

3D ADI-FDTD FOR THE DISPERSIVE MEDIA USING PLRC AND RC METHOD

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Abstract: A 3D alternating direction implicit finite-difference time-domain(ADI-FDTD) algorithm was presented for dispersive, conductive media. A formulation was presented by using the recursive convolution(RC) and piecewise linear recursive convolution(PLRC) approaches to handle the dispersive features of the media. A simple numerical example of ground penetrating radar(GPR) application was included to validate the proposed formulation.

Keywords: ADI-FDTD method; PLRC; RC; Debye model;

1. Introduction

In recent years, an alternating direction implicit finite difference time domain(ADI-FDTD) Method[1] has been used for solving electromagnetic problems. An ADI-FDTD method is not restricted by the Courant Friedrich Levy(CFL) stability condition since it is unconditionally stable[2] and has a potential increasing the computational efficiency of a FDTD method especially in fine geometrical details. The dielectric constant and conductivity of the dispersive media are function of frequency. So, an ADI-FDTD algorithm has to handle the dispersive media. There are conventional methods to incorporate the dispersion of medium in the time-domain solutions: Recursive Convolution(RC), Piecewise Linear Recursive Convolution(PLRC)[3], Auxiliary Differential Equation(ADE)[4], and Z-Transform(ZT)[5]. In this paper, we propose the 3D dispersive ADI-FDTD using the RC and PLRC.

2. ADI-FDTD Formulation

The PLRC method is presented in the previous researches. In a linear dispersive medium, the relation between the electric flux density and the electric field can be written in time-domain form as

$$D(t) = \varepsilon_0 \varepsilon_\infty E(t) + \varepsilon_0 \int_0^t E(t-\tau) \chi(\tau) d\tau \quad (1)$$

where ε_0 is the permittivity of free space, ε_∞ is the dielectric constant of the medium at infinite frequency, and $\chi(t)$ is the susceptibility function of the medium. To take a discrete form, we extended equation (1) into an ADI-FDTD by using the piecewise linear time-reversed method.

$$\begin{aligned}
D^n &= \varepsilon_0 \varepsilon_\infty E^n + \varepsilon_0 \sum_{m=0}^{2n-1} E^{n-m/2} \int_{m\Delta t/2}^{(m+1)\Delta t/2} \chi(\tau) d\tau \\
&+ \varepsilon_0 \sum_{m=0}^{2n-1} (E^{n-(m+1)/2} - E^{n-m/2}) \frac{2}{\Delta t} \int_{m\Delta t/2}^{(m+1)\Delta t/2} (\tau - m\Delta t/2) \chi(\tau) d\tau
\end{aligned} \tag{2}$$

The equation (2) is a discrete form of the equation (1) in time-domain.

$$\begin{aligned}
&\left[1 + \frac{C_b|_{i+1/2,j,k}}{\Delta y} \frac{1}{\Delta y} (D_b|_{i+1/2,j+1/2,k} + D_b|_{i+1/2,j-1/2,k}) \right] E_x|_{i+1/2,j,k}^{n+1/2} \\
&- \frac{C_b|_{i+1/2,j,k}}{\Delta y} \frac{D_b|_{i+1/2,j+1/2,k}}{\Delta y} E_x|_{i+1/2,j+1,k}^{n+1/2} - \frac{C_b|_{i+1/2,j,k}}{\Delta y} \frac{D_b|_{i+1/2,j-1/2,k}}{\Delta y} E_x|_{i+1/2,j-1,k}^{n+1/2} \\
&= C_a|_{i+1/2,j,k} E_x|_{i+1/2,j,k}^n + \frac{C_b|_{i+1/2,j,k}}{\Delta y} D_a|_{i+1/2,j+1/2,k} H_z|_{i+1/2,j+1/2,k}^n \\
&- \frac{C_b|_{i+1/2,j,k}}{\Delta y} D_a|_{i+1/2,j-1/2,k} H_z|_{i+1/2,j-1/2,k}^n \\
&- \frac{C_b|_{i+1/2,j,k}}{\Delta z} (H_y|_{i+1/2,j,k+1/2}^n - H_y|_{i+1/2,j,k-1/2}^n) \\
&- \frac{C_b|_{i+1/2,j,k}}{\Delta y} \frac{D_b|_{i+1/2,j+1/2,k}}{\Delta x} (E_y|_{i+1,j+1/2,k}^n - E_y|_{i,j+1/2,k}^n) \\
&+ \frac{C_b|_{i+1/2,j,k}}{\Delta y} \frac{D_b|_{i+1/2,j-1/2,k}}{\Delta x} (E_y|_{i+1,j-1/2,k}^n - E_y|_{i,j-1/2,k}^n) \\
&+ C_c|_{i+1/2,j,k} \Psi_x|_{i+1/2,j,k}^n
\end{aligned} \tag{3}$$

$$\begin{aligned}
C_a|_{i,j,k} &= \left(\varepsilon_\infty - \xi_{i,j,k}^0 - \frac{\sigma_{i,j,k} \Delta t}{4\varepsilon_0} \right) / \left(\varepsilon_\infty + \chi_{i,j,k}^0 - \xi_{i,j,k}^0 + \frac{\sigma_{i,j,k} \Delta t}{4\varepsilon_0} \right), \\
C_b|_{i,j,k} &= \left(\frac{\Delta t}{2\varepsilon_0} \right) / \left(\varepsilon_\infty + \chi_{i,j,k}^0 - \xi_{i,j,k}^0 + \frac{\sigma_{i,j,k} \Delta t}{4\varepsilon_0} \right), \\
C_c|_{i,j,k} &= 1 / \left(\varepsilon_\infty + \chi_{i,j,k}^0 - \xi_{i,j,k}^0 + \frac{\sigma_{i,j,k} \Delta t}{4\varepsilon_0} \right), \\
\Psi_{i,j,k}^n &= \sum_{m=0}^{2n-1} \left\{ \begin{aligned} &E_x|_{i+1/2,j,k}^{n-m/2} \Delta \chi|_{i+1/2,j,k}^m \\ &+ (E_x|_{i+1/2,j,k}^{n-(m+1)/2} - E_x|_{i+1/2,j,k}^{n-m/2}) \Delta \xi|_{i+1/2,j,k}^m \end{aligned} \right\}
\end{aligned} \tag{4}$$

By using the equation (2) we can derive the field update equation for the dispersive ADI-FDTD. Equations (3) and (4) are E_x update equation and coefficients, respectively. If all ξ and $\Delta \xi$ terms in equation (4) are set to zero, the equation (3) is modified to RC approach with a loss term [3]. The quantity Ψ is a recursive accumulator. In the second half step, the E_x field update equation can be derived by following the similar procedure at the first time half step.

3. Formulation for Debye media

The equation (1) is the relation between the electric flux density and the electric field in

dispersive media. The $\chi(t)$ in equation (1) plays an important role in deciding a specific media. For the first-order Debye media the $\chi(t)$ is given by equation (5).

$$\chi(t) = [(\varepsilon_s - \varepsilon_\infty) / t_0] e^{t/t_0} u(t) \quad (5)$$

where ε_s is the zero frequency dielectric constant, ε_∞ is the infinite frequency dielectric constant, t_0 is the relaxation time of the medium, $u(t)$ is the unit-step function. A recursive relation Ψ is given by equation (6).

$$\Psi_{i,j,k}^n = \left(\Delta\chi_{i,j,k}^0 - \Delta\xi_{i,j,k}^0 \right) E_{i,j,k}^n + \Delta\xi_{i,j,k}^0 E_{i,j,k}^{n-1/2} + \Psi_{i,j,k}^{n-1/2} e^{-\Delta t/2t_0} \quad (6)$$

3. Numerical results

Fig. 1 shows a simple problem model. The grid size is 1.666cm. The current source is differential Gaussian pulse with 400MHz. Table 1 gives the parameters of two Debye media [3]. Fig. 2 shows the ADI-FDTD results compared with the FDTD results for (a) $\Delta t_{ADI} = 5\Delta t_{max}$ and (b) $\Delta t_{ADI} = 10\Delta t_{max}$.

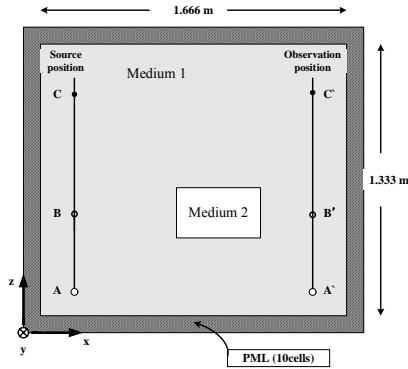


Fig.1. Geometry of the problem

TABLE 1
PARAMETERS FOR DEBYE MEDIA

Medium1	Medium 2
$\varepsilon_\infty = 3$	$\varepsilon_\infty = 3.7677$
$\varepsilon_s = 4.5$	$\varepsilon_s = 20.2677$
$t_0 = 6.4 \times 10^{-10} s$	$t_0 = 1.1614 \times 10^{-11} s$
$\sigma = 0.005 S/m$	$\sigma = 0.1165 S/m$

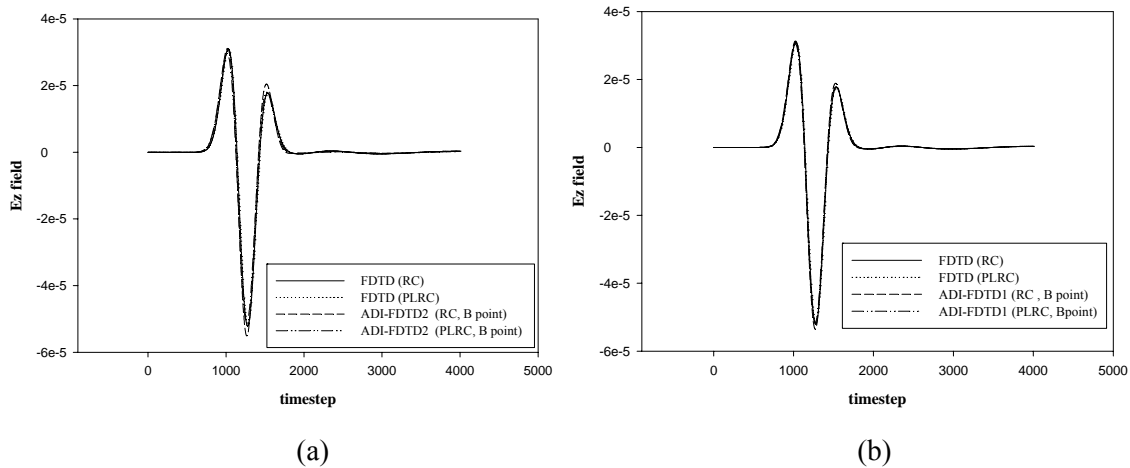


Fig. 2 The ADI-FDTD results for (a) $\Delta t_{ADI} = 5\Delta t_{max}$ and (b) $\Delta t_{ADI} = 10\Delta t_{max}$.

Fig. 2 shows that the PLRC approach is better than the RC approach in the ADI-FDTD method because it is basically developed to reduce the computation time by increasing the time step size [1],[2].

TABLE 2 COMPARISON OF THE CPU TIME

	FDTD	ADI-FDTD1	ADI-FDTD2
Δt (ps)	16.03	$10 \Delta t_{\text{FDTD}}$	$20 \Delta t_{\text{FDTD}}$
CPU time(s)	38604	12581	6371
Normalized CPU time	1.0	0.326	0.165

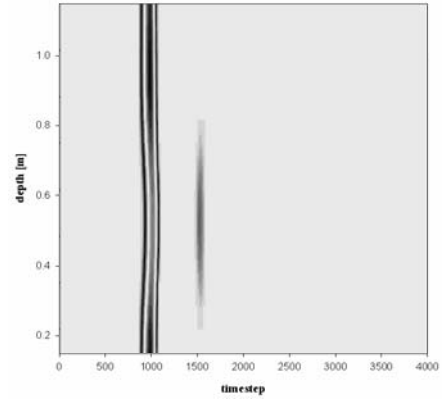


Fig. 3. The reconstructed image

Table 2 gives the comparison data of the CPU time and time step size. As we can see, the designed ADI-FDTD method can substantially reduce a CPU time. Fig. 3 shows the proposed ADI-FDTD (PLRC) results at $\Delta t_{\text{ADI}} = 20\Delta t_{\text{FDTD}}$. The source and observation position are simultaneously shifted from the point A(A') to C(C') onto the positive z-direction in Fig. 1. We tried to reconstruct the image of an arbitrary cubic through the received pulse array and can observe the image of an arbitrary cubic (Medium 2).

4. CONCLUSIONS

We validated the algorithm for the simple GPR problem including the two dispersive medias by comparing with the conventional FDTD algorithm. Excellent agreement between a FDTD and the ADI-FDTD results using the PLRC is obtained. The Our ADI-FDTD algorithm gives not only the decreasing computational time but also the desired accuracy level. The PLRC approach is especially much better than the RC approach because the maximum time step is only constrained by the required accuracy in the ADI-FDTD method.

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