Analysis of thin-film waveguides with partial periodic structure of the grooves

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Abstract: In this paper, a method based on mode-matching method in the sense of least squares is proposed for analyzing the thin-film waveguides with partial periodic structure of the grooves. For example, this method is applied for fundamental analyses of grating couplers, DFB and DBR lasers.

1. Introduction

Thin-film waveguides having a periodic structure have recently been of considerable interest in connection with DFB and DBR lasers, optical filters, grating couplers, and so on[1,2]. In this paper, we shall present a method based on the mode-matching method in the sense of least squares[3] for analyzing the electromagnetic field of the thin-film waveguide with partial periodic structure of the grooves. For example, this method is applied to the fundamental analyses of the grating coupler, DBR and DFB lasers when the Bragg condition is satisfied. In this method, the approximate wave functions of the cover, core and substrate region of the waveguide are described by the superposition of plane waves with band-limited spectra, respectively. These superpositions of plane waves can be regarded as modal expansions and the expansion theorem of mode matching method in the sense of least squares can be applied[4,5]. These approximate wave functions are determined in such way that the mean-square boundary residual is minimized. This method results in simultaneous integral equations of the second kind Fredholm type in respect of unknown spectra of the plane waves after all[5]. The radiation field and the amplitudes of the guided modes which are generated by partial periodic structure can be obtained from the spectra directly in this method. The results of analyses are presented on the basis of the first order approximate solutions in this paper.

2. Formulation of the Problems and Algorithm

The problems is formulated about the symmetric thin-film waveguide with partial periodic structure of the grooves. The waveguide shown by Fig.(1) is uniform in the direction of y axis and consist of dielectric media of I, II and III with refractive index n_1, n_2, n_1, respectively. The regions occupied by respective media are assumed to be cover, core and substrate regions and they are denoted with S_I, S_{II} and S_{III}, respectively. The waveguide has partial periodic structure

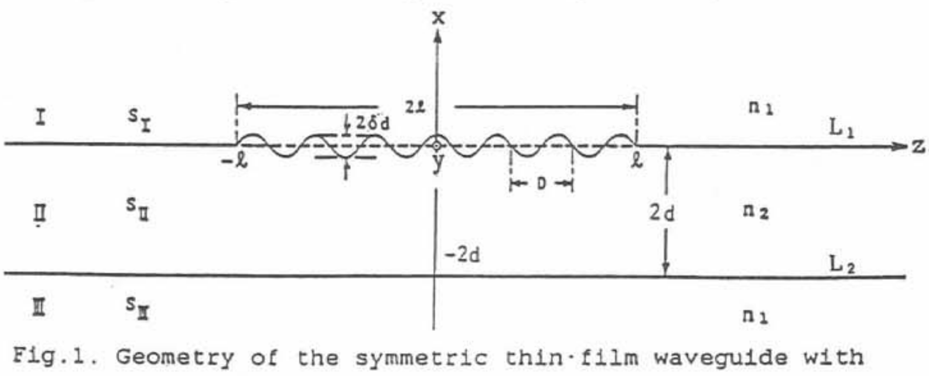

Fig.1. Geometry of the symmetric thin-film waveguide with partial periodic structure of the grooves.

of the grooves in the region ($|z| \leq \ell$). The boundary L_1 is given by

$$x=\xi(z)=\begin{cases} \delta d\eta(z) & ,|z|\leq \ell \\ 0 & ,|z|>\ell \end{cases} \quad , \ \eta(z)=\cos Kz \ , \ K=\frac{2\pi}{D} \qquad , (1)$$

where δ is the perturbation parameter and $2d$ is the thickness of the unperturbed part of the core region. K is the grating constant and D is the periodic length of the gratings. 2ℓ is the length of the part of the periodic structure of the waveguide. The time factor is understood to be $\exp(-j\omega t)$ throughout this paper. The wave number in free space is assumed to be k_0 and wave length is λ_0. In this paper, two problem are analyzed. One of them is the problem that TE plane wave polarized y direction is incident onto the surface of the core region from the cover region of the waveguide. Another is the prob lem that the lowest order even TE mode is incident into periodic part from the unperturbed part of the waveguide. Since these prob- lem are analyzed similarly, the common algorithm to these problems are discussed. In these problems, the approximate wave functions for scattered fields in each region are defined with band-limited superposition of plane waves as follows:

$$\Psi_{mw}(z,x)=\frac{1}{2\pi}\int_{-w}^{w}\psi_{mw}(h)\ \phi_m(h,z,x)\ dh\ , \ (m=1,2,3)$$

$$\phi_m(h,z,x)=\begin{cases} \exp\{j\kappa_1(h)x+jhz\} & , m=1 \\ \cos\{\kappa_2(h)(x+d)\}-p(h)\sin\{\kappa_2(h)(x+d)\}]\exp(jhz) & , m=2 \\ \exp\{-j\kappa_1(h)x+jhz\} & , m=3 \end{cases} \qquad , (2)$$

where $\phi_2(h,z,x)$ is defined taking boundary condition on L_2 into account and $P(h)$ and wave numbers $\kappa_m(h)$ $(m=1,2)$ are given by

$$\begin{cases} P(h)=\dfrac{I_e(h)}{I_o(h)}=\dfrac{j\kappa_1(h)\cos\{\kappa_2(h)d\}+\kappa_2(h)\sin\{\kappa_2(h)d\}}{\kappa_2(h)\cos\{\kappa_2(h)d\}-j\kappa_1(h)\sin\{\kappa_2(h)d\}} \\ \kappa_m(h)=\{(n_m k_0)^2-h^2\}^{1/2} \quad , \ (m=1,2) \end{cases} \qquad . (3)$$

In Eq.(2), $\psi_{mw}(h)$ $(m=1,2,3)$ are band-limited spectra of each approximate wave function. $\psi_{3w}(h)$ is connected with $\psi_{2w}(h)$ by the boundary condi- tion on L_2. The approximate wave functions satisfy Helmholtz equa- tions in each region and $\Psi_{1w}(z,x)$ and $\Psi_{3w}(z,x)$ satisfy outgoing radia- tion condition at $x\to\pm\infty$. The expansion theorem of mode matching method in the sense of least squares[3] which is applied to the prob- lem for scattering body with finite size can be extended to the problem for the infinite boundary with partial periodic structure of the grooves. That is, for the scattered wave functions $\Psi_m(z,x)$, $(m=1,2,3)$, we have the sequences of approximate wave functions $\{\Psi_{mw}(z,x), w\to\infty\}$, $(m=1,2,3)$, which uniformly converge to $\Psi_m(z,x)$, $(m=1,2,3)$ in any closed subdomains of each region S_I, S_{II} and S_{III}[4]. The spectra of these approximate wave functions are determined in such way that the mean-square boundary residual on L_1 is minimized. This method results in Fredholm integral equation of the second kind in respect to the vector whose components are two unknown spectra. The inte- gral equation is given by

$$(\mathbf{A}+\alpha^2\ \mathbf{B})\cdot\boldsymbol{\psi}_w(h)=\int_{-w}^{w}\mathbf{K}(h,h',\delta)\cdot\boldsymbol{\psi}_w(h')dh'+\mathbf{F}(h)+\alpha^2\mathbf{G}(h) \qquad , (4)$$

where "·" denotes an inner product. In Eq.(4), $\boldsymbol{\psi}_w(h)$ and $\mathbf{F}(h), \mathbf{G}(h)$ are vectors, and $\mathbf{A}$, $\mathbf{B}$ and integral kernel $\mathbf{K}(h,h',\delta)$ are dyadic. The vector $\boldsymbol{\psi}_w(h)$ which is the solution of Eq.(4) has components as follows:

$$\boldsymbol{\psi}_w(h) = (\psi_{1w}(h), \psi_{2w}(h))^t \qquad .(5)$$

When perturbation method is applied to Eq.(4) and the first order solution $\boldsymbol{\psi}_w^{(1)}(h)$ is obtained, its components are given by

$$\psi_{1w}^{(1)}(h) = \begin{cases} \left\{ \dfrac{\cos(\kappa_2(h)d)}{2I_e(h)} - \dfrac{\sin(\kappa_2(h)d)}{2I_o(h)} \right\} \zeta^{(1)}(h) & , |h| \le w \\ 0 & , |h| > w \end{cases} \qquad , (6)$$

$$\psi_{2w}^{(1)}(h) = \begin{cases} \dfrac{\zeta^{(1)}(h)}{2I_e(h)} & , |h| \le w \\ 0 & , |h| > w \end{cases} \qquad , (7)$$

where $\zeta^{(1)}(h)$ is given in the case of guided mode incidence as follows:

$$\zeta^{(1)}(h) = -(\delta d) A^e \, \mathcal{F}\{\beta_i - h, \eta(z)\} \cos\{\kappa_2(\beta_i)d)\} \frac{V^2}{d^2} \qquad . (8)$$

In Eq.(8), β_i, A^e and V are the propagation constant, the amplitude of the incident wave and the normalized frequency, respectively. The term $\mathcal{F}\{\beta_i - h, \eta(z)\}$ in Eq.(8) denotes the integral transform of $\eta(z)$ and it is given by

$$\mathcal{F}\{\beta_i - h, \eta(z)\} \equiv \int_{-\ell}^{\ell} \cos(Kz)\exp\{j(\beta_i - h)z\}\, dz = \left\{ \frac{\sin(K+\beta_i-h)\ell}{K+\beta_i-h} + \frac{\sin(K+h-\beta_i)\ell}{K+h-\beta_i} \right\} \qquad . (9)$$

$\psi_{3w}^{(1)}(h)$ is connected with $\psi_{2w}^{(1)}(h)$ by boundary condition on L_2 and given by similar equation to Eqs.(6) and (7). In the case of TE plane wave incidence, $\zeta^{(1)}(h)$ is obtained in the same way as the case of the guided mode incidence. $I_e(h)$ and $I_o(h)$ of Eqs.(6) and (7) are given by Eq.(3). The eigenvalue equations of even and odd TE modes are given, respectively, as follows:

$$I_e(h) = 0, \quad I_o(h) = 0 \qquad .. (10)$$

Therefore it is shown that the even and odd TE guided modes are generated by the partial periodic structure by substituting Eqs.(6) and (7) to Eq.(2). The amplitudes of the guided modes generated by the periodic structure is given by calculation of the residues of the approximate wave functions expressed by integral transform of Eq.(2). The far-field is obtained by applying the method of steepest descent to Eq.(2). In this paper, two problems mentioned above are analyzed in the case that the incident wave and the lowest order even TE guided mode which is excited and propagates negative z direction are coupled under the Bragg condition. The results of analyses based on the first order perturbation is presented.

3. Results of Analyses and Conclusion

The results of our method shown by Fig.(2)-(5) have good agreement with physical consideration. Therefore our method is available in the analyses of thin-film waveguides with partial periodic structure of the grooves.

References

1. K.Ogawa, et al.,"Theoretical analysis of etched grating coupler for integrated optics", IEEE.J.Quantum Electron,QE-9,1,29-42(1973).
2. D. Marcus, "Mode conversion by surface imperfections of a di-

electric slab waveguide", Bell Syst. tech. J.,48,3187-3215 (1969)
3. K. Yasuura and M. Tomita, "Numerical analysis of plane wave scattering from dielectric gratings(in Japanese)", Trans. Inst. Electron. Commun. Eng. Jpn., J61-B(7),132-139,1979.
4. K. Yasuura and T. Miyamoto, "Numerical analysis of an embedded optical waveguide", Radio Sci., 17, 93-98(1982).
5. M.Tomita,Y.Miyata and M.Ueda,"Analysis of light waveguide with partial periodic structure", Trans.Inst.Electron.Inform.Commun. Eng. Jpn., E71,10,926-930(1988).

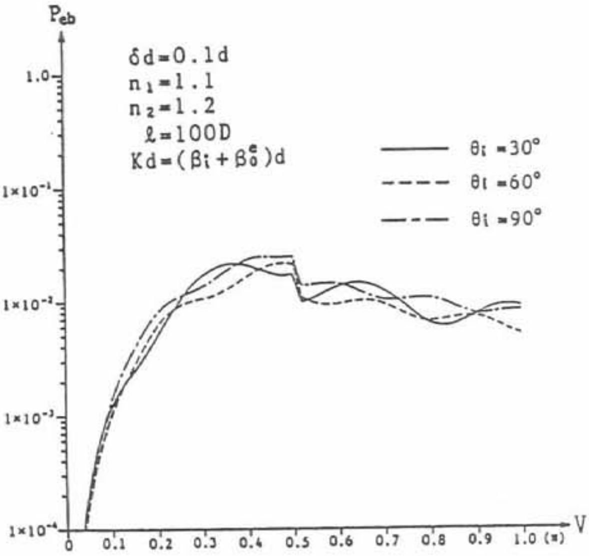

Fig.2. Normalized power, P_{eb} as functions of V when incident angle θ_i varies as parameter. P_{eb} is the power of the lowest order TE even mode which is coupled with the incident plane wave by the partial periodic structure of the symmetric waveguide and propagates backward in the case that the Bragg condition is satisfied.

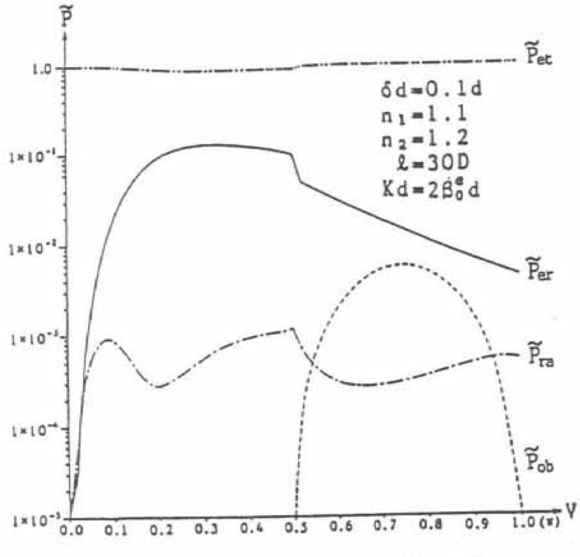

Fig.3. Normalized power, $\tilde{P}_{et}$, $\tilde{P}_{er}$, $\tilde{P}_{ra}$ and $\tilde{P}_{ob}$ as functions of V. $\tilde{P}_{et}$ and $\tilde{P}_{er}$ is the transmitted, reflected powers of incident guided mode and $\tilde{P}_{ra}$ is radiated power. $\tilde{P}_{ob}$ is the power carried backward by the lowest order odd TE mode. They are powers generated by the periodic structure in the case that the lowest order even TE mode is incident into periodic part from the unperturbed part of the symmetric waveguide when the Bragg condition is satisfied.

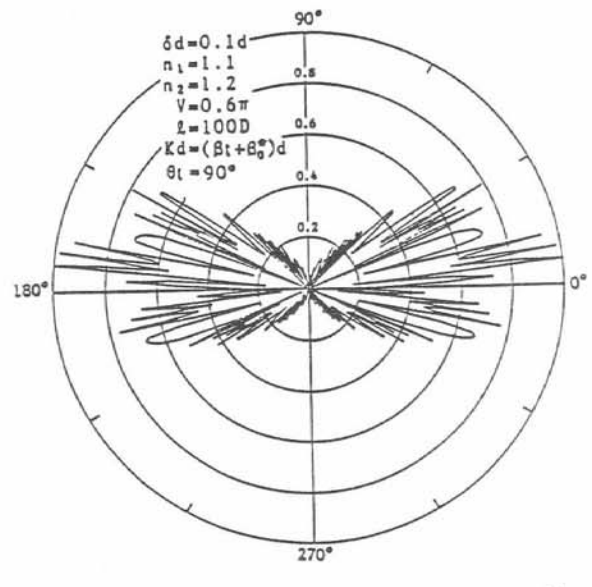

Fig.4. Far-field pattern of the scattered field in the case that the plane wave is incident onto the surface of the core region of the symmetric waveguide at the angle θ_i =90° when the Bragg condition is satisfied.

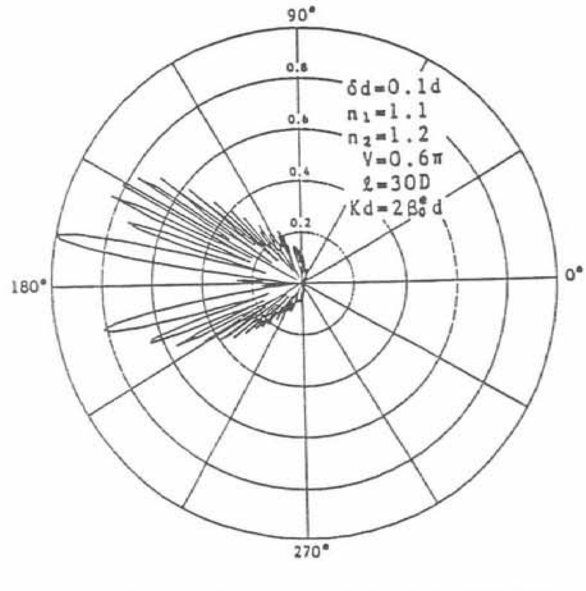

Fig.5. Far-field pattern of the scatterd field in the case that the lowest order guided mode is incident into periodic part from the unperturbed part of the symmetric waveguide when the Bragg condition is satisfied.