique. In this method, the FDTD update equations were modified by introducing a spatial distribution of quasi-static field near the antenna conductor into the FDTD update equations using the integral form of Faraday's law. We have indicated that the accuracy is significantly improved and agree very well with measured data without reducing cell size.

In this paper, the method is applied to a microstrip line on dielectric substrate backed by perfect conductor. In the first half of this paper, we briefly review our proposed method and indicate the modified FDTD update equation. In the second half a characteristic impedance of straight microstrip line and the reflection coefficient of a bend line at right-angle will be calculated and be compared with the original FDTD method. The proposed method is extremely accurate but a time step must be reduced. Its condition is also indicated.

**2 Quasi-static approximation**

The electric and magnetic fields in the cross section of the microstrip line shown in Fig.1 are well approximated by the TEM fields even if the slight discontinuities are involved within the line. The fields are expressed by scalar potential  $\Psi$  which correspond to an  $x$  component magnetostatic vector potential when the current on the conductor is in  $x$  direction. from which Then the electric and magnetic fields are calculated by

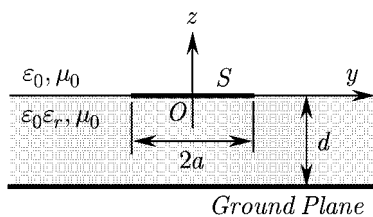


Figure 1: Cross section of microstrip line

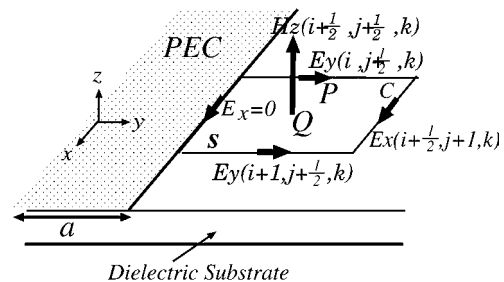


Figure 2: FDTD cell

$$\begin{cases} \mathbf{H}(y, z) = \frac{1}{\mu} \nabla_t \times (\Psi(y, z) \hat{x}) \\ \quad = \frac{1}{\mu} \left( \frac{\partial \Psi}{\partial z} \hat{y} - \frac{\partial \Psi}{\partial y} \hat{z} \right) \\ \mathbf{E}(y, z) = \sqrt{\frac{\mu}{\varepsilon}} \mathbf{H}(y, z) \times \hat{x} \end{cases} \quad (1)$$

The integral expression  $\Psi$  can be obtained by introducing a two dimensional Green's function  $G(\mathbf{r}, \mathbf{r}')$  that represents the potential from a line source at  $\mathbf{r}'$  as follows.

$$\Psi(\mathbf{r}) = \mu_0 \int_S G(\mathbf{r}, \mathbf{r}') J_x(\mathbf{r}') dS' \quad (2)$$

where  $S$  is the surface of the microstrip conductor shown in Fig.1. The Green's function  $G(\mathbf{r}, \mathbf{r}')$  of eq.(2) is easily found using the boundary conditions at the air-dielectric interface and at the ground surface.

The current distribution  $J_x(y)$  is well approximated by

$$J_x(y) = \frac{I_0}{\sqrt{a^2 - y^2}} \quad (3)$$

Substituting (3) to (2) we obtain  $\Psi$  analytically. Therefore, the  $\mathbf{E}^{stat}$  and  $\mathbf{H}^{stat}$  are easily obtained using eq.(1).

Next, we modify the original FDTD update equations using the spatial distribution of the static electromagnetic field. Fig. 2 shows an electric cell edge on the interface. In this region, quasi-static fields are considered dominant, and well approximated by  $\mathbf{E}^{stat}$  and  $\mathbf{H}^{stat}$ . Using these fields, temporal electric and magnetic fields on FDTD cells can be approximated as

$$E_y(y, z, t) \simeq E_y^{FDTD}(P, z, t) \frac{1}{B_P} E_y^{stat}(y, z) \quad (4)$$

$$H_z(y, z, t) \simeq H_z^{FDTD}(Q, z, t) \frac{1}{B_Q} H_z^{stat}(y, z) \quad (5)$$

where  $B_P(\mathbf{r}) = E_y^{stat}(P, z)$ ,  $B_Q(\mathbf{r}) = H_z^{stat}(Q, z)$

After, this approximation, applying the Faraday's law

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mu \mathbf{H} \cdot d\mathbf{S} \quad (6)$$

to contour path  $C$  in Fig.2, the modified FDTD update equation can be derived.

In this approach, the quasi-static field distribution is introduced to the FDTD update equation as described above. Therefore, the stability condition on the Courant condition should be changed. By applying the same manner for deriving the Courant condition to the modified update equation, we obtain the modified Courant condition as follow

$$v \Delta t \leq \frac{1}{\sqrt{C \left(\frac{1}{\Delta x}\right)^2 + D \left(\frac{1}{\Delta y}\right)^2 + \left(\frac{1}{\Delta z}\right)^2}} \quad (7)$$

where

$$C = B(Q)/B(P)$$

$$D = \frac{1}{\{\Psi(\Delta y, 0) - \Psi(a, 0)\} / \{B(Q) \Delta y\}}$$

The coefficient  $C$  and  $D$  are large than 1 in general, so that the time step must be change slightly below the original Courant limit  $\Delta t_c$ .

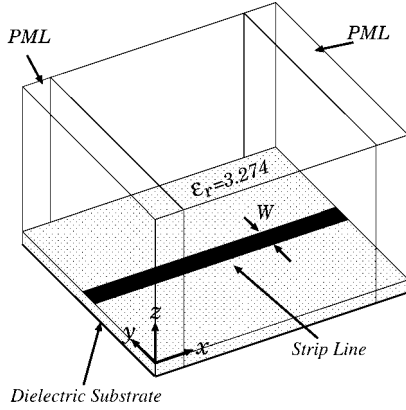


Figure 3: Microstrip line

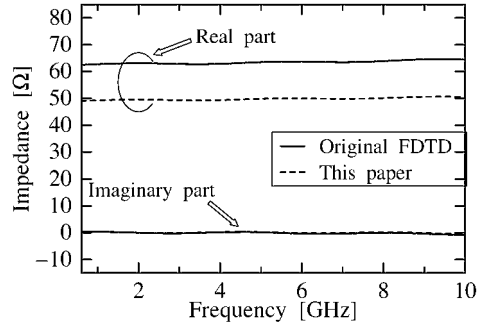


Figure 4: Characteristic impedance of the microstrip line

### 3 Results

The first example is a straight strip line whose geometry is shown in Fig.3. The thickness of the substrate is  $0.827\text{mm}$ , and relative permittivity is  $\epsilon_r = 3.274$ . The width of the strip line set to  $1.8\text{mm}$ . The characteristic impedance is designed as  $52\Omega$ . The calculated results is shown in Fig.4. In this calculation cell size are set as  $\Delta x = 0.251\text{mm}$ ,  $\Delta y = 0.2\text{mm}$  and  $\Delta z = 0.27566\text{mm}$ . The solid line shows the impedance calculated by the original FDTD. The broken line is the one obtained by the proposed method. It is found that the significant improvement of the calculation accuracy would be achieved even though the same cell size is used.

The right-angled bend microstrip line shown Fig.5 is also calculated. Fig.6 shows the current distribution on the microstrip line calculated by original FDTD. These figures shows that the  $J_x$  and  $J_y$ . Therefore the same approximation described above will also be effective for this example. The reflection coefficient is shown Fig.7. It is found that our method agrees very well with the original FDTD with very fine cell ( $\Delta x = \Delta y = 0.1125\text{mm}$ ,  $\Delta z = 0.068915\text{mm}$ ) for a wide frequency range.

### 4 Conclusion

In this paper, high accuracy FDTD method which utilize the quasi-static approximation was applied to microstrip structures including the right-angled microstrip line. It has been shown that the higher accurate result can be obtained without reducing cell size. The modified Courant condition was also indicated.

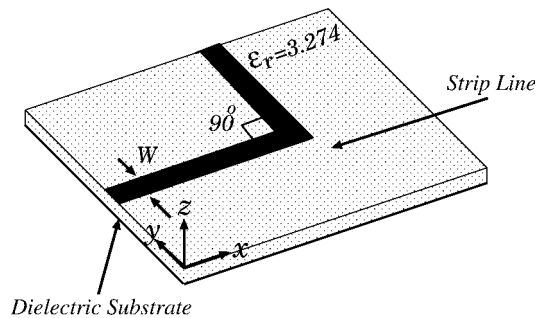


Figure 5: Right-angled bend microstrip line

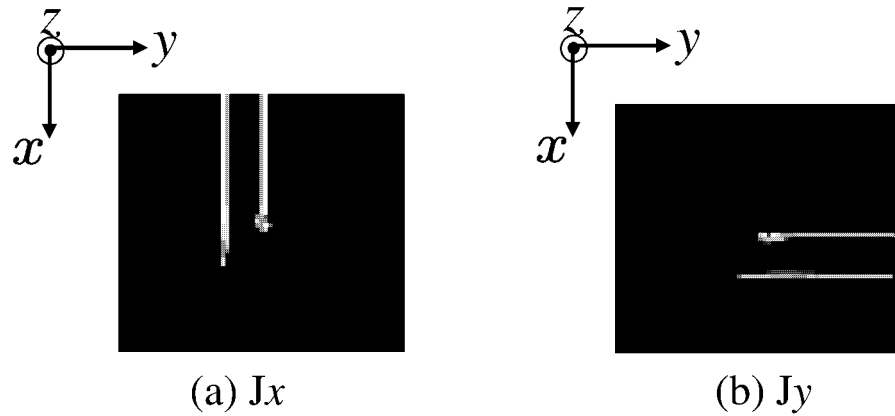


Figure 6: Current distribution on the microstrip line

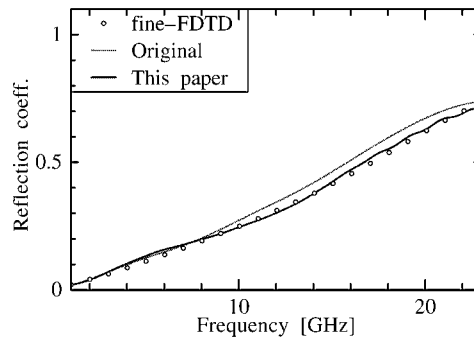


Figure 7: Reflection coefficient of the microstrip line

## References

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- [2] A. Taflove, Ed., "Advances in Computational Electromagnetics, The Finite-Difference Time-Domain Method," Boston: Artech House, 1998.
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- [4] Takuji Arima, Toru Uno and Masaharu Takahashi, "FDTD analysis of printed antenna on thin dielectric sheet incorporating with quasi-static field spatial distribution," Proc. International Symposium on Antenna and Propagat. 2004, pp.481-484, sendai Japan, Aug. 2004,