Analyses of Nonlinearly Loaded Antennas and Antenna Arrays Using Particle Swarm Algorithm

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Abstract

In this paper, the particle swarm algorithm is applied to the analysis of nonlinearly loaded antennas and antenna arrays. In most cases, the particle swarm algorithm is utilized for optimization problem. However, the particle swarm algorithm is utilized in this study for analyzing a nonlinear electromagnetic problem, but not for optimization. Initially, the scattering of nonlinear loaded antennas is viewed as an equivalent microwave circuit with the circuit parameters representing the antenna structure, incident wave and nonlinear load. The analysis of this equivalent microwave circuit is then transformed into the optimization of a scalar fitness function to which the particle swarm algorithm can be applied. Numerical examples show that the results calculated in this study are consistent with those given by other existing studies. The analysis of such a nonlinear problem using the particle swarm algorithm is very straightforward and is found to be more efficient than other stochastic approaches. Therefore, it can also be applied to the analyses of many other nonlinear problems in electromagnetic waves.

1. Introduction

Nonlinearly loaded antennas mean that the input terminals of antennas are attached by nonlinear electronic devices to yield the desired scattering characteristics. There have been many studies for the analyses of a single nonlinearly loaded antenna element [1-9] and antenna arrays [10-13]. In general, the analysis for such a problem is difficult due to the addition of device nonlinearity to the inherently complicated antenna theory. This motivates us to develop a general and straightforward approach for solving this type of nonlinear problems.

In this paper, analyses of nonlinearly loaded antennas are given using the particle swarm algorithm [14]. Initially, the analysis is replaced by solving the equivalent problem of a nonlinear microwave circuit with the circuit parameters representing the antenna structure, incident wave and nonlinear electronic device [10-13]. These equivalent circuit parameters representing the antenna structure and incident wave can be calculated by the moment method [15]. By applying Kirchhoff's circuit law, this equivalent circuit can be formulated into an optimization problem with a scalar fitness (or cost) function. This fitness function represents the sum of currents flowing outward at a specified node and thus should be close to zero finally. The particle swarm algorithm is then utilized to find a set of antenna terminal voltage at different harmonics that makes the fitness function as close to zero as possible.

As the array structure is considered, the equivalent microwave circuit becomes a multi-port linear network with each port attached by a nonlinear electronic device [10-13]. It should be noted that the array mutual coupling effects are included in the multi-port linear network. The mutual coupling mechanisms within the array structure are given and interpreted in detail in [13]. The flow chart for analyzing the problem of such an array structure is very similar to that of the single element case except that the fitness function is properly modified.

The particle swarm algorithm [14] has widespread applications in engineering. This is a new stochastic evolutionary computation technique based on the movement and intelligence from a swarm of particles. It has been shown in certain instances to outperform other stochastic methods of optimization like genetic algorithms [16]. The illustration of particle swarm algorithm and the related applications in electromagnetics were described in [17] in detail. Most of the applications are for optimizing, but not for analyzing nonlinear engineering problems. However, the particle swarm algorithm utilized in this study is for analyzing a nonlinear electromagnetic problem, but not for optimization. To our knowledge, this paper is the first study to apply the particle swarm algorithm to the analysis of such a nonlinear electromagnetic problem. Numerical examples show that the results calculated by the particle swarm algorithm are consistent with those of other existing studies. The analysis in this paper does not require a suitable guess of an initial solution and there exists no gradient operations in the iteration procedures. Therefore, it becomes very straightforward, and easy in formulation and programming. In addition, it is found to be more efficient than other stochastic approaches.

In Section 2, the analysis of a single nonlinearly loaded antenna is investigated using the particle swarm algorithm. In Section 3, the analysis for single element case is extended to array structure. Numerical examples are illustrated in Section 4. Finally, the conclusion is given in Section 5.

2. FORMULATIONS OF SINGLE ELEMENT

Consider a single nonlinearly loaded antenna illuminated by a plane wave E_i, as shown in Fig.1. Similar to the treatment in [3][7][10-13], the analysis becomes an equivalent microwave circuit problem, as shown in Fig.2. The goal is to find a terminal voltage $\overline{V}_s$ at different harmonic frequencies and all the scattering characteristics can be obtained through $\overline{V}_s$. In Fig.2, the $\overline{I}_{eq}$ is the short-circuit current at the antenna terminal due to the incident wave, the $\overline{I}_{sN}$ is the current of the nonlinear load, and $\overline{\overline{Y}}$ denotes the antenna input admittances at different harmonic frequencies. From Kirchhoff's circuit law, we have an error vector $\overline{\varepsilon}$

$$\overline{\varepsilon} = \overline{\overline{Y}}\,\overline{V}_s - \overline{I}_{eq} + \overline{I}_{sN} \rightarrow \overline{0} \qquad (1)$$

where

$$\overline{V}_s = [V_{s,0} \quad V_{s,1} \quad V_{s,2} \quad \cdots \quad V_{s,2P-1} \quad V_{s,2P}]^T, \qquad (2)$$

$$\overline{I}_{eq} = [0 \quad I_{eq,1} \quad I_{eq,2} \quad \cdots \quad 0 \quad 0]^T, \qquad (3)$$

$$\overline{I}_{sN} = [I_{sN,0} \quad I_{sN,1} \quad I_{sN,2} \quad \cdots \quad I_{sN,2P-1} \quad I_{sN,2P}]^T, \qquad (4)$$

and

$$\overline{\overline{Y}} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & G(\omega_0) & B(\omega_0) & 0 & \vdots & 0 \\ 0 & -B(\omega_0) & G(\omega_0) & \ddots & 0 & \vdots \\ \vdots & 0 & 0 & \ddots & \ddots & 0 \\ 0 & \vdots & \vdots & 0 & G(P\omega_0) & B(P\omega_0) \\ 0 & 0 & \cdots & 0 & -B(P\omega_0) & G(P\omega_0) \end{bmatrix}. \qquad (5)$$

The integer P in (2)-(5) denotes the number of harmonics. The variables in the above equations are defined from

$$v_s(t) = V_{s,0} + \sum_{p=1}^{P} \left\{ V_{s,2p-1}\cos(p\omega_0 t) + V_{s,2p}\, p\sin(p\omega_0 t) \right\}, \qquad (6)$$

$$i_{eq}(t) = I_{eq,1}\cos\omega_0 t + I_{eq,2}\sin\omega_0 t \qquad (7)$$

$$i_{sN}(t) = I_{sN,0} + \sum_{p=1}^{P} \left\{ I_{sN,2p-1}\cos(p\omega_0 t) + I_{sN,2p}\sin(p\omega_0 t) \right\} \qquad (8)$$

and

$$Y(p\omega_0) = G(p\omega_0) + jB(p\omega_0), \quad p = 1, \ldots, P. \qquad (9)$$

where ω_0 is the frequency of the incident wave and p denotes the corresponding harmonic.

Assume the time-domain and frequency-domain variables are related by

$$\begin{cases} \overline{v}_s(t) = \overline{\overline{T}}\,\overline{V}_s \\ \overline{I}_{sN} = \overline{\overline{D}}\,\overline{i}_{sN}(t) \end{cases} \qquad (10)$$

where $\overline{\overline{T}}$ and $\overline{\overline{D}}$ are the related transformation matrices between time and frequency domain. From (1) and (10), we have

$$\overline{\varepsilon} = \overline{\overline{Y}}\,\overline{V}_s - \overline{I}_{eq} + \overline{\overline{D}} f(\overline{\overline{T}}\,\overline{V}_s) \rightarrow \overline{0} \qquad (11)$$

where $f(\cdot)$ is the i-v characteristics of the nonlinear load, i.e.,

$$i_{sN}(t) = f(v_s(t)). \qquad (12)$$

The problem then becomes to find an optimum vector $\overline{V}_s$ in (11) that makes the error vector $\overline{\varepsilon}$ approach zero.

In this study, the particle swarm algorithm is used to find an optimum $\overline{V}_s$ that satisfies (11). The flow chart of the algorithm is shown in Fig.3 and each iteration step is described as below.

Step-1. Determine solution dimension and range

According to (2), the solution of $\overline{V}_s$ has dimension of $N_D = 2P+1$. The range for these N_D (=2P+1) variables of $V_{s,0}$, $V_{s,1}$, …, and $V_{s,2P+1}$ should be determined initially.

Step-2. Determine number of particles

Assume there are M particles in the iteration procedures, i.e., the population size is M. Each particle has its location and velocity with the vector dimension of $N_D = 2P+1$. The location vector of each particle represents $\overline{V}_s$ in our problem, i.e., the solution we want to find. The velocity vector of each particle represents the magnitude and direction for the location variation in the next iteration step. Parametric studies have found that a population size less than 30 is suitable for many engineering problem, i.e., $M \leqq 30$.

Step-3. Determine fitness function

Based on (11), the fitness or cost function of the iteration procedure is defined as

$$fitness = \left\|\overline{\varepsilon}\right\|^2 = \left\|\overline{\overline{Y}}\,\overline{V}_s - \overline{I}_{eq} + \overline{\overline{D}} f(\overline{\overline{T}}\,\overline{V}_s)\right\|^2. \qquad (13)$$

The goal is then to find an optimal set of $\overline{V}_s$ that makes the value of *fitness* in (13) minimum, i.e., positive and as close to 0 as possible.

Step-4. Determine initial locations and velocities of particles

The initial location and velocity (both with N_D =2P+1 dimension) of all the M particles are given randomly.

Step-5. Evaluate the value of fitness

The value of *fitness* for each particle is evaluated by (13).

Step-6. Record the best locations of particles

For the m-th particle, its personal best location, i.e., the location of the m-th particle that produced minimum *fitness* during the current and past iteration loops, is recorded as $pbest_m$. The global best location, i.e., the best location among $pbest_m$, m=1, 2, ⋯, M, is recorded as $gbest$. In other words, $gbest$ is the best location that the M particles ever encountered. This best location produced the globally minimum *fitness* during the current and past iteration loops of all the M particles.

Step-7. Update velocities and locations of particles

According to [18], the velocity v_m and the location x_m of the m-th particle are updated as

$$\begin{cases} v_m = K[v_m + \varphi_1 \cdot rand() \cdot (pbest_m - x_m) + \\ \qquad \varphi_2 \cdot rand() \cdot (gbest - x_m)] \\ x_m = x_m + v_m \cdot \Delta t \end{cases}, \tag{14}$$

where Δt is the time-setp and is usually chosen as 1 second, *rand*() is a random value in the interval of 0 to 1, and K is the constriction factor determined from

$$\begin{cases} \varphi = \varphi_1 + \varphi_2 > 4 \\ K = \dfrac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \end{cases}. \tag{15}$$

In this study, we choose φ_1=2.8 and φ_2=1.3 according to the suggestion of [19].

***Step-8. Check if the stop condition is reached*?**

Go to ***Step-5*** until the stop condition is reached. The stop condition usually means the maximum iteration loop or the minimum *fitness* threshold. In other words, the iteration will continue until the specified maximum loop or minimum *fitness* threshold is reached.

3. FORMULATIONS OF ARRAY STRUCTURES

Consider an N-element nonlinearly loaded antenna array illuminated by a plane wave E_i, as shown in Fig.4. Similar to the treatment in [10-13], the equivalent circuit can be expressed as Fig.5, where port n represents the n-th antenna terminal. The mutual coupling effects are included in the linear network. The circuit equation is the same as (11) and the fitness function is the same as (13) except the definitions of the variables are modified as

$$\overline{V}_s = [V_{s,1,0} \;\; V_{s,1,1} \;\; V_{s,1,2} \;\; \cdots \;\; V_{s,1,2P-1} \;\; V_{s,1,2P} \;\; ;\cdots\cdots\cdots; \;\; V_{s,N,0} \;\; V_{s,N,1} \;\; V_{s,N,2} \;\; \cdots \;\; V_{s,N,2P-1} \;\; V_{s,N,2P}]^T \tag{16}$$

$$\overline{I}_{eq} = [0 \;\; I_{eq,1,1} \;\; I_{eq,1,2} \;\; \cdots \;\; 0 \;\; 0 \;\; ;\cdots\cdots\cdots; \;\; 0 \;\; I_{eq,N,1} \;\; I_{eq,N,2} \;\; \cdots \;\; 0 \;\; 0]^T \tag{17}$$

$$\overline{I}_{sN} = [I_{sN,1,0} \;\; I_{sN,1,1} \;\; I_{sN,1,2} \;\; \cdots \;\; I_{sN,1,2P-1} \;\; I_{sN,1,2P} \;\; ;\cdots\cdots\cdots; \;\; I_{sN,N,0} \;\; I_{sN,N,1} \;\; I_{sNN,2} \;\; \cdots \;\; I_{sN,N,2P-1} \;\; I_{sN,N,2P}]^T \tag{18}$$

and

$$\overline{\overline{Y}} = \begin{bmatrix} \overline{\overline{Y}}_{11} & \cdots & \overline{\overline{Y}}_{1N} \\ \vdots & \ddots & \vdots \\ \overline{\overline{Y}}_{N1} & \cdots & \overline{\overline{Y}}_{NN} \end{bmatrix} \tag{19}$$

where $\overline{\overline{Y}}_{ij}$ denotes the mutual admittances between the i-th and the j-th antenna elements at different harmonics. In (16), the subscripts of $V_{s,n,2p-1}$ and $V_{s,n,2p}$ denote the cosine and sine components of terminal voltage for the n-th antenna at the p-th harmonic. Similarly, subscripts in (17) and (18) also have the same meanings. After the fitness function of (13) is determined, the particle swarm algorithm can be used to find the optimum $\overline{V}_s$ of (16) that satisfies (11) or minimizes (13). The iteration procedures are the same as those given in Section 2.

4. NUMERICAL SIMULATION

In this section, numerical examples are given to illustrate the methods described above. Without loss of generality, the dipole antennas are considered for simplicity since there is no limitation on types of antennas in this study. In the following examples, the dipole antenna has length-to-diameter ratio of 74.2. The incident plane wave in Fig.1 is assumed to be $\overline{E}_i = e^{j(\omega t - kx)}\hat{z}$, where k is the wave number. In other words, the incident wave has E-field direction parallel to the dipole and propagation direction perpendicular to the dipole. Each dipole antenna is loaded with a nonlinear load with the i-v characteristics $f(\cdot)$ given as

$$i = f(v) = \frac{1}{75}v + 4v^3 \cdot \tag{20}$$

The equivalent circuit parameters in Fig.2 are solved by the Pocklington's equation and moment methods [15]. In the particle swarm algorithm, the number of particles is chosen to be M=20. The number of maximum iteration loops is set to be 600.

In the first example, a single nonlinearly loaded antenna is considered. Following the particle swarm algorithm based analysis in Section 2, the final antenna terminal voltage components for different dipole lengths at frequencies of ω_0, $2\omega_0$ and $3\omega_0$ are shown in Fig.6. For comparison, the results calculated from the harmonic balance technique [7,12] are also given. It shows that they are in good agreement. From this example, we are convinced that particle swarm algorithm gives accurate results in the problems of nonlinearly loaded antennas.

In the second example, two parallel dipole antennas with each antenna loaded with a nonlinear load are considered. The nonlinear load is the same as that of the previous example. The dipole length is chosen to be 0.47 λ. Following the particle swarm algorithm based analysis in Section 2 and formulations of array structures in Section 3, the final antenna terminal voltage components for different element spacing at frequencies of ω_0, $2\omega_0$ and $3\omega_0$ are shown in Fig.7. For comparison, the results calculated from the harmonic balance technique [12] are also given. It shows that they are in good agreement. It should be emphasized that the mutual coupling effects between antenna elements are included in the analysis.

5. CONCLUSIONS

In this paper, the particle swarm algorithm is applied to the analyses of nonlinearly loaded antennas and antenna arrays including array mutual coupling effects. The analyses of these structures can be transformed into an optimization problem with a scalar fitness function. This scalar fitness function should be positive and as close to 0 as possible. The final terminal voltages will be obtained as the scalar fitness function is minimized by the particle swarm algorithm. Numerical simulation shows that the results calculated from the proposed methods in this study are consistent with those using the harmonic balance techniques. Since the particle swarm algorithm is inherently a stochastic optimization technique, it does not require a suitable guess of an initial solution and there exists no gradient operations in the iteration procedures. The utilization of the particle swarm algorithm in nonlinear problems of this type is very straightforward, and easy in both formulation and programming. Therefore, the proposed methods in this study can be extended to the analyses of many other nonlinear problems in electromagnetics.

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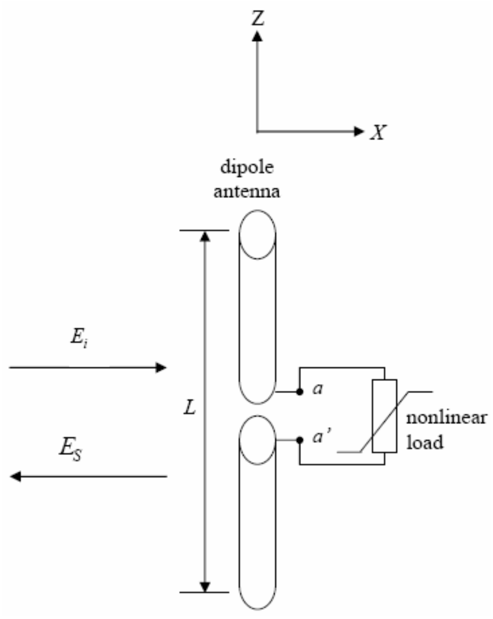

Fig.1 Schematic diagram of a nonlinearly loaded antenna.

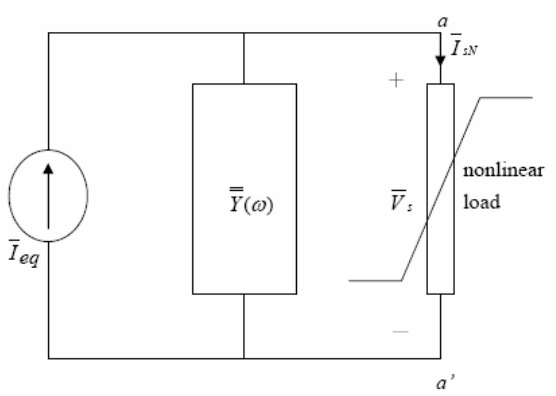

Fig.2 The equivalent microwave circuit of Fig.1.

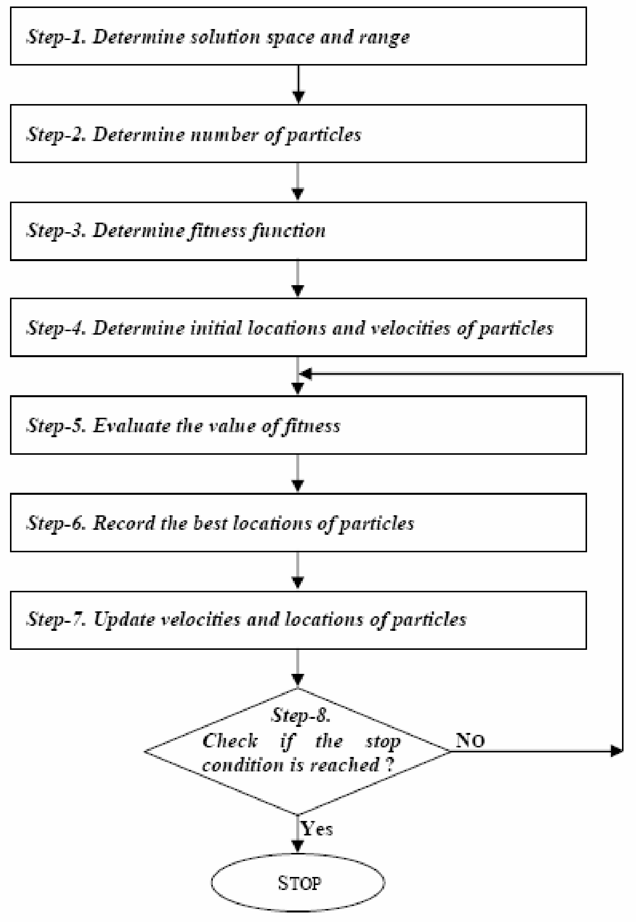

Fig.3 The flow chart of the particle swarm algorithm.

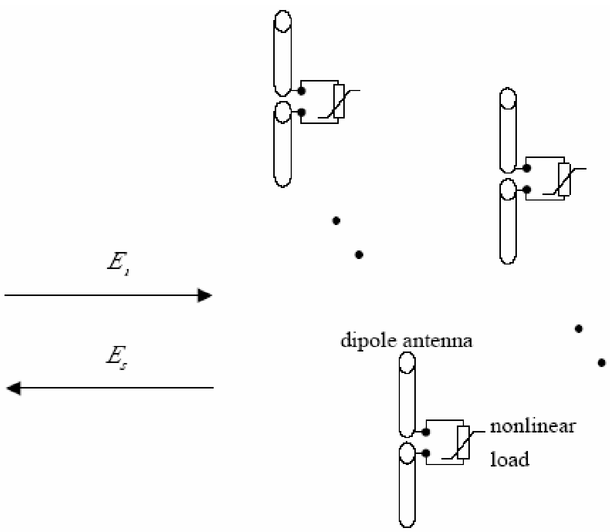

Fig.4 Schematic diagram of a finite nonlinearly loaded antenna array.

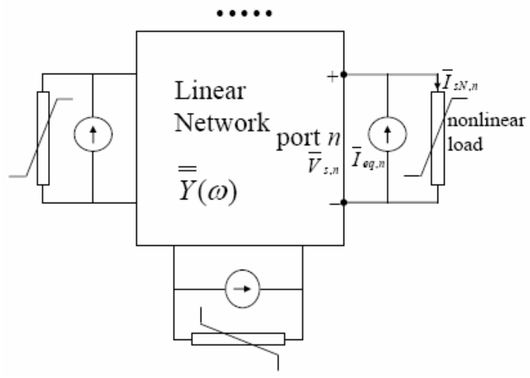

Fig.5 The equivalent microwave circuit of Fig.4.

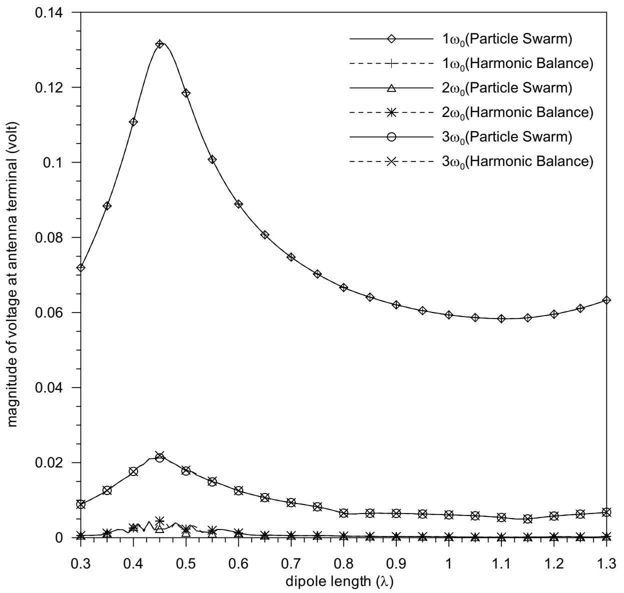

Fig.6 Magnitude of voltage at the input terminal of a single nonlinearly loaded dipole antenna for different dipole lengths at different harmonic frequencies of $1\omega_0$, $2\omega_0$ and $3\omega_0$ by using particle swarm algorithm and harmonic balance technique.

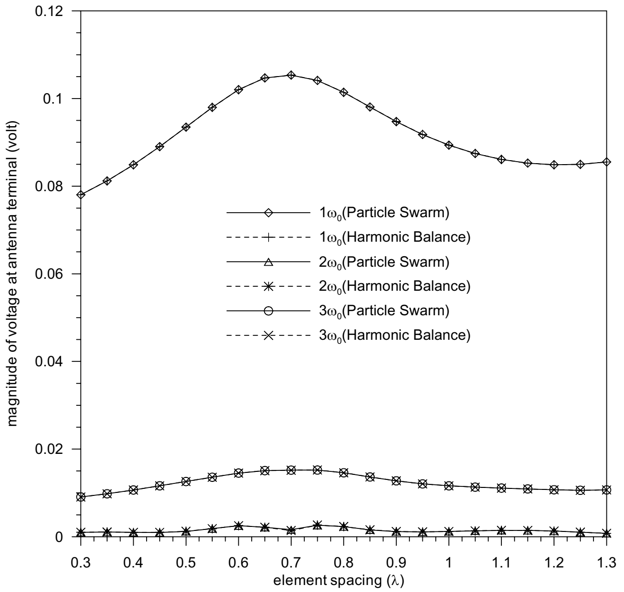

Fig.7 Magnitude of voltage at each input terminal of a nonlinearly loaded dipole antenna array with two parallel dipoles for different array element spacing at different harmonic frequencies of $1\omega_0$, $2\omega_0$ and $3\omega_0$ by using particle swarm algorithm and harmonic balance technique.