TIME DOMAIN BORN APPROXIMATION FOR LOW FREQUENCY SCATTERING

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1. Introduction

For past decades many numerical methods has been developed in electromagnetics for time and frequency domains. The capability of individual method has continuously been increased, and so numerical techniques have been successfully applied to a wide range of applications from a calculation of radar cross section of a complicated object to a RF circuit optimization. However, due to computer capability numerical methods have been usually used for a moderate frequency range and/or scatterers of a moderate size. In very high or low frequencies, analytical techniques may have advantages over numerical methods yet.

Low frequency band has been widely used in many applications such as medical imaging since the band provides some advantages, for example good penetration through highly lossy media such as human body. However, at very low frequencies it is well known that the conventional numerical methods fail to generate correct results so that to solve the problem a special basis function such as loop-star basis function has to be used [1]. In this frequency range, any time domain technique such as finite difference time domain (FDTD) also may fail since to accurately model a scatterer, a very small cell should be used and so the time step is automatically very small, but global simulation time may be very large. Therefore due to the dispersion error of FDTD algorithm, the overall simulation accuracy is drastically degenerated.

Since in very low frequencies a scatterer may be electrically very small, interaction inside the scatterer may be very weak, and thus negligible. Hence in frequency domain, (distorted) Born approximation has been widely used for the frequencies [2]. To calculate time domain response of a complex scatterer, first a frequency domain response is computed and then using Fourier transform, the desired time domain solution is constructed. However, this procedure is very inefficient with respect to computational complexity. Hence in this paper, an efficient time domain Born approximation is formulated, and to maximize time step, sampling theorem is used.

2. Formulation

Figure 1 shows the problem geometry where a field is incident on an inhomogeneous dielectric scatterer with a dielectric constant $\varepsilon_r(x',y')$. Frequency of interest is very low (< kHz) so that the size of the scatterer may be electrically very small. For this kind of situation, Born approximation can accurately estimate a polarization current inside the dielectric object, which gives $\vec{J} \approx -ik_0 Y_0(\varepsilon_r - 1)\vec{E}^i(\varpi, \vec{r}\,')$ in frequency domain [2].

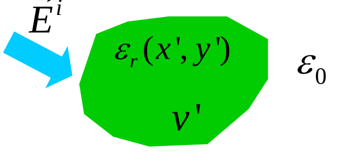

Figure 1: Problem Geometry

Therefore an electric Hertz vector induced by the current can be simply written as

ISBN: 89-86522-77-2 94460©KEES

$$\vec{\Pi}_e(\varpi,\vec{r}) = -\frac{1}{4}\int_{v'}(\varepsilon_r - 1)\frac{e^{ik_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\vec{E}^i(\varpi,\vec{r}')dv'$$
$$= -\frac{1}{8\pi}\int_{v'}(\varepsilon_r - 1)\frac{e^{ik_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\int_{-\infty}^{\infty}\vec{E}^i(t,\vec{r}')e^{i\varpi t}dtdv'$$

where $\vec{E}^i(t,\vec{r}')$ is the incident field in time domain. To obtain the time domain Hertz vector representation, Fourier transform is taken in the both sides of the above equation. The Hertz potential in time domain can be expressed as

$$\vec{\Pi}_e(t,\vec{r}) = -\frac{1}{8\pi}\int_{v'}dv'\int_{-\infty}^{\infty}dt\vec{E}^i(t,\vec{r}')\frac{1}{|\vec{r}-\vec{r}'|}\int_{-\infty}^{\infty}d\varpi(\varepsilon_r - 1)e^{-i\varpi t'}e^{ik_0|\vec{r}-\vec{r}'|}e^{i\varpi t} \tag{1}$$

where $\varepsilon_r = \varepsilon' + i\dfrac{\sigma}{\varpi}$, σ is a conductivity of the scatterer. Using the properties of Fourier transform [3], (1) can be evaluated into a more concise form as

$$\vec{\Pi}_e(t,\vec{r}) = -\frac{1}{4}\int_{v'}(\varepsilon' - 1)\frac{\vec{E}^i(t-|\vec{r}-\vec{r}'|/c,\vec{r}')}{|\vec{r}-\vec{r}'|}dv' - \frac{1}{4}\int_0^t d\xi\int_{v'}dv'\sigma\frac{\vec{E}^i(\xi-|\vec{r}-\vec{r}'|/c,\vec{r}')}{|\vec{r}-\vec{r}'|} \tag{2}$$

where c is a light velocity of the host medium. To obtain (2) it is assumed that $\vec{E}^i$ exists from $t = 0$. Since the scatterer may have very complicated shape, it is very hard to numerically evaluate the integral with respect to v' in (2). Hence the scatterer is divided into many small volumes, the integral is performed over the cells, and then summarized. Since the electric field may be a smooth function, the field can be assumed a constant over each cell, and so the integral over the scatterer can be rewritten as

$$\int_{v'}\frac{\vec{E}^i(t-|\vec{r}-\vec{r}'|/c,\vec{r}')}{|\vec{r}-\vec{r}'|}dv' \approx \sum_n \vec{E}^i(t-|\vec{r}-\vec{r}_{nc}'|/c,\vec{r}')\int_{v_n'}\frac{1}{|\vec{r}-\vec{r}'|}dv_n'$$

where $\vec{r}_{nc}$ is a center of the nth cell.

To efficiently deal with an arbitrary incident field, $\vec{E}^i$, sampling theorem can be used. In general, $\vec{E}^i$ is a bandlimted process, which means that $|\vec{E}^i(\varpi)| = 0$ for $|\varpi| > B$. Therefore $\vec{E}^i$ can be exactly interpolated using a few samples as

$$\vec{E}^i(t) = \sum_{n=-\infty}^{\infty}\vec{E}^i(nT)\,\mathrm{sinc}[B(t-nT)] \tag{3}$$

where $T = \pi/B$, and $\mathrm{sinc}(x) = \sin(x)/x$. Since $\vec{E}^i$ is localized in time domain, the summation in (3) can be truncated. In (3) $\vec{E}^i(nT)$ are samples at $t = nT$, and so these are not a function of time. Only $\mathrm{sinc}(\cdot)$ is dependant on time in (3). After some algebraic manipulations, the Hertz vector can be given by

$$\vec{\Pi}_e(t,\vec{r}) = -\frac{1}{4}\int_{v'}dv'\frac{1}{|\vec{r}-\vec{r}'|}\sum_{n=-\infty}^{\infty}\vec{E}^i(nT)\left[\begin{array}{l}(\varepsilon'-1)\,\mathrm{sinc}\left\{B(t-|\vec{r}-\vec{r}'|/c - nT)\right\} + \\ \dfrac{\sigma}{B}\left\{\mathrm{Si}\left[t-|\vec{r}-\vec{r}'|/c - nT\right] + \mathrm{Si}\left[|\vec{r}-\vec{r}'|/c + nT\right]\right\}\end{array}\right] \tag{4}$$

where $\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}dt$ is known as sine integral [3].

The scattered electric field can be easily computed from the obtained Hertz potential as

$$\vec{E}=\nabla\times\nabla\times\vec{\Pi}_e=\begin{pmatrix} -\frac{\partial^2}{\partial^2 y}-\frac{\partial^2}{\partial^2 z} & \frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial x\partial z} \\ \frac{\partial^2}{\partial x\partial y} & -\frac{\partial^2}{\partial^2 x}-\frac{\partial^2}{\partial^2 z} & \frac{\partial^2}{\partial y\partial z} \\ \frac{\partial^2}{\partial x\partial z} & \frac{\partial^2}{\partial y\partial z} & -\frac{\partial^2}{\partial^2 x}-\frac{\partial^2}{\partial^2 y} \end{pmatrix}\vec{\Pi}_e$$

If the Hertz vector is written in a form of $\vec{\Pi}_e(t,\vec{r})=-\frac{1}{4}\sum_{n=-\infty}^{\infty}\vec{E}^i(nT)F\cdot G$, each component of electric field can be written as

$$\frac{\partial\vec{\Pi}_e}{\partial p\partial q}=-\frac{1}{4}\sum_{n=-\infty}^{\infty}\vec{E}^i(nT)\left[F_{pq}\cdot G+F_p\cdot G_q+F_q\cdot G_p+F\cdot G_{pq}\right]$$

where $F=\int_{v'}dv'\frac{1}{|\vec{r}-\vec{r}\,'|}$, and G is the combination of $\sin\mathrm{c}(\cdot)$ and $\mathrm{Si}(\cdot)$ in (4). The derivatives of F and G can be computed analytically. Based on the above equations, the electric field can be computed at the same sampling time, $t=nT$, and then using sampling theorem the electric field can be exactly recomputed at any wanted time. Therefore the required time step may be sufficiently large, and so the total computational complexity is very low compared with other time domain methods.

3. Numerical Results

To verify the proposed expressions, the operating frequency and the bandwidth are set at 100Hz and 50Mz, respectively. First scattering by a small cubic box located at a origin is considered whose size is $12.5\times12.5\times12.5cm$, and dielectric constant and conductivity are 60 and 10, respectively. The scattered field is computed at (0, 0, -1) when a Gaussian plane wave is incident at $\theta_i=0^o$ and $\varphi_i=0^o$ whose magnitude is $1/\sqrt{3}(\vec{x}+\vec{y}+\vec{z})$. Figure 2 shows two x-directed scattered electric field components in time domain which are calculated by two methods: the proposed formulation and the conventional frequency domain Born approximation. As seen in the figure the two results are in excellent agreement. Figure 3 shows the total field of z-directed component. As expected the scattered field reduces the incident field since the observation point is inside the shadow region. Next scattering from a small dielectric sphere with 5cm radius is considered since a very accurate approximate solution, Rayleigh solution, is known for the structure. For the calculation, the dielectric constant is reduced to 1.1, but conductivity is kept at 10. To accurately model the sphere, 552 very small cells $1\times1\times1cm$ are used. The observation point and plane wave incidence angles are the same as the previous computation, but an h-polarized incident wave is assumed for this case. The proposed solution generates a very accurate result as seen in the figure 4. The observed small discrepancies are caused by the discretization error.

4. Conclusions

In this paper, an efficient time domain formulation is proposed for low frequency scattering by a very complex shaped inhomogeneous dielectric object. The formulation is based on Born approximation and sampling theorem. By comparisons with a known frequency Born approximation and Rayleigh formulation, the proposed equations are verified for several examples

5. Reference

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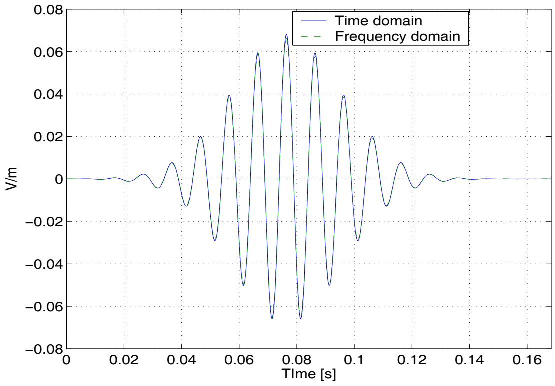

Figure 2: E_x components for scattering by a cubic box

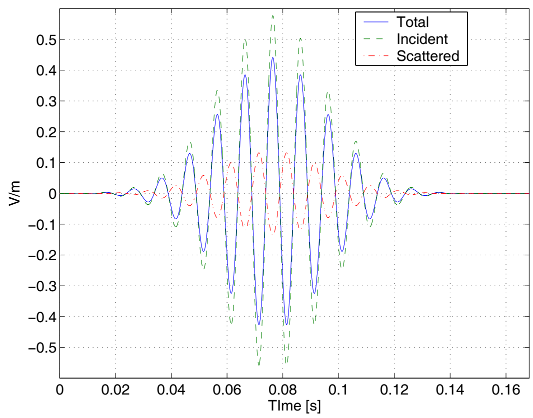

Figure 3: E_z components for scattering by a cubic box

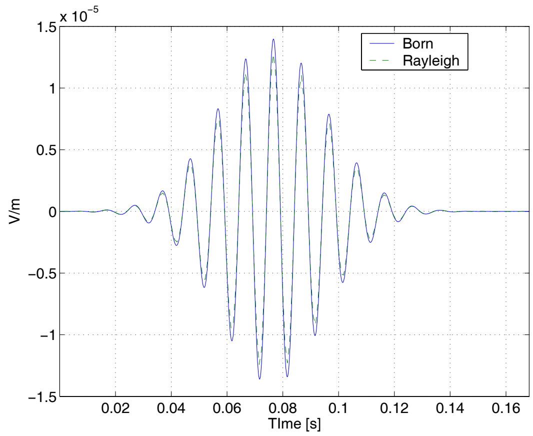

Figure 4: E_y components for scattering by a sphere