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# ON THE GENERALIZATION OF SOMMERFELD INTEGRALS FOR MULTIPOLES


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Recent development of the Unimoment Method [1, 2, 3, 4] has enabled the efficient calculation of the scattering and radiation problems involving material bodies, either in 2-space or 3-space. The success of the method relies on the Finite Element Method for the solution inside a mathematical surface, and on the proper field expansions in the exterior of the surface. In order to implement the unimoment method to solve the scattering of buried material bodies, it is necessary to find a set of exterior model fields, each of which satisfies the boundary conditions of the air-ground interface.

The well known Sommerfeld integrals provide such field solutions for dipole sources. The purpose of this paper is to generalize the Sommerfeld integrals to higher order multipoles such that the radiated fields of an arbitrary localized source above (or under) the ground can be expanded.

The key to the generalization of Sommerfeld integrals is the following Fourier Bessel integrals of the spherical Hankel-Legendre functions

$$h_n^{(2)}(kr)\; P_n^m(\cos\theta) = \int_0^\infty f_{m,n}(\lambda)\; J_m(\lambda\rho) e^{-\mu|z|} d\lambda \tag{1}$$

where $\mu = \sqrt{\lambda^2-k^2}$ with real part of $\mu > 0$. The amplitude functions $f_{m,n}(\lambda)$ are found in the form of the following recurrence relations

$$f_{m,m}(\lambda) = j\left(\frac{\lambda}{k}\right)^{m+1} \frac{1}{\mu} P_m^m(0); \qquad f_{m,m-1}(\lambda) \equiv 0 \tag{2}$$

and

$$f_{m,n+1}(\lambda) = \frac{2n+1}{(n-m+1)} \left[\frac{\mu}{k} f_{m,n}(\lambda) + \frac{(n+m)}{(2n+1)} f_{m,n-1}(\lambda)\right] \tag{3}$$

for $m = 0, 1, 2, \ldots,$ and $n = m, m+1, m+2, \ldots$ .

Using the integrals of (1) the following vector potentials are obtained, where the multipoles are situated in region II as shown in Figure 1.

I) The vertical electric multipoles ($TM_{m,n}$ modes):

In region I:

$$\vec{A}_e^I = \hat{z}\, e^{\pm jm\phi} \int_0^\infty \frac{2\varepsilon_1\mu_2}{\varepsilon_1\mu_2 + \varepsilon_2\mu_1} f_{m,n}(\lambda)\, J_m(\lambda\rho) e^{-\mu_2 d - \mu_1(z-d)} d\lambda \tag{4}$$

In region II:

$$\vec{A}_e^{II} = \hat{z}\, e^{\pm jm\phi} \Big[ h_n^{(2)}(k_2 r)\, P_n^m(\cos\theta) + \int_0^\infty \frac{\varepsilon_1\mu_2 - \varepsilon_2\mu_1}{\varepsilon_1\mu_2 + \varepsilon_2\mu_1} f_{m,n}(\lambda)\, J_m(\lambda\rho)\, e^{-\mu_2(2d-z)} d\lambda \Big] \tag{5}$$

II) The horizontal rotating electric multipoles ($RTM_m$ modes, m = 1, 2, ...):

In region I:

$$\vec{A}_e^I = (\hat{x} \pm j\,\hat{y}) e^{\pm j(m-1)\phi} \int_0^\infty \frac{2\mu_2}{\mu_2 + \mu_1} f_{m-1,m-1}(\lambda) J_{m-1}(\lambda\rho) e^{-\mu_2 d - \mu_1(z-d)} d\lambda$$

$$+ \hat{z}\, e^{\pm jm\phi} \int_0^\infty \frac{(\varepsilon_1 - \varepsilon_2)\; 2\mu_2\lambda}{(\mu_1 + \mu_2)(\varepsilon_2\mu_2 + \varepsilon_1\mu_2)} f_{m-1,m-1}(\lambda) J_m(\lambda\rho) e^{-\mu_2 d - \mu_1(z-d)} d\lambda \tag{6}$$

In region II:

$$\vec{A}_e^{II} = (\hat{x} \pm j\,\hat{y})\, h_{m-1}^{(2)}(k_2 r)\, P_{m-1}^{m-1}(\cos\theta) e^{\pm j(m-1)\phi}$$

$$+ (\hat{x} \pm j\,\hat{y}) e^{\pm j(m-1)\phi} \int_0^\infty \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} f_{m-1,m-1}(\lambda) J_{m-1}(\lambda\rho) e^{-\mu_2(2d-z)} d\lambda$$

$$+ \hat{z}\, e^{\pm jm\phi} \int_0^\infty \frac{(\varepsilon_1 - \varepsilon_2) 2\mu_2\lambda}{(\mu_1 + \mu_2)(\varepsilon_2\mu_1 + \varepsilon_1\mu_2)} f_{m-1,m-1}(\lambda) J_m(\lambda\rho) e^{-\mu_2(2d-z)} d\lambda \tag{7}$$

In these equations, we have used $\mu_1 = \sqrt{\lambda^2 - k_1^2}$ and $\mu_2 = \sqrt{\lambda^2 - k_2^2}$ with non-negative real parts. The vector potential $\vec{A}_e$ is related to the Hertz potential by

$$\vec{A}_e = j\,\omega\,\varepsilon\,\vec{\pi}_e \tag{8}$$

The formulations of the magnetic multipoles can be found similar to those of the electric multipoles. It was originally believed that the vertical electric and vertical magnetic multipoles were sufficient to expand an arbitrary radiating field, but our experience proved to be otherwise, i.e., the horizontal multipole fields has to be included to insure rapid convergence [5, 6].

The vector wave expansion, by using the multipoles and their generalized Sommerfeld integrals, constitutes a complete expansion for the scattering and radiating waves in the presence of the lossy-ground. They have been successfully applied to the calculation of scattering by buried obstacles [5].

References


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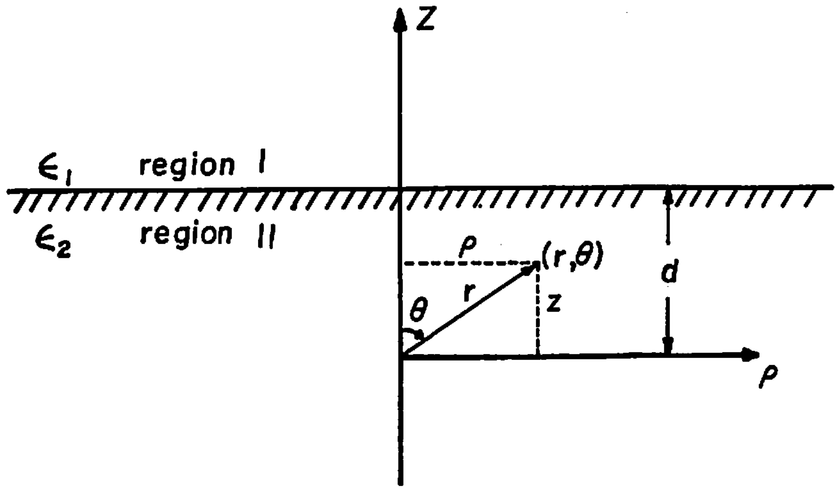


Figure 1. Coordinates in the Meridional Plane.