A Novel Interpolation Scheme for Full-Wave Electromagnetic Simulations

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Abstract

The conventional Lagrange interpolation scheme, which has demonstrated a very high efficiency in approximating the quasi-static Greens' functions, is not very effective for interpolating full-wave Green's functions due to their highly oscillatory characteristics of the phase term. To alleviate this difficulty, we propose a new full-wave interpolation scheme in which a well-designed phase compensation technique is employed. Applying this technique in 2-D non-uniform Lagrange interpolations, we can approximate the Green's functions in an area of 100×100 λ^2 *with an accuracy of* 10^{-3}*, using only* 90×90 *Chebyshev interpolation points. The proposed scheme can be adopted to build a fast solver for full-wave electromagnetic simulations.*

1. Introduction

Because of their simplicity and efficiency, interpolation schemes enjoy an increasing popularity in Computational Electromagnetics (CEM). For instance, in the Finite-Element Method (FEM) and Method of Moments (MoM), higher-order interpolatory basis functions are respectively used in discretizing the differential and integral equations for obtaining a higher accuracy with less number of unknowns [1]; in the Multilevel Fast Multipole Algorithm (MLFMA), the interpolation is used in the spectral domain [2]; in the Precorrected-FFT Method [3], [4] and Multilevel Green's Function Interpolation Method (MLGFIM) [5], interpolation schemes are adopted to approximate the spatial Green's functions. In MLGFIM, combining with the multilevel discretization and using the conventional uniform Tartan grid, the Lagrange interpolation is applied to approximate the Green's functions, obtaining an $O(N)$ complexity. The theoretical derivations and numerical experiments in [5] sufficiently demonstrated that this approach is highly efficient for solving quasi-static problems. To interpolate the Green's function in a cube generated by a point that is one cube away from this cube, only $3\times3\times3=27$ Tartan grid points are needed. Theoretically, the interpolation scheme can be used for approximating any kind of Green's functions, and therefore MLGFIM can be used for solving any kind of integral equation problems. However, due to the highly-oscillatory nature of the phase term of the Green's functions in full-wave electromagnetic problems, to approximate the Green's function in a cube that is greater than one wavelength (1λ) in edge size, the number of Lagrange interpolation points should be increased [6]. Following the increase of the electrical size of the cube, the number of interpolation points drastically increases to ensure a preset accuracy. This will greatly affect the interpolation efficiency of MLGFIM. To overcome this difficulty, here we introduce a simple but effective phase compensation (PC) scheme, in which the rapidly oscillating phase term of the Green's function is compensated by a corrected phase and thus the oscillating feature is dramatically reduced. Numerical experiments demonstrated that for approximating Green's function values in a 100×100 λ^2 area with an accuracy of 10^{-3}, only 90×90 interpolation points are needed. If we use the conventional Lagrange interpolation without phase compensation, in a 32 bit computer using double precision, the interpolation will diverge when the edge size of the square is greater than 8 λ.

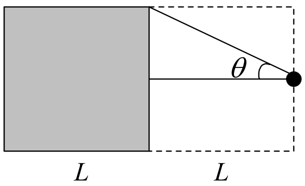
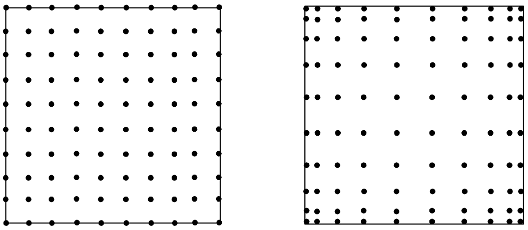


Fig. 1: The numerical experiment sketch.

2. Conventional Green's Function Interpolation

Fig. 1 illustrates the numerical experiment that was performed repeatedly in this paper. The shadowed square with an edge length L is filled with field points. Assuming that there are κ interpolation points along one direction in this square and the source point is one L length away from this square, the conventional Green's function interpolation can be written as



(a) (b)

Fig. 2: Interpolation points. (a) Uniformly spaced grids and (b) Chebyshev grids.

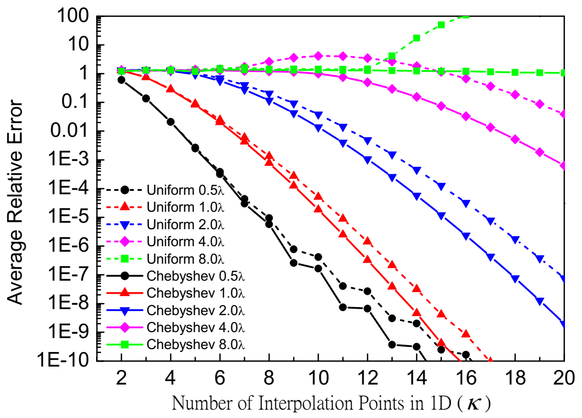


Fig. 3: Average relative error versus the number of interpolation points in one direction κ using the conventional Lagrange interpolation.

$$G(\boldsymbol{r},\boldsymbol{r}') = \sum_{i=1}^{K} \omega_i(\boldsymbol{r}) G(\boldsymbol{r}_i,\boldsymbol{r}'), \tag{1}$$

where the total number of interpolation points $K = \kappa \times \kappa$, the i^{th} interpolation point $i = p \times \kappa + q$, the orthonormal Lagrange basis $\omega_i(\boldsymbol{r}) = L(\kappa, p, x) \cdot L(\kappa, q, y)$, and $L(\cdot)$ is the one dimensional Lagrange basis function and expressed as

$$L(\kappa,\ u,\ t) = \prod_{\substack{s=1 \\ u \neq s}}^{\kappa} (t - t_s)/(t_u - t_s) \cdot \tag{2}$$

Here we use two kinds of interpolation points, viz., uniformly spaced grid and Chebyshev grid as depicted in Figs. 2a and 2b, respectively. The grid points of the former can be expressed as

$$t_s = (s-1)L/(\kappa - 1), \tag{3}$$

and the latter as

$$t_s = \frac{L}{2}\left(1 - \cos\frac{(2s-1)\pi}{2\kappa} \Big/ \cos\frac{\pi}{2\kappa}\right). \tag{4}$$

Fig. 3 shows the average relative error of the conventional Green's function interpolation. We observe that: 1) the interpolation efficiency is low, for instance, for $L = 4\lambda$, 20 Chebyshev interpolation points along one direction is needed to ensure the accuracy of 10^{-3}; 2) in these conventional Lagrange interpolations, using Chebyshev interpolation grids will enhance the accuracy but not significantly; 3) when $L = 8$ λ, the conventional Lagrange interpolation using uniformly spaced grids diverges.

To alleviate these difficulties, we develop a simple but efficient phase compensation scheme, which is shown in next section.

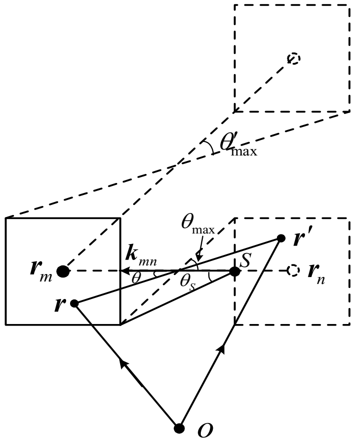


Fig. 4: The Green's function interpolation problem appeared in MLGFIM.

3. Phase Compensation for Green's Function Interpolation

Let's consider the Green's function interpolation problem appeared in MLGFIM in Fig. 4. The source point $\boldsymbol{r}'$ is in square n centred at $\boldsymbol{r}_n$, while the field point $\boldsymbol{r}$ is in square m centred at $\boldsymbol{r}_m$. To interpolate $G(\boldsymbol{r},\boldsymbol{r}') = \exp(ikR)/R$ (where $R = |\boldsymbol{r} - \boldsymbol{r}'|$) in square m, one can use (1). However, here we adopt the following steps instead. First, the Green's function is rewritten as

$$G(\boldsymbol{r},\boldsymbol{r}') = \frac{\exp(ik|\boldsymbol{r}-\boldsymbol{r}'|)}{|\boldsymbol{r}-\boldsymbol{r}'|} \cdot \exp(-ik\boldsymbol{k}_{mn} \cdot (\boldsymbol{r}-\boldsymbol{r}')) \cdot \exp(ik\boldsymbol{k}_{mn} \cdot (\boldsymbol{r}-\boldsymbol{r}')) = \tilde{G} \cdot \exp(ik\boldsymbol{k}_{mn} \cdot (\boldsymbol{r}-\boldsymbol{r}')) \tag{5}$$

In (5),

$$\tilde{G}(\boldsymbol{r},\boldsymbol{r}') = \frac{\exp(ik|\boldsymbol{r}-\boldsymbol{r}'|)}{|\boldsymbol{r}-\boldsymbol{r}'|} \cdot \exp(-ik\boldsymbol{k}_{mn} \cdot (\boldsymbol{r}-\boldsymbol{r}')), \tag{6}$$

where $\boldsymbol{k}_{mn} = (\boldsymbol{r}_m - \boldsymbol{r}_n)/|\boldsymbol{r}_m - \boldsymbol{r}_n|$. $\tilde{G}(\boldsymbol{r},\boldsymbol{r}')$ in (5) and (6) can be viewed as the phase corrected Green's function. Consequently, instead of interpolating the original Green's function, we now interpolate the phase corrected one. Thus (1) can be rewritten as

$$G(\boldsymbol{r},\boldsymbol{r}') = \exp(ik_{mn} \cdot (\boldsymbol{r}-\boldsymbol{r}')) \sum_{i=1}^{K} \omega_i(\boldsymbol{r}) \tilde{G}(\boldsymbol{r}_i,\boldsymbol{r}') \cdot \tag{7}$$

To see if (7) is more efficient than (1), we should study (6), which is rewritten as

$$\tilde{G}(\boldsymbol{r},\boldsymbol{r}') = \frac{\exp(ik|\boldsymbol{r}-\boldsymbol{r}'| - ik\boldsymbol{k}_{mn} \cdot (\boldsymbol{r}-\boldsymbol{r}'))}{|\boldsymbol{r}-\boldsymbol{r}'|} = \frac{\exp(ik|\boldsymbol{r}-\boldsymbol{r}'|(1-\cos\theta))}{|\boldsymbol{r}-\boldsymbol{r}'|} \tag{8}$$

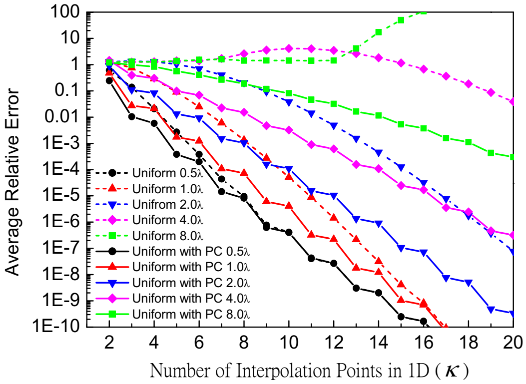


Fig. 5: The average relative errors versus κ for various L using phase compensation with uniform grid points.

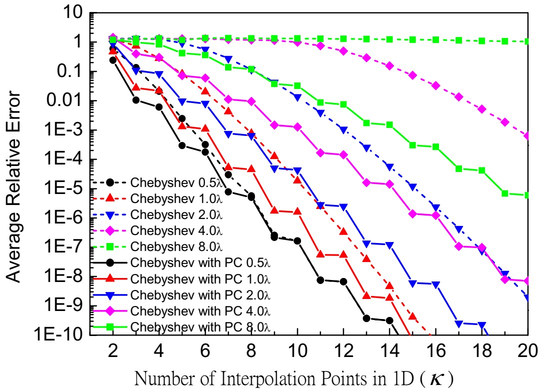


Fig. 6: The average relative errors versus κ for various L ($\leq 8\,\lambda$) using phase compensation with Chebyshev interpolation points.

where θ is the angle between $\boldsymbol{k}_{mn}$ and $\boldsymbol{r}-\boldsymbol{r}'$. Through this phase correction, we expect that the phase-corrected Green's function will oscillate more slowly than the original one. From Fig. 4, we can see that $\theta_{\max}=45^{\circ}$ and $1-\cos\theta \leq 1-\cos\theta_{\max} < 0.293$, implying that the phase term of $\tilde{G}$ is about three times smaller than that of G. If the source point $\boldsymbol{r}'=S$, then $\theta=\theta_s=\tan^{-1}(1/2)$ and $1-\cos\theta_S \approx 0.106$, which means that about 90% of the phase variation is eliminated. For the case that the source square n is shifted to the upper right of Fig. 4, the maximum angle $\theta'_{\max} < \theta_{\max}$. Thus, study of the un-shifted case is sufficient. Clearly, with the same accuracy, interpolating $\tilde{G}$ needs much less number of interpolation points than to interpolate G. We call this the phase compensation scheme in Green's function interpolation.

Because in (7) the remainder part $\exp(ik\boldsymbol{k}_{mn}\cdot(\boldsymbol{r}-\boldsymbol{r}'))$ can be easily expressed as the multiplication between the function of the field point and that of the source point, thus, the phase compensation scheme can be used to develop fast algorithms for full-wave electromagnetic simulations.

4. Numerical Results

To confirm arguments discussed in Section 3, we repeat the experiment shown in Fig. 1. (This can also be viewed as the Green's function in square m generated by S as shown in Fig. 4). Fig. 5 shows the average relative errors versus κ for various L with and without phase compensation. We can see that the interpolation efficiency is significantly enhanced with phase compensation. When L=4λ, adopting phase compensation, only 12 interpolation points are needed in one linear direction to achieve the accuracy of 10^{-3}. Without the compensation, more than 20 interpolation points are needed. When L increases to 8 λ, 20 interpolation points are used with phase compensation. Without it, the results diverge due to the finite machine precision.

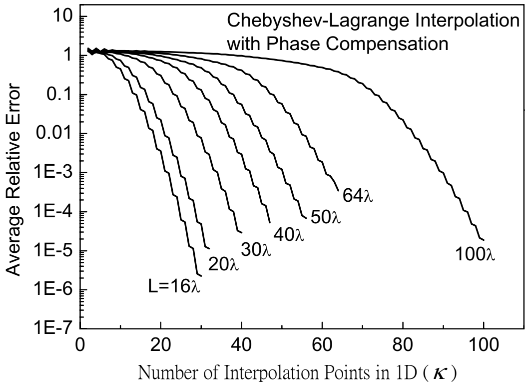


Fig. 7: The average relative errors versus κ for various L ($\geq 16\,\lambda$) using phase compensation with Chebyshev interpolation points.

To further enhance the interpolation efficiency, we also try non-uniform interpolation points, i.e., Chebyshev interpolation points.

Fig. 6 shows the results for cases that L is less than or equal to 8 λ. In this figure, interpolations with phase compensation again obtain a higher accuracy than that without it. We also see that applying non-uniform Chebyshev interpolation points, the interpolation efficiency is enhanced when compared with the results in Fig. 5 in which uniformly spaced interpolation points were used. For instance, only 14 Chebyshev interpolation points are needed as opposed to 20 uniformly spaced points to ensure the 10^{-3} accuracy.

Fig. 7 shows the average relative errors of phase compensation interpolation with Chebyshev interpolation points versus κ when L is greater than or equal to 16 λ. We can see that for a square with an edge size 100 λ, only 90 interpolation points per linear direction are needed to achieve the accuracy of 10^{-3}. This soundly demonstrates the efficacy of the proposed phase compensation scheme for Green's function interpolation.

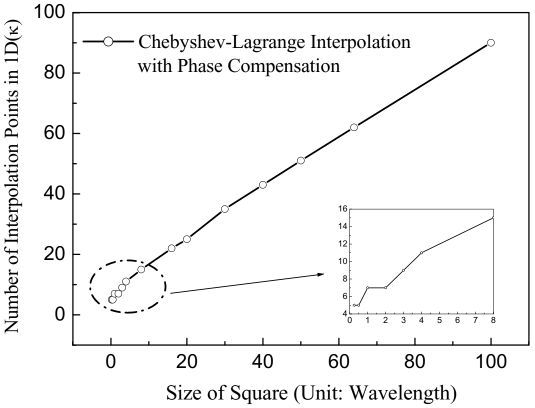


Fig. 8: Number of interpolation points versus L for the accuracy of 10^{-3} using the phase compensation interpolation scheme with Chebyshev grids.

Fig. 8 shows the relationship between edge size of the square and the number of interpolation points needed per linear direction for the interpolation accuracy of 10^{-3} with Chebyshev grids. A linear relationship $\kappa = 0.9L$ is observed.

5. CONCLUSION

In this paper, a simple and efficient phase compensation scheme is proposed to address the highly oscillatory nature of full-wave Green's function. Numerical results demonstrate the efficiency of this scheme. Because of its simplicity and efficiency, it can be used to develop fast solvers for full-wave electromagnetic simulations.

ACKNOWLEDGEMENT

This work is supported by the Hong Kong Research Grant Council Grant No. CityU110606.

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