An Application of Wavelet-Based Method of Moments to Scattering Problems with Discontinuous Impedance Matrix

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Abstract

A huge computation time is required for a method such as the method of moments (MoM) which estimates a physical quantity by descretizing an integral equation to a set of simultaneous linear equations, because its system matrix, impedance matrix in MoM, is dense in general. It has been reported that the wavelet transform can be used to make dense impedance matrix sparse for reduction of the calculation complexity [1]-[6]. *However, if the impedance matrix has discontinuity in the space domain, it oscillates in the spectral domain. Consequently enough sparcification can not be realized by the wavelet transform. In this paper, a new transformation based on a wavelet transform is proposed to overcome the case where the impedance matrix becomes discontinuous in the space domain with validation of its effectiveness.*

1. SPARSIFICATION BY THE WAVELET TRANSFORM

Wavelets are applied in direct and indirect way, that is, the direct way uses wavelets as the testing and the trial functions and the indirect way is performed by the wavelet transform on the impedance matrix. This paper deals with the latter.

In MoM, the integral equation described as

$$\int_S \overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_s(\mathbf{r}')d\mathbf{r}' = -\mathbf{E}^{inc}(\mathbf{r}) \qquad (1)$$

is discretized into the following simultaneous equations

$$\mathbf{ZJ} = \mathbf{E}, \qquad (2)$$

where $\overline{\mathbf{G}}$ is the dyadic green's function, $\mathbf{J}_s$ is the unknown induced current, $\mathbf{E}^{inc}$ is the incident electric field, $\mathbf{Z}$ is the impedance matrix, and $\mathbf{J}$, $\mathbf{E}$ are discretized vector of the induced current and the incident electric field respectively. Since the impedance matrix is usually dense, eq.(2) is transformed into wavelet domain as

$$\mathbf{Z}'\mathbf{J}' = \mathbf{E}'. \qquad (3)$$

Each matrix (vector) is obtained by the discrete wavelet transform as the following equations.

$$\mathbf{Z}' = \mathbf{WZW}^T$$
$$\mathbf{J}' = \mathbf{WJ} \qquad (4)$$
$$\mathbf{E}' = \mathbf{WE}$$

where $\mathbf{W}$ is the wavelet transform matrix which is an orthogonal matrix satisfying $\mathbf{WW}^T = \mathbf{I}$. For the wavelet transform matrix, Daubechies wavelets with the order of 8 are used throughout all the calculations in this paper. Due to the wavelet transform, the impedance matrix in the wavelet domain $\mathbf{Z}'$ becomes sparse, that is, most of its elements are approximately zero. Therefore it can then be thresholded, that is, those elements which have magnitude less than the defined threshold τ are discarded, and the remaining elements of $\mathbf{Z}'$ are stored in sparse matrix form. Although there are several methods for defining of τ, it is defined as in [5],

$$\tau = \alpha \cdot \max_m \left[\sum_n |Z(n,m)| / N \right], \qquad (5)$$

where N is the number of unknowns, and α is a constant value, defined 0.1 in this paper. If the impedance matrix after thresholding is described as $\mathbf{Z}''$, the unknown induced current can be calculated by the following equation,

$$\mathbf{J} = \mathbf{Z}''^{-1}\mathbf{E}. \qquad (6)$$

This kind of equation is usually performed with iteration methods for sparse matrix such as the conjugate gradient method.

2. PROPOSAL

Referring to Kim[1] and Steinberg[2], sparsification by the wavelet transform is achieved when the following two conditions are satisfied:

(a) wavelets have compact support in both the space and the spectral domain.

(b) impedance matrix varies slowly in the spectral domain.

(a) is always satisfied since it is one of the features of the wavelets. However, (b) is not realized when the impedance matrix has discontinuity in the space domain. To explain this situation, a scattering problem with several linear conducting elements is considered as shown in Fig.1. P_x, P_y represents the number of conducting elements for the direction of x, y axes respectively and the total number of the conducting elements P is $P = P_x \times P_y$. Spacing between the elements is λ/2 for x, y axes respectively. All the elements are discretized with spacing of λ/10, its length is 12.8λ, and the total number of sampling points per an element is 128. Electric field is incident from $-y$ direction as a plane wave, and its polarization is $+z$ direction. Fig.2 shows the spectral characteristics of impedance matrix when P = 1, 2. It is confirmed that the result of 1 element case varies slowly, while the result of 2 elements case indicates significant oscillation.

In the above mentioned example, the impedance matrix is written as the following form.

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} & \cdots & \boldsymbol{Z}_{1P} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} & \cdots & \boldsymbol{Z}_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{Z}_{P1} & \boldsymbol{Z}_{P2} & \cdots & \boldsymbol{Z}_{PP} \end{bmatrix} \qquad (7)$$

where $\mathbf{Z}_{pq}$ is the direct or the mutual impedance matrix between p th and q th elements. The boundary between each partial matrices results in discontinuity, and that leads to oscillation in the spectral domain. As a result, sparsification of the impedance matrix can not be achieved enough.

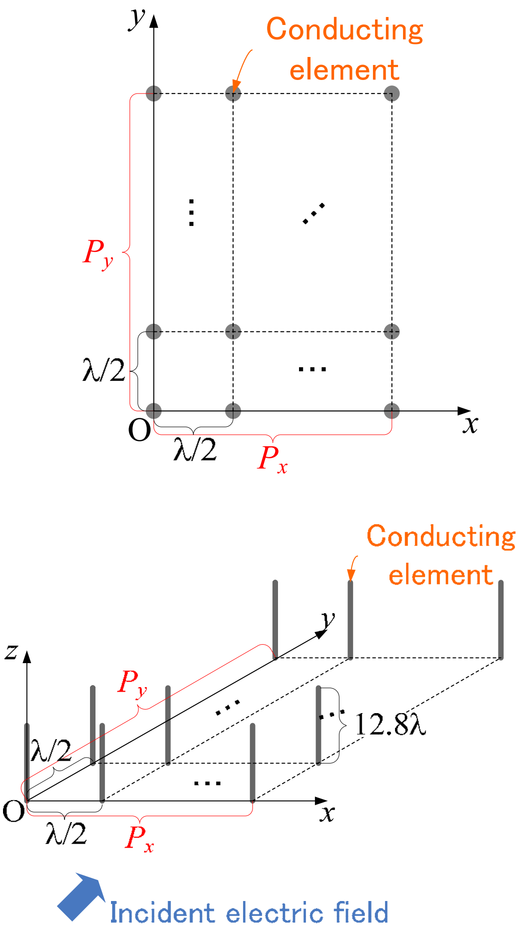
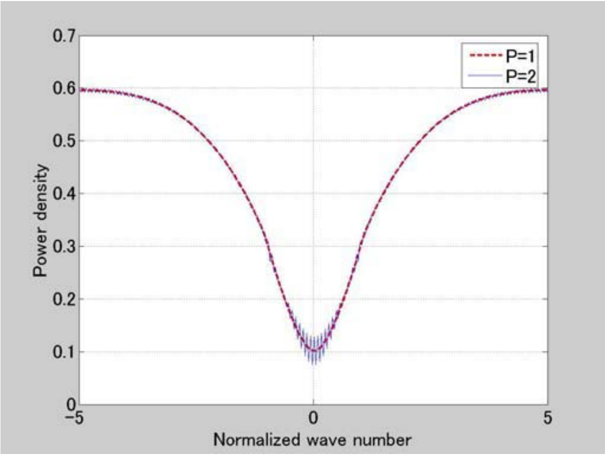
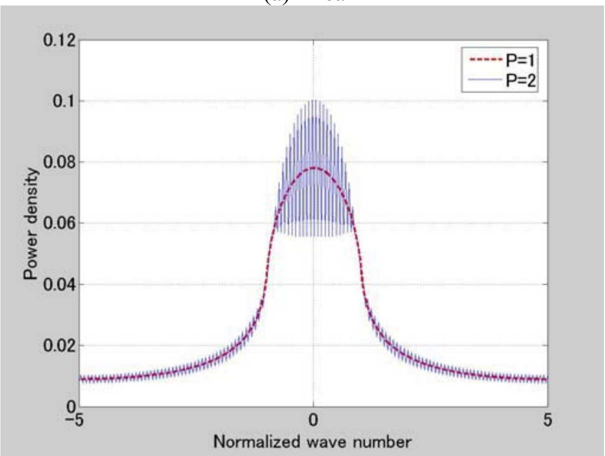


Fig. 1: Calculation model

To overcome this problem, if the impedance matrix can be divided into partial matrices which has no discontinuity as in eq.(7) and the structure of scatterers is periodic, the transformation is not performed with NP x NP wavelet transform matrix $\mathbf{W}_{NP\times NP}$, but performed with the modified transform matrix $\mathbf{W'}_{NP\times NP}$ which consists of P x P partial matrix $\mathbf{W}_{N\times N}$ which is N x N wavelet transform matrix and operates on each partial matrix $\mathbf{Z}_{pq}$ in $\mathbf{Z}$. Note that, to keep



(a) Real



(b) Imaginary

Fig. 2: Spectral characteristics of impedance matrix

$\mathbf{W'}_{NP\times NP}$ being an orthogonal matrix, its partial matrices $\mathbf{W}_{N\times N}$ are multiplied by the elements of Hadamar matrix as

$$\mathbf{W}'_{NP\times NP} = \frac{1}{\sqrt{P}} \begin{bmatrix} h_{11}\mathbf{W}_{N\times N} & h_{12}\mathbf{W}_{N\times N} & \cdots & h_{1P}\mathbf{W}_{N\times N} \\ h_{21}\mathbf{W}_{N\times N} & h_{22}\mathbf{W}_{N\times N} & \cdots & h_{2P}\mathbf{W}_{N\times N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{P1}\mathbf{W}_{N\times N} & h_{P1}\mathbf{W}_{N\times N} & \cdots & h_{PP}\mathbf{W}_{N\times N} \end{bmatrix} \qquad (8)$$

where h_{pq} indicates the element of Hadamar matrix. The orthogonality of $\mathbf{W'}_{NP\times NP}$ is easily proved as following.

TABLE 1: NUMERICAL RESULTS

P (P_x, P_y)	NP	Method	% of nonzero elements (Real	Imag)	Condition number	CPU time [sec]	Error in current density (Real	Imag)
1 (1, 1)	128	Brute force	-		102.74	0.094	-	
		Conventional	18.31	5.04	105.42	0.047	0.0093	0.012
		Proposal	18.31	5.04	105.42	0.047	0.018	0.033
2 (2, 1)	256	Brute force	-		127.37	0.28	-	
		Conventional	13.34	5.94	115.86	0.14	0.023	0.0070
		Proposal	10.45	3.70	118.66	0.11	0.015	0.0066
4 (2, 2)	512	Brute force	-		135.85	3.20	-	
		Conventional	11.35	7.48	144.65	1.55	0.023	0.011
		Proposal	8.19	3.84	142.32	0.59	0.0068	0.0094
8 (4, 2)	1024	Brute force	-		153.15	16.94	-	
		Conventional	11.90	9.64	165.84	7.84	0.0044	0.0067
		Proposal	4.77	2.99	151.38	1.52	0.0035	0.0066
16 (4, 4)	2048	Brute force	-		168.57	139.34	-	
		Conventional	13.12	12.08	176.30	60.25	0.0053	0.0035
		Proposal	4.30	3.47	174.17	10.19	0.0024	0.0028

$$\mathbf{W}'_{NP\times NP}\mathbf{W}'^{T}_{NP\times NP} = \frac{1}{P}\begin{bmatrix} P\mathbf{I}_{N\times N} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & P\mathbf{I}_{N\times N} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & P\mathbf{I}_{N\times N} \end{bmatrix} = \mathbf{I}_{NP\times NP}$$

$$\left(\because \mathbf{W}_{N\times N}\mathbf{W}^{T}_{N\times N} = \mathbf{I}_{N\times N}\right)$$

3. NUMERICAL RESULTS

The proposed method explained in the previous chapter is validated with the several parameters shown in Table 1: Numerical results as a function of problem size. The problem size studied here ranges from NP = 128 to 2048 which corresponds to the range of the number of conducting elements P varying from 1 to 16. Three methods are compared here, one is referred to "brute force" and is original method of moment without any transformations, other one is "conventional" wavelet method, and the last one is the proposed method shown as "proposal".

In all the results, the proposed method shows less percentage of non-zero elements than the conventional method. This means that the sparsification is more effective for the proposed method than the conventional one. It can also be said that the higher the number of unknowns N becomes, the larger the difference between the conventional and the proposed method is. Next, with regard to the condition number, significant difference can not be seen among the three methods for all the cases. For the results of CPU time, remarkable difference can be seen between the conventional and the proposal. Detail consideration for this will be given below with Fig.3. It also turns out that the estimation error of induced current is around 2% for all the results, and it can also be seen that the results for the proposal is slightly less than those for the conventional.

Fig.3 presents a comparison of CPU time required to solve the moment equations with the bi-conjugate gradient method. Brute force is approximately on the order of O(N^3), the conventional is O($N^{2.5}$), and the proposal is about O(N^2). This clearly indicates the effectiveness of the proposed method.

Fig.4 describes an example of calculation of induced current density. It can be objectively confirmed that the results for the three methods show very good agreement with each other.

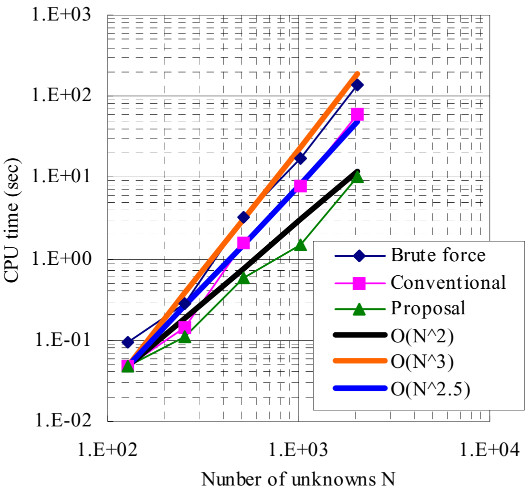


Fig. 1:CPU run time required for solving moment equation with the bi-conjugate gradient method

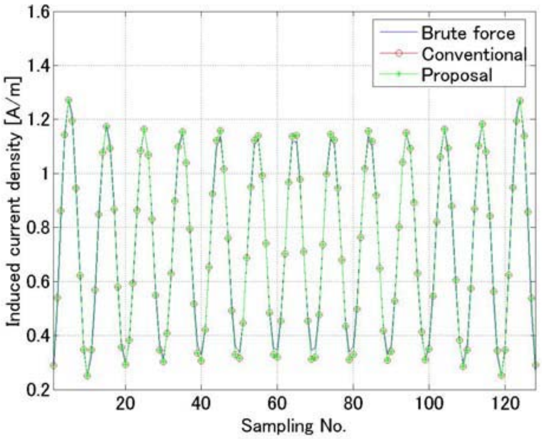


Fig. 2:Induced current density on one of conducting elements

4. CONCLUSIONS

It was shown that the sparcification of the impedance matrix using the wavelet transform is not satisfactorilly achieved if the impedance matrix becomes discontinuous in the space domain and oscillatory in the spectral domain. To overcome this problem, a new transformation based on a wavelet transform was proposed in this paper. A scattering problem with several conducting elements was considered for the analysis of the problem. It was proved that the proposed method gave much more effective sparcification of the impedance matrix than the conventional method did, and CPU time was approximately less than the order of O(N^2) for the proposed method while around O($N^{2.5}$) for the conventional one. It was also revealed that the proposed method achieved similar accuracy of estimating the induced currents on scatterers to conventional method.

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