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1. Cylindrical multilayered surface-wave systems with thin layers (Fig. 1). Circularly symmetrical transverse magnetic mode

$E_{00}$  is considered. The field components may be written as<sup>1</sup>

$$(1) E_r = \frac{\gamma}{k^2} E'_z ; H_\varphi = j \frac{\omega \epsilon}{k^2} E'_z$$

where  $E'_z = \frac{dE_z}{dr} ; k^2 = \gamma^2 + \omega^2 \epsilon \mu$ .

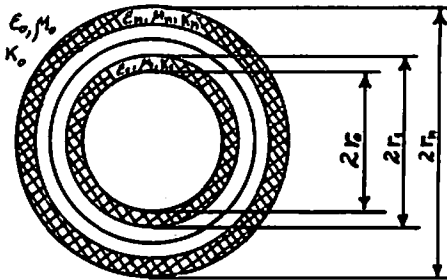


Fig. 1

In the dielectric layers with number  $i (i=1, 2, \dots, n)$   $E_z$  is given by the equation

$$(2) E_z = A_i \left[ J_0(k_i r) + B_i N_0(k_i r) \right] \times e^{-\gamma z + j \omega t}$$

In outer space

$$(3) E_{z(i+1)} = A_{n+1} K_0(k_o r) e^{-\gamma z + j \omega t}$$

In inner space in the case of dielectric "O-guide"

$$(4) E_{z0} = A_0 I_0(k_o r) e^{-\gamma z + j \omega t}$$

$$-k_o^2 = \gamma^2 + \omega^2 \epsilon \mu ;$$

$J_0$  is the Bessel function,  
 $N_0$  is the Neumann function,  
 $K_0$  is the MacDonalld function,  
 $I_0$  is the modif. Bessel function.

The boundary conditions are

$$(5) E_{zi}(r_{i+1}) = E_{z(i+1)}(r_{i+1})$$

$$(6) H_{\varphi i}(r_{i+1}) = H_{\varphi(i+1)}(r_{i+1})$$

For dielectrical "O-guide"  $i=0, 1, \dots, n$ , for G-line  $i=1, 2, \dots, n$  and  $E_{z1}(r_1) = 0$ .

As the layers are thin we can write  $d_i = r_i - r_{i-1} \ll r_i$  and

$$(7) E_{zi}(r_i) = E_{zi}(r_{i-1}) + d_i E'_z(r_{i-1})$$

From Eq. (1), (5), (6) and (7) we obtain

$$(8) \frac{k_{i+1}^2}{\epsilon_{i+1}} \frac{E_{z(i+1)}(r_i)}{E'_{z(i+1)}(r_i)} = \frac{k_i^2}{\epsilon_i} d_i + \frac{k_i^2 E_{zi}(r_{i-1})}{\epsilon_i E'_{zi}(r_{i-1})}$$

After several following substitutions for dielectrical "O-guide" we obtain equation for  $k_o$

$$(9) \frac{x}{4\pi^2} \left[ \frac{K_0(x)}{K_1(x)} - \frac{I_0(x)}{I_1(x)} + xb \right] = \frac{r_n}{\lambda^2} \sum_{i=1}^n \left( \mu_{ir} - \frac{1}{\epsilon_{ir}} \right) d_i$$

with  $b = \frac{1}{r_n} \sum_{i=1}^n \frac{d_i}{\epsilon_{ir}} ; x = k_o r_n$

For G-line analogically we obtain

$$(10) \frac{x}{4\pi^2} \left[ \frac{K_0(x)}{K_1(x)} - xb \right] = \frac{r_n}{\lambda^2} \sum_{i=1}^n (M_{ir} - \frac{1}{\epsilon_{ir}}) d_i$$

The left sides of Eq(9) and (10) are plotted as  $S_1$  and  $S_2$  respectively in Fig.2.

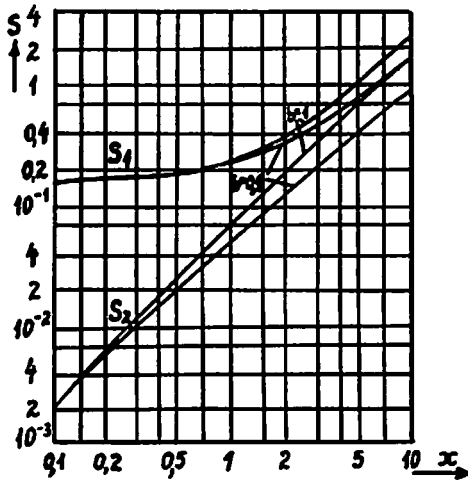


Fig.2

2. Multilayered G-line in case  $k_i r_n \ll 1^2$ . The functions J, N, K and I can be replaced by their asymptotic representations. With (1), (2), (5) and (6) the constants  $B_i$  may be expressed by  $B_{i-1}$

$$(11) \frac{k_i^2}{\epsilon_i} B_i = \frac{k_{i-1}^2}{\epsilon_{i-1}} \left[ B_{i-1} - \frac{2}{\pi} \ln(0,89 k_{i-1} r_i) \right] + \ln(0,89 k_i r_i)$$

$$(12) B_1 = \frac{2}{\pi} \ln(0,89 k_1 r_0).$$

From the ratio  $E_z/H_\varphi$  at the surface of the coat ( $r=r_n$ ), (11) and (12) we get the relation

$$(13) \left( \frac{k_0 r_n}{2\pi} \right)^2 \ln(0,89 k_0 r_n) =$$

$$= \left( \frac{r_n}{\lambda} \right)^2 \sum_{i=1}^n (M_{ir} - \frac{1}{\epsilon_{ir}}) \ln \frac{r_i}{r_{i-1}}$$

The left side of (13) is plotted by Goubau<sup>2</sup>.

3. Flat multilayered metal-dielectrical surface-wave system in case  $k_i y_n \ll 1$  (Fig.3). By substituting in the upper equations

$\varphi$  with  $x$ ,  $r$  with  $y$  and Bessel functions with exponents we obtain analogically

$$(14) E_{zi} = A_i \left[ e^{-jk_i y} + B_i e^{jk_i y} \right] e^{-\gamma z + j\omega t}$$

$$(15) \frac{k_{i+1}}{\epsilon_{i+1}} (B_{i+1} - 1) = j2y_i \frac{k_{i+1}^2}{\epsilon_{i+1}} - j2y_i \frac{k_i^2}{\epsilon_i} + \frac{k_i}{\epsilon_i} (B_i - 1)$$

$$(16) k_0 = \left( \frac{2\pi}{\lambda} \right)^2 \sum_{i=1}^n (M_{ir} - \frac{1}{\epsilon_{ir}}) d_i$$

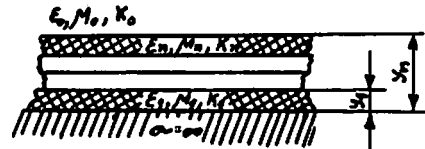


Fig.3

4. Conclusions. It is seen that each layer affects independently from other layers the propagation constants  $k_0$ . The Eq.(13) may be used in meter, decimeter and centimeter range and Eq.(10) in millimeter range.

#### 5. References.

1. J.A. Stratton, Electromagnetic Theory (Mc-Graw-Hill Book Company, Inc. New York, 1941) pp. 516-517.
2. G. Goubau, J. Appl. Phys. 21. 1119 (1950).