3-D ADI-FDTD Method with Fewer For-Loops

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Abstract

This paper presents an alternative implementation of the three-dimensional (3-D) alternating direction implicit finite-difference time-domain (ADI-FDTD) method. The method has fewer for-loops than the conventional ADI-FDTD method. Furthermore, it involves the same total flops count like conventional ADI-FDTD, which is less than both Crank-Nicolson direct-splitting (CNDS) and Crank-Nicolson-cycle sweep-uniform (CNCSU) methods. The formulation of the present method with fewer for-loops is described and detailed comparison among various methods in flops and loops count is discussed.

1. Introduction

There has been considerable interest in the numerical solutions of electromagnetic wave problems based on finite-difference time-domain (FDTD) methods [1]. One of the celebrated FDTD method is that developed by Yee [2], which is an explicit but conditionally stable scheme with time-step size constrained by Courant-Friedrich-Levy (CFL) condition. To remove the CFL constraint, the unconditionally stable FDTD method based on the alternating direction implicit (ADI) technique has been developed [3], [4]. This method has been demonstrated to be useful for problems with very fine meshes relative to wavelength. Recently, alternative unconditionally stable methods have been devised especially those based on Crank-Nicolson (CN) schemes [5]. In particular, the Crank-Nicolson direct-splitting (CNDS) and Crank-Nicolson-cycle sweep-uniform (CNCSU) methods have been developed and investigated. The former method has the same numerical dispersion as ADI-FDTD while the latter method can have smaller anisotropy.

Apart from different numerical accuracy or anisotropy, the computation efficiency of ADI and CN methods is also different. Often the count of floating-point operations (flops) is used to describe their computation costs (and hence efficiency). In addition, one should also consider the for-loop overheads incurred in most programming languages [5]. A for-loop, especially that of a nested one, consumes some CPU time and has been used occasionally as a simple delay in some programming instances. It has been found that both CNDS and CNCSU methods have fewer for-loops than ADI-FDTD even though they require more flops count. For maximum efficiency, it will be desirable to have as few loops as possible in implementing a FDTD method, and preferably not at the expense of more arithmetic operations.

In this paper, an alternative implementation of ADI-FDTD method is presented. The method has fewer for-loops than the conventional ADI-FDTD method of [3], [4]. Furthermore, it involves the same total flops count like conventional ADI-FDTD, which is less than both CNDS and CNCSU methods. Section II describes the formulation of the present method with fewer for-loops, while Section III provides a detailed comparison among various methods in flops and loops count. For generality and completeness, we shall consider the method for three-dimensional (3-D) case in the following.

2. Formulation

The conventional ADI-FDTD method involves two updating procedures for advancement of time steps from n to $n+\frac{1}{2}$ and from $n+\frac{1}{2}$ to $n+1$. Each procedure may consist of implicit updating of electric (magnetic) field components and explicit updating of magnetic (electric) field components. For instance, the implicit updating equations for E_x read

$$\begin{aligned}
&-\frac{a_1a_2}{\Delta y^2}E_x\big|_{i+\frac{1}{2},j-1,k}^{n+\frac{1}{2}} + \Big(1+\frac{2a_1a_2}{\Delta y^2}\Big)E_x\big|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} \\
&\quad -\frac{a_1a_2}{\Delta y^2}E_x\big|_{i+\frac{1}{2},j+1,k}^{n+\frac{1}{2}} = E_x\big|_{i+\frac{1}{2},j,k}^{n} \\
&\quad + \frac{a_1}{\Delta y}\big(H_z\big|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n} - H_z\big|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n}\big) \\
&\quad - \frac{a_1}{\Delta z}\big(H_y\big|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n} - H_y\big|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n}\big) \\
&\quad - \frac{a_1a_2}{\Delta x\Delta y}\big(E_y\big|_{i+1,j+\frac{1}{2},k}^{n} - E_y\big|_{i,j+\frac{1}{2},k}^{n} \\
&\qquad - E_y\big|_{i+1,j-\frac{1}{2},k}^{n} + E_y\big|_{i,j-\frac{1}{2},k}^{n}\big) \qquad (1)
\end{aligned}$$

$$\begin{aligned}
&-\frac{a_1a_2}{\Delta z^2}E_x\big|_{i+\frac{1}{2},j,k-1}^{n+1} + \Big(1+\frac{2a_1a_2}{\Delta z^2}\Big)E_x\big|_{i+\frac{1}{2},j,k}^{n+1} \\
&\quad -\frac{a_1a_2}{\Delta z^2}E_x\big|_{i+\frac{1}{2},j,k+1}^{n+1} = E_x\big|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} \\
&\quad + \frac{a_1}{\Delta y}\big(H_z\big|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z\big|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}\big) \\
&\quad - \frac{a_1}{\Delta z}\big(H_y\big|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y\big|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}\big) \\
&\quad - \frac{a_1a_2}{\Delta x\Delta z}\big(E_z\big|_{i+1,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_z\big|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} \\
&\qquad - E_z\big|_{i+1,j,k-\frac{1}{2}}^{n+\frac{1}{2}} + E_z\big|_{i,j,k-\frac{1}{2}}^{n+\frac{1}{2}}\big) \qquad (2)
\end{aligned}$$

while the explicit updating equations for H_x are

$$H_x|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} = H_x|^{n}_{i,j+\frac{1}{2},k+\frac{1}{2}} + \frac{a_2}{\Delta z}\big(E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1} - E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k}\big) - \frac{a_2}{\Delta y}\big(E_z|^{n}_{i,j+1,k+\frac{1}{2}} - E_z|^{n}_{i,j,k+\frac{1}{2}}\big) \quad (3)$$

$$H_x|^{n+1}_{i,j+\frac{1}{2},k+\frac{1}{2}} = H_x|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} + \frac{a_2}{\Delta z}\big(E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1} - E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k}\big) - \frac{a_2}{\Delta y}\big(E_z|^{n+1}_{i,j+1,k+\frac{1}{2}} - E_z|^{n+1}_{i,j,k+\frac{1}{2}}\big) \quad (4)$$

where

$$a_1 = \frac{\Delta t}{2\epsilon}, \qquad a_2 = \frac{\Delta t}{2\mu}. \quad (5)$$

The updating equations for other field components may be referred to [1], [3], [4]. There are altogether 12 for-loops needed to perform the sweep along x, y and z-directions for all implicit and explicit updating equations in both procedures. To reduce the for-loops, some of the explicit updating equations may be omitted.

Consider the right-hand side of (2), there are magnetic field components H_y and H_z evaluated at time step $n+\frac{1}{2}$. These field components may be eliminated by substituting their explicit updating equations for

$$H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} = H_y|^{n}_{i+\frac{1}{2},j,k+\frac{1}{2}} + \frac{a_2}{\Delta x}\big(E_z|^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}} - E_z|^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}}\big) - \frac{a_2}{\Delta z}\big(E_x|^{n}_{i+\frac{1}{2},j,k+1} - E_x|^{n}_{i+\frac{1}{2},j,k}\big) \quad (6)$$

$$H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} = H_z|^{n}_{i+\frac{1}{2},j+\frac{1}{2},k} + \frac{a_2}{\Delta y}\big(E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k} - E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k}\big) - \frac{a_2}{\Delta x}\big(E_y|^{n}_{i+1,j+\frac{1}{2},k} - E_y|^{n}_{i,j+\frac{1}{2},k}\big), \quad (7)$$

and correspondingly for $H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k-\frac{1}{2}}$, $H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}$. Upon this substitution and some simplifications, the implicit updating equation for $E_x|^{n+1}$ can be obtained as

$$\begin{aligned}
&-\frac{a_1a_2}{\Delta z^2}E_x|^{n+1}_{i+\frac{1}{2},j,k-1} + \Big(1+\frac{2a_1a_2}{\Delta z^2}\Big)E_x|^{n+1}_{i+\frac{1}{2},j,k} - \frac{a_1a_2}{\Delta z^2}E_x|^{n+1}_{i+\frac{1}{2},j,k+1} = \Big(1-\frac{2a_1a_2}{\Delta y^2}\Big)E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} \\
&+ \frac{a_1a_2}{\Delta y^2}\big(E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k} + E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-1,k}\big) \\
&- \frac{2a_1a_2}{\Delta x\Delta z}\big(E_z|^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}} - E_z|^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}} - E_z|^{n+\frac{1}{2}}_{i+1,j,k-\frac{1}{2}} + E_z|^{n+\frac{1}{2}}_{i,j,k-\frac{1}{2}}\big) \\
&+ \frac{a_1a_2}{\Delta z^2}\big(E_x|^{n}_{i+\frac{1}{2},j,k+1} - E_x|^{n}_{i+\frac{1}{2},j,k} - E_x|^{n}_{i+\frac{1}{2},j,k} + E_x|^{n}_{i+\frac{1}{2},j,k-1}\big) \\
&+ \frac{a_1}{\Delta y}\big(H_z|^{n}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z|^{n}_{i+\frac{1}{2},j-\frac{1}{2},k}\big) - \frac{a_1}{\Delta z}\big(H_y|^{n}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y|^{n}_{i+\frac{1}{2},j,k-\frac{1}{2}}\big) \\
&- \frac{a_1a_2}{\Delta x\Delta y}\big(E_y|^{n}_{i+1,j+\frac{1}{2},k} - E_y|^{n}_{i,j+\frac{1}{2},k} - E_y|^{n}_{i+1,j-\frac{1}{2},k} + E_y|^{n}_{i,j-\frac{1}{2},k}\big). \quad (8)
\end{aligned}$$

Similar manipulations can be performed to derive the implicit updating equations for other electric field components $E_y|^{n+1}$ and $E_z|^{n+1}$.

Although (8) does not involve magnetic field components at $n+\frac{1}{2}$, it still contains many more terms that call for substantial arithmetic operations. To reduce the terms, we recognize that some of them appear concurrently in the right-hand sides of both (1) and (8). By keeping these terms to avoid recalculations, we arrive at the implicit updating equations for E_x as

$$\begin{aligned}
e_x|^{n}_{i+\frac{1}{2},j,k} = {}& \frac{a_1}{\Delta y}\big(H_z|^{n}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z|^{n}_{i+\frac{1}{2},j-\frac{1}{2},k}\big) - \frac{a_1}{\Delta z}\big(H_y|^{n}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y|^{n}_{i+\frac{1}{2},j,k-\frac{1}{2}}\big) \\
&- \frac{a_1a_2}{\Delta x\Delta y}\big(E_y|^{n}_{i+1,j+\frac{1}{2},k} - E_y|^{n}_{i,j+\frac{1}{2},k} - E_y|^{n}_{i+1,j-\frac{1}{2},k} + E_y|^{n}_{i,j-\frac{1}{2},k}\big) \quad (9)
\end{aligned}$$

$$-\frac{a_1a_2}{\Delta y^2}E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-1,k} + \Big(1+\frac{2a_1a_2}{\Delta y^2}\Big)E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} - \frac{a_1a_2}{\Delta y^2}E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k} = E_x|^{n}_{i+\frac{1}{2},j,k} + e_x|^{n}_{i+\frac{1}{2},j,k} \quad (10)$$

$$\begin{aligned}
&-\frac{a_1a_2}{\Delta z^2}E_x|^{n+1}_{i+\frac{1}{2},j,k-1} + \Big(1+\frac{2a_1a_2}{\Delta z^2}\Big)E_x|^{n+1}_{i+\frac{1}{2},j,k} - \frac{a_1a_2}{\Delta z^2}E_x|^{n+1}_{i+\frac{1}{2},j,k+1} = \Big(1-\frac{2a_1a_2}{\Delta y^2}\Big)E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} \\
&+ \frac{a_1a_2}{\Delta y^2}\big(E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k} + E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-1,k}\big) \\
&- \frac{2a_1a_2}{\Delta x\Delta z}\big(E_z|^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}} - E_z|^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}} - E_z|^{n+\frac{1}{2}}_{i+1,j,k-\frac{1}{2}} + E_z|^{n+\frac{1}{2}}_{i,j,k-\frac{1}{2}}\big) \\
&+ \frac{a_1a_2}{\Delta z^2}\big(E_x|^{n}_{i+\frac{1}{2},j,k+1} - E_x|^{n}_{i+\frac{1}{2},j,k} - E_x|^{n}_{i+\frac{1}{2},j,k} + E_x|^{n}_{i+\frac{1}{2},j,k-1}\big) \\
&+ e_x|^{n}_{i+\frac{1}{2},j,k}. \quad (11)
\end{aligned}$$

Note that (9) should be incorporated within the loop of (10) to avoid introducing additional loops. In the similar manner, other implicit updating equations can be derived for E_y as

$$\begin{aligned}
e_y|^{n}_{i,j+\frac{1}{2},k} = {}& \frac{a_1}{\Delta z}\big(H_x|^{n}_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_x|^{n}_{i,j+\frac{1}{2},k-\frac{1}{2}}\big) - \frac{a_1}{\Delta x}\big(H_z|^{n}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z|^{n}_{i-\frac{1}{2},j+\frac{1}{2},k}\big) \\
&- \frac{a_1a_2}{\Delta y\Delta z}\big(E_z|^{n}_{i,j+1,k+\frac{1}{2}} - E_z|^{n}_{i,j,k+\frac{1}{2}}
\end{aligned}$$

$$- E_z|^n_{i,j+1,k-\frac{1}{2}} + E_z|^n_{i,j,k-\frac{1}{2}}\big) \quad (12)$$

$$-\frac{a_1a_2}{\Delta z^2}E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-1} + \Big(1+\frac{2a_1a_2}{\Delta z^2}\Big)E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k} - \frac{a_1a_2}{\Delta z^2}E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1} = E_y|^n_{i,j+\frac{1}{2},k} + e_y|^n_{i,j+\frac{1}{2},k} \quad (13)$$

$$\begin{aligned}-\frac{a_1a_2}{\Delta x^2}E_y|^{n+1}_{i-1,j+\frac{1}{2},k} &+ \Big(1+\frac{2a_1a_2}{\Delta x^2}\Big)E_y|^{n+1}_{i,j+\frac{1}{2},k} - \frac{a_1a_2}{\Delta x^2}E_y|^{n+1}_{i+1,j+\frac{1}{2},k} = \Big(1-\frac{2a_1a_2}{\Delta z^2}\Big)E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k}\\ &+ \frac{a_1a_2}{\Delta z^2}\big(E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1} + E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-1}\big)\\ &- \frac{2a_1a_2}{\Delta x\Delta y}\big(E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k} - E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} - E_x|^{n+\frac{1}{2}}_{i-\frac{1}{2},j+1,k} + E_x|^{n+\frac{1}{2}}_{i-\frac{1}{2},j,k}\big)\\ &+ \frac{a_1a_2}{\Delta x^2}\big(E_y|^n_{i+1,j+\frac{1}{2},k} - E_y|^n_{i,j+\frac{1}{2},k} - E_y|^n_{i,j+\frac{1}{2},k} + E_y|^n_{i-1,j+\frac{1}{2},k}\big)\\ &+ e_y|^n_{i,j+\frac{1}{2},k} \quad (14)\end{aligned}$$

and for E_z as

$$\begin{aligned}e_z|^n_{i,j,k+\frac{1}{2}} &= \frac{a_1}{\Delta x}\big(H_y|^n_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y|^n_{i-\frac{1}{2},j,k+\frac{1}{2}}\big) - \frac{a_1}{\Delta y}\big(H_x|^n_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_x|^n_{i,j-\frac{1}{2},k+\frac{1}{2}}\big)\\ &- \frac{a_1a_2}{\Delta x\Delta z}\big(E_x|^n_{i+\frac{1}{2},j,k+1} - E_x|^n_{i+\frac{1}{2},j,k} - E_x|^n_{i-\frac{1}{2},j,k+1} + E_x|^n_{i-\frac{1}{2},j,k}\big) \quad (15)\end{aligned}$$

$$-\frac{a_1a_2}{\Delta x^2}E_z|^{n+\frac{1}{2}}_{i-1,j,k+\frac{1}{2}} + \Big(1+\frac{2a_1a_2}{\Delta x^2}\Big)E_z|^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}} - \frac{a_1a_2}{\Delta x^2}E_z|^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}} = E_z|^n_{i,j,k+\frac{1}{2}} + e_z|^n_{i,j,k+\frac{1}{2}} \quad (16)$$

$$\begin{aligned}-\frac{a_1a_2}{\Delta y^2}E_z|^{n+1}_{i,j-1,k+\frac{1}{2}} &+ \Big(1+\frac{2a_1a_2}{\Delta y^2}\Big)E_z|^{n+1}_{i,j,k+\frac{1}{2}} - \frac{a_1a_2}{\Delta y^2}E_z|^{n+1}_{i,j+1,k+\frac{1}{2}} = \Big(1-\frac{2a_1a_2}{\Delta x^2}\Big)E_z|^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}}\\ &+ \frac{a_1a_2}{\Delta x^2}\big(E_z|^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}} + E_z|^{n+\frac{1}{2}}_{i-1,j,k+\frac{1}{2}}\big)\\ &- \frac{2a_1a_2}{\Delta y\Delta z}\big(E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1} - E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k} - E_y|^{n+\frac{1}{2}}_{i,j-\frac{1}{2},k+1} + E_y|^{n+\frac{1}{2}}_{i,j-\frac{1}{2},k}\big)\\ &+ \frac{a_1a_2}{\Delta y^2}\big(E_z|^n_{i,j+1,k+\frac{1}{2}} - E_z|^n_{i,j,k+\frac{1}{2}} - E_z|^n_{i,j,k+\frac{1}{2}} + E_z|^n_{i,j-1,k+\frac{1}{2}}\big)\\ &+ e_z|^n_{i,j,k+\frac{1}{2}}. \quad (17)\end{aligned}$$

For the magnetic field components, their explicit updating equations are

$$\begin{aligned}H_x|^{n+1}_{i,j+\frac{1}{2},k+\frac{1}{2}} &= H_x|^n_{i,j+\frac{1}{2},k+\frac{1}{2}} + \frac{2a_2}{\Delta z}\big(E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1} - E_y|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k}\big)\\ &- \frac{a_2}{\Delta y}\big(E_z|^{n+1}_{i,j+1,k+\frac{1}{2}} - E_z|^{n+1}_{i,j,k+\frac{1}{2}} + E_z|^n_{i,j+1,k+\frac{1}{2}} - E_z|^n_{i,j,k+\frac{1}{2}}\big) \quad (18)\end{aligned}$$

Table 1: Flops and Loops Count for FDTD Methods

FDTD Method		(9)-(20)	ADI	CNDS	CNCSU
Implicit	M/D	21	18	21	30
	A/S	57	48	63	96
Explicit	M/D	6	12	6	6
	A/S	18	24	24	24
Total	M/D	27	30	27	36
	A/S	75	72	87	110
Loops		9	12	9+1	9+1

$$\begin{aligned}H_y|^{n+1}_{i+\frac{1}{2},j,k+\frac{1}{2}} &= H_y|^n_{i+\frac{1}{2},j,k+\frac{1}{2}} + \frac{2a_2}{\Delta x}\big(E_z|^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}} - E_z|^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}}\big)\\ &- \frac{a_2}{\Delta z}\big(E_x|^{n+1}_{i+\frac{1}{2},j,k+1} - E_x|^{n+1}_{i+\frac{1}{2},j,k} + E_x|^n_{i+\frac{1}{2},j,k+1} - E_x|^n_{i+\frac{1}{2},j,k}\big) \quad (19)\end{aligned}$$

$$\begin{aligned}H_z|^{n+1}_{i+\frac{1}{2},j+\frac{1}{2},k} &= H_z|^n_{i+\frac{1}{2},j+\frac{1}{2},k} + \frac{2a_2}{\Delta y}\big(E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k} - E_x|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k}\big)\\ &- \frac{a_2}{\Delta x}\big(E_y|^{n+1}_{i+1,j+\frac{1}{2},k} - E_y|^{n+1}_{i,j+\frac{1}{2},k} + E_y|^n_{i+1,j+\frac{1}{2},k} - E_y|^n_{i,j+\frac{1}{2},k}\big). \quad (20)\end{aligned}$$

Note that there is no more explicit updating equation required at half time step like (3), (6) or (7). Equations (9)-(20) constitute the ADI-FDTD method with nine for-loops, which are fewer than those required in the conventional method [3], [4].

3. Discussion and Comparison

To provide more detailed assessment of the method above, let us acquire the flops count taking into account the number of multiplications/divisions (M/D) and additions/subtractions (A/S) required for one complete time step. Table 1 lists the flops count for the present and conventional methods [3], [4], based on the right-hand sides of their respective updating equations, cf. (9)-(20) and (1)-(4) etc. For simplicity, the number of electric and magnetic field components in all directions have been taken to be the same. It is also assumed that all multiplicative factors have been precomputed and stored. From the table, it is clear that the present method does not cost more total flops count despite having fewer for-loops than the conventional method. Also listed in the table are the flops and loops requirements for CNDS and CNCSU methods [5]. Both methods also have fewer for-loops but require more flops than conventional ADI-FDTD.

In addition, according to [5], one extra loop is needed to store the field components at time step n. In the present method, the array pointers to the field components at time step n and $n+1$ are alternated and indexed according to whether n is odd or even. For instance, the code for such indexing may take the form

$$\mathrm{n}_0 = 2 - \mathrm{mod}(n, 2) \tag{21}$$

$$\mathrm{n}_1 = 2 - \mathrm{mod}(n + 1, 2). \tag{22}$$

Then subsequent field array variables, say $\mathrm{E}(\mathrm{n}_0)$ and $\mathrm{E}(\mathrm{n}_1)$, will refer to $E|^n$ and $E|^{n+1}$ respectively. This circumvents the need to transfer the field values and saves the extra loop cost. It should be noted that although the present method has fewer loops than conventional ADI and other CN methods while involving the same or fewer flops, such advantage should not be over-emphasized [5]. There are other computation costs including the solutions of tridiagonal systems for implicit schemes as well as memory indexing and access, etc., which often depend on the actual code arrangement and the level of compiler optimization. Reducing all these costs especially during run-time remains a challenging topic to be investigated further.

4. Conclusion

This paper has presented an alternative implementation of the 3-D ADI-FDTD method. The method has fewer for-loops than the conventional ADI-FDTD method. Furthermore, it involves the same total flops count like conventional ADI-FDTD, which is less than both CNDS and CNCSU methods. The formulation of the present method with fewer for-loops has been described and detailed comparison among various methods in flops and loops count has been discussed. Apart from for-loops, other aspects of implementation should also be enhanced whenever possible in order to improve the overall performance of the ADI-FDTD method.

References

[1] A. Taflove and S. C. Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method (Boston, M. A.: Artech House, 2000).

[2] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," IEEE Trans. Antennas Propagat., vol. 14, pp. 302-307, 1966.

[3] F. Zheng, Z. Chen and J. Zhang, "Toward the Development of a Three-Dimensional Unconditionally Stable Finite-Difference Time-Domain Method," IEEE Trans. Microwave Theory Tech., vol. 48, pp. 1550-1558, 2000.

[4] T. Namiki, "3-D ADI-FDTD Method – Unconditionally Stable Time-Domain Algorithm for Solving Full Vector Maxwell's Equations," IEEE Trans. Microwave Theory Tech., vol. 48, pp. 1743-1748, 2000.

[5] G. Sun and C. W. Trueman, "Efficient Implementations of the Crank-Nicolson Scheme for the Finite-Difference Time-Domain Method," IEEE Trans. Microwave Theory Tech., vol. 54, pp. 2275-2284, 2006.