

## 1-III C2

### TRANSVERSE-NETWORK REPRESENTATION FOR SLOTTED WAVEGUIDE HAVING A RECTANGULAR CROSS SECTION

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Many investigations of longitudinally slotted rectangular waveguides as flush-mounted antennas are well known. In particular, Oliner<sup>1)</sup> has used a transverse-resonance method to determine the complex propagation constant of the leaky wave modes.

Although these have been carried out in existence of a ground plane, no study appears in the case of no ground plane. This paper extends a transverse-resonance method to slotted rectangular waveguide of no ground plane and shows the insight into the derivation of slot admittances and propagation constants for the leaky waves.

Fig.1 shows the coordinate system, in which the problem is idealized by assuming that the slotted waveguide extends uniformly along the  $z$ -axis, and an incident H-type parallel-plate mode with no variation in the  $y$ -direction is considered for the transverse discontinuity problem.

Since this problem is that of a waveguide radiating into a space (not a half space), it will be reasonable to expect that a

well approved elliptic cylinder becomes a good approximation for the conducting surface of a rectangular cross section (refer to Fig.1). Then, by introducing the approximations discussed in the previous paper<sup>2)</sup>, one can transform the original (parallel plate) discontinuity problem to that in the elliptic cylinder.

The fields in the vicinity of the slot ( $|x| \leq a/2$ ) for the original problem will not depend on the nature of the space region, and the internal susceptance is given by Eq.1. In the derivation of the external admittance, let us suppose that both the complex power flows across the slots on  $x=a/2$  and  $\xi=\xi_0$  will be identical, provided that the successful replacement between both problems is made in the points of shape, slot location and slot field. Applying the boundary conditions at  $x=a/2$  and  $\xi=\xi_0$  to the internal and the external fields of the guide, respectively, the required admittance may be expressed in the variational form as Eqs.2-3. In Eqs.2-3,  $D(\eta)$  is the phase transformation factor.<sup>2)</sup>

$$\frac{B_{ext}}{Y_0} = \frac{-b}{a} \left( \frac{\pi}{d^2} \right) \sum_{m=0}^{\infty} \frac{J_{em}(\xi_0) \cdot \dot{J}_{em}(\xi_0) + N_{em}(\xi_0) \cdot \dot{N}_{em}(\xi_0)}{N_m^e \cdot |H_{em}^{(2)}(\xi_0)|^2} \cdot |I_{qm}^c|^2 \quad (3)$$

Fig.2 shows the computed and the observed results for the propagation constant in the case of standard waveguide of X-band.

We will find the successful advantages for the technique proposed in this paper. Now, the slot admittance will be influenced considerably by the shape or ellipticity of a substituted boundary and there are many ways to define it. For instance, it may be adequate to consider the  $eH_1$ -mode in an elliptic cylinder waveguide for an  $H_{10}$ -mode in the rectangular waveguide. Fig.3 shows the computed results for various shapes of the elliptic cylinder whose cut-off wavenumber is kept equal to that of  $H_{10}$ -mode.

From these, it will be noted that the effective approximation for the boundary will be realized when the ellipticity  $\tanh$  may be defined as the ratio  $b/a$  of a rectangular cross section.

• References

1. L.O.Goldstone and A.A.Oliner; IRE, Trans. PGAP-7, Oct., 1959.
2. H.Shigesawa and K.Takiyama; IEEE, Trans. PGAP-17, Jan., 1969.

$$\frac{B_{int}}{Y_0} = \frac{b}{a} \ln \csc \left( \frac{\pi d}{2b} \right) \quad (1)$$

$$\frac{B_{ext}}{Y_0} = \frac{b}{a} \left( \frac{\pi}{d^2} \right) \sum_{m=0}^{\infty} \frac{|I_{qm}^c|^2}{N_m^e |H_{em}^{(2)}(\xi_0)|^2} \quad (2)$$

where

$$I_{qm}^c = \int_{-z_a}^{z_a} S_{em}(\zeta) \cdot h_1 \cdot \exp(-jD(\zeta)) d\zeta$$

$$Y_0 = \frac{K}{\omega \mu_0}; \quad h_1 = C_0 \sqrt{\cosh^2 \xi_0 - \cos^2 \zeta}$$

$S_{em}$  ; even angular Mathieu function

$H_{em}^{(2)}$  ; even radial Mathieu function

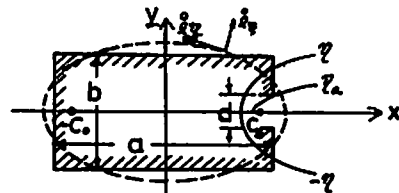


Fig.1. Coordinate system employed in calculation.

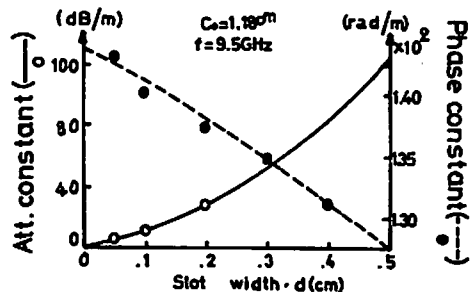


Fig.2. Complex propagation constant.

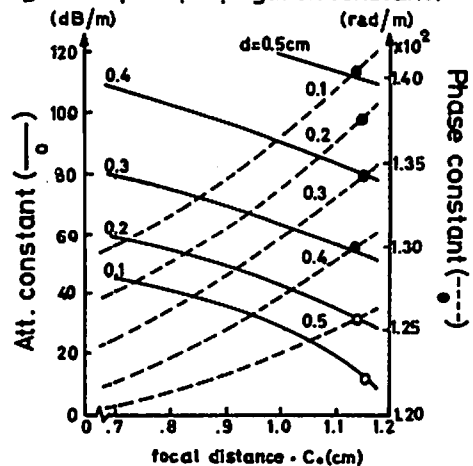


Fig.3. Dependence of the propagation constant on the focal distance.