1-III C1

# MULTILAYERED SURFACE-WAVE SYSTEMS


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**1.Cylindrical multilayered surface-wave systems with thin layers(Fig.1).** Circularly symmetrical transverse magnetic mode $E_{00}$ is considered. The field components may be written as[1]

(1) $E_r = \frac{\gamma}{k^2} E'_z$ ; $H_\varphi = j\frac{\omega\varepsilon}{k^2} E'_z$

where

$$E'_z = \frac{dE_z}{dr} \ ; \ k^2 = \gamma^2 + \omega^2\varepsilon\mu .$$

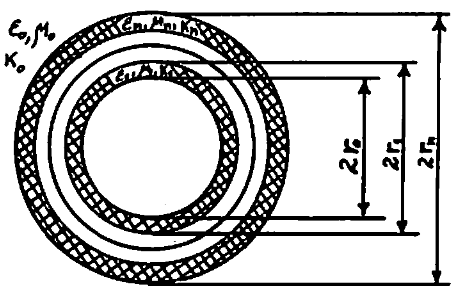

Fig.1

In the dielectric layers with number $i(i=1,2,..,n)$ $E_z$ is given by the equation

(2) $$E_z = A_i\left[J_o(k_i r) + B_i N_o(k_i r)\right] \times e^{-\gamma z + j\omega t}.$$

In outer space

(3) $$E_{z(i+1)} = A_{n+1} K_o(k_o r) e^{-\gamma z + j\omega t}.$$

In inner space in the case of dielectric "O-guide"

(4) $$E_{zo} = A_o I_o(k_o r) e^{-\gamma z + j\omega t},$$

$$-k_o^2 = \gamma^2 + \omega^2\varepsilon\mu ;$$

$J_o$ is the Bessel function,
$N_o$ is the Neumann function,
$K_o$ is the MacDonald function,
$I_o$ is the modif.Bessel function.

The boundary conditions are

(5) $E_{zi}(r_{i+1}) = E_{z(i+1)}(r_{i+1})$

(6) $H_{\varphi i}(r_{i+1}) = H_{\varphi(i+1)}(r_{i+1})$

For dielectrical "O-guide" $i=0, 1,..,n$, for G-line $i=1,2,..,n$ and $E_{z1}(r_1) = 0$.

As the layers are thin we can write $d_i = r_i - r_{i-1} \ll r_i$ and

(7) $E_{zi}(r_i) = E_{zi}(r_{i-1}) + d_i E'_z(r_{i-1})$.

From Eq.(1),(5),(6) and (7) we obtain

(8) $$\frac{k_{i+1}^2}{\varepsilon_{i+1}} \cdot \frac{E_{z(i+1)}(r_i)}{E'_{z(i+1)}(r_i)} = \frac{k_i^2}{\varepsilon_i} d_i + \frac{k_i^2}{\varepsilon_i} \cdot \frac{E_{zi}(r_{i-1})}{E'_{zi}(r_{i-1})} .$$

After several following substitutions for dielectrical "O-guide" we obtain equation for $k_o$

(9) $$\frac{x}{4\pi^2}\left[\frac{K_o(x)}{K_1(x)} - \frac{I_o(x)}{I_1(x)} + xb\right] = \frac{r_n}{\lambda^2}\sum_{i=1}^{n}(\mu_{ir} - \frac{1}{\varepsilon_{ir}})d_i$$

with

$$b = \frac{1}{r_n}\sum_{i=1}^{n}\frac{d_i}{\varepsilon_{ir}} \ ; \quad x = k_o r_n$$

For G-line analogically we obtain

$$(10)\ \frac{x}{4\pi^2}\left[\frac{K_0(x)}{K_1(x)}-xb\right]=\frac{r_n}{\lambda^2}\sum_{i=1}^{n}(\mu_{ir}-\frac{1}{\varepsilon_{ir}})d_i$$

The left sides of Eq.(9) and (10) are plotted as $S_1$ and $S_2$ respectively in Fig.2.

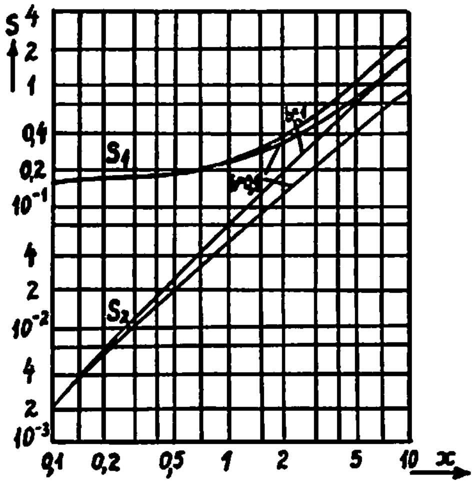


Fig.2

<u>2.Multilayered G-line in case $k_ir_n \ll 1$</u>[2]. The functions J,N,K and I can be replaced by their asymptotic representations. With (1), (2),(5) and (6) the constante $B_i$ may be expressed by $B_{i-1}$

$$(11)\ \frac{k_i^2}{\varepsilon_i}B_i=\frac{k_{i-1}^2}{\varepsilon_{i-1}}\left[B_{i-1}-\frac{2}{\pi}\ln(0,89k_{i-1}r_i)\right]+$$
$$+\ln(0,89k_ir_i)$$

and

$$(12)\quad B_1=\frac{2}{\pi}\ln(0,89k_1r_0).$$

From the ratio $E_z/H_\varphi$ at the surface of the coat($r=r_n$),(11) and (12) we get the relation

$$(13)\ \left(\frac{k_or_n}{2\pi}\right)^2\ln(0,89k_or_n)=$$
$$=\left(\frac{r_n}{\lambda}\right)^2\sum_{i=1}^{n}(\mu_{ir}-\frac{1}{\varepsilon_{ir}})\ln\frac{r_i}{r_{i-1}}$$

The left side of (13) is plotted by Goubau[2].

<u>3.Flat multilayered metal-dielectrical surface-wave system in case $k_iy_n \ll 1$</u>(Fig.3). By substituting in the upper equations $\varphi$ with x, r with y and Bessel functions with exponents we obtain analogically

$$(14)\ E_{zi}=A_i\left[e^{-jk_iy}+B_ie^{jk_iy}\right]e^{-\gamma z+j\omega t}$$

$$(15)\ \frac{k_{i+1}}{\varepsilon_{i+1}}(B_{i+1}-1)=j2y_i\frac{k_{i+1}^2}{\varepsilon_{i+1}}-$$
$$-j2y_i\frac{k_i^2}{\varepsilon_i}+\frac{k_i}{\varepsilon_i}(B_i-1)$$

$$(16)\quad k_o=\left(\frac{2\pi}{\lambda}\right)^2\sum_{i=1}^{n}(\mu_{ir}-\frac{1}{\varepsilon_{ir}})d_i$$

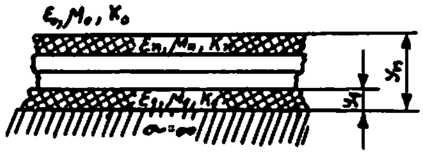

Fig.3

<u>4.Conclusions.</u> It is seen that each layer affects independently from other layers the propagation constante $k_o$. The Eq.(13) may be used in meter, decimeter and centimeter range and Eq.(10) in milimeter range.

<u>5. References.</u>


1.J.A.Stratton,Electromagnetic Theory(Mc-Graw-Hill Book Company, Inc.New York,1941)pp.516-517.

2.G.Goubau,J.Appl.Phys.21.1119 (1950).