One-dimensional Simulation of Electromagnetic Waves Reflected from Perfect Planes Vibrating with Different Forms

Mingtsu Ho
Department of Electronic Engineering, WuFeng Institute of Technology
117 Jian Kuo Road Section 2, Min Shong, Taiwan 621, homt@mail.wfc.edu.tw

Abstract

The one-dimensional simulation results of the reflection of electromagnetic waves from vibrating perfectly electric conducting planes are presented in this paper. The perfect conducting plane is set to vibrate sinusoidally or in zigzag with an impractically high frequency so that the instantaneous velocity at the equilibrium position is equal to either 10 or 20 percent of the speed of light. The reason of so doing is for the purpose of easy observation of the Doppler effects on the reflected waves. The computational results are obtained using the method of characteristics through the application of both characteristic variable and relativistic boundary conditions. By comparing the computational results with the theoretical Doppler shift values, the accuracy of the numerical method is investigated. It is found that the predicted data are in good agreement with the exact values.

Keywords: method of characteristics, relativistic boundary conditions, Doppler shift, vibrating conductor

1. Introduction

The objective of this paper is to report an accurate numerical method for the computational electromagnetic scattering problems in one dimension and to validate the scheme accuracy by comparing the simulation results with the theoretical Doppler shift values. Several one-dimensional simulations are investigated where the electromagnetic waves reflected from perfect conducting plane that is vibrating either sinusoidally or in zigzag. The computational results are obtained using the method of characteristics with the aid of characteristic variable and relativistic boundary conditions.

Several analytical studies of electromagnetic wave scattering from moving conductors can be dated back to as early as 1979 [1-3]. The following observations are given as conclusive remarks: the translational motion of perfect conductor results in the well-known Doppler shifts in the magnitude of the scattered fields while the oscillation of target gives rise to the changes in phase and magnitude of the scattered fields.

For the past half-century two most popular used numerical techniques for modeling electromagnetic scattering problems have been the method of moments (MoM) and the finite-difference time-domain (FDTD) technique. Another approach was developed to directly approximate the time-domain Maxwell curl equations is the method of characteristics. The method of characteristics was reported by Whitfield and Janus for the numerical solution of the Navier-Stokes equations [4]. Shang applied the explicit finite-difference approximation to solve the Maxwell's equations [5]. The implicit formulation was developed for the same purpose in conjunction with the lower-upper approximate factorization method and found in good agreement with results generated by FDTD [6]. Unlike MoM and FDTD where the field components are allocated at grid nodes, the method of characteristics positions all field components in the center of grid cell. That is, each field variable in characteristics method is an averaged quantity over the entire computational cell. The method of characteristics is consequently considered a suitable approach for problems involving time-varying cells. To solve the problem, the method of characteristics casts the Maxwell's equations in the form of Euler equation, transforms them into curvilinear coordinate system, and then directly approximates the Maxwell's equations by balancing the flux within each computational cell.

2. Boundary Condition Treatment

In order to solve electromagnetic scattering problems, one must specify the problem with particular initial values and apply proper boundary conditions since the time-domain Maxwell curl equations constitute a hyperbolic system. That the initial values must be specified implies both the electric and magnetic fields are given prior to the progress of numerical procedure. There are two types of computational cell arrays to which boundary conditions are applied: around the object's surfaces and the truncated computational boundaries. In both areas of the computational domain field components are maneuvered according to physics during the process. As an example, the tangential component of the electric field intensity must vanish on the surface of a stationary perfect conductor and there's no penetration of field into the conductor. On the outer computational boundaries, the Sommerfeld's radiation condition must hold. The proper boundary conditions ensure that there is no reflecting of fields from this layer of cell.

Since the perfect conducting plane is vibrating with a relatively high frequency, the relativistic boundary conditions must be considered. The boundary conditions used in the present method are the combination of the characteristic variable (CV) boundary conditions and relativistic boundary conditions. The CV boundary conditions are inherent from the nature of the method of characteristics. CV is defined as the product of the instantaneous variable vector and one row eigenvector associated with one particular eigenvalue. Every eigenvalue designates the direction and velocity of the information propagating across the cell face. The helpfulness of CV for the evaluation of boundary variables is evident in interpreting physics.

Since the conductor is vibrating with extremely high frequency and in order to predict the relativistic effects, the relativistic relation is considered. It is given as

$$\mathbf{n} \times \mathbf{E}^* = (\mathbf{v} \cdot \mathbf{n})\, \mathbf{B}^* \tag{1}$$

where **n** and **v** are respectively the unit normal vector and instantaneous velocity of the perfect conductor. Symbols $\mathbf{E}^*$ and $\mathbf{B}^*$ represent the boundary values of the electric field intensity and magnetic flux density, respectively. The CV arriving on the boundary (designated as CV^*) is the one carries information approaching the vibrating boundary from the adjacent cell and is given by

$$CV^* = n \times \mathbf{B} + \eta_o\, \mathbf{D} \tag{2}$$

with η_o being the impedance of free space and symbols **B** and **D** are the electric and magnetic flux densities of the cell adjacent to the moving boundary. The boundary values $\mathbf{E}^*$ and $\mathbf{B}^*$ can be solved through equations (1) and (2).

A stationary grid system is used for a motionless boundary where both the cell number and cell size are time-invariant and uniform as depicted in Figure 1(a). If the boundary is in motion and moves to the left, portion of the N^{th} cell is truncated by the boundary as in Figure 1(b). Reversely, as shown in Figure 1(c), when the boundary travels to the right, an extra fractional cell, the $(N+1)^{th}$ cell, is introduced into the grid system. Therefore, both cell number and cell size are time-dependent and the determination of the numerical time step is so important that the numerical field must not to pace or skip any grid during the simulation. This can be done by cautiously updating the effective cell and accordingly adjusting the numerical time step.

3. The Problem

Plane electromagnetic wave is used in the present simulation as the incident excitation as specified below. It is monochromatic with a frequency of 0.1 GHz. Each incident wave train consists of five complete wavelengths. For practical reason, a Gaussian window is applied to each end of the wave train with a cutoff level of 100 dB with respect to the peak. The length from the peak to the truncated point is measured one wavelength as depicted in Figure 2. The incidence initially propagates in the positive-x direction in free space and normally illuminates upon a perfect conducting plane that is either at rest or in motion. The wave train has only $\mathbf{D}_z$ and $\mathbf{B}_y$ components whose electric field intensity is normalized to unity. The grid density is 500 points per wavelength and the uniform numerical time step is set so that the numerical electromagnetic wave takes twenty steps to march one cell size.

For easy examination on the effects of the vibrating object on the reflected waves, the perfect conductor is set to move as described as follows. The perfect conducting plane vibrates either sinusoidally or in zigzag with an impractical high frequency and constant amplitude so that the extreme instantaneous velocity equals ± 0.1 C or ± 0.2 C. Note that the letter C stands fro the light speed, and that when the perfect conducting plane vibrates in zigzag, there is no acceleration associated with the motion except abrupt change in direction. If the vibration frequency and amplitude are respectively 0.1 GHz and 95.49 mm peak-to-peak for sinusoidal oscillation, then the resulted instantaneous velocity is ± 0.1 C near the equilibrium position. It is 150 mm peak-to-peak for zigzag path to have a speed of 0.1 C for the same vibration frequency. Under the same circumstance, if the vibration amplitude is doubled then the instantaneous velocity is doubled as well. In this report the symbol b_v is used to represent the ratios of the extreme instantaneous velocity to the speed of light. Since b_v may range from – 0.2 and + 0.2, $| b_v |$ is used to disregard the change of motion direction. The sign of b_v is designated as positive if the perfect conducting plane and the incident waves move in the same direction and negative if they are approaching each other. The variation in the reflected electric field strength is investigated by the theoretical Doppler shift values

$$| E_i | \frac{1 - b_v}{1 + b_v} \tag{3}$$

where $| E_i |$ is the normalized electric field strength of the incidence.

4. Results

In order to observe how the electromagnetic fields are affected by the vibrating perfect conducting plane, two time-sequences of the electric field intensity are illustrated in Figures 3 and 4. The former is reflected from a sinusoidal vibrating boundary with a frequency as high as the incident waves ($f_v = f_i = 0.1$ GHz). The latter is for the case where the perfect conducting plane vibrates in zigzag with a frequency being five times as high as the incidence ($f_v = 5\, f_i = 0.5$ GHz). It can be obviously seen that the reflected electric fields reveal the oscillatory behavior of the perfect conducting plane; that is, the Doppler effects on the wave form and amplitude of the reflected fields.

For closer observations, another set of result is plotted in

Figure 5 where the vibration amplitude is doubled and so is the instantaneous velocity. The exact values in the reflected electric field intensity are given as indicated. The calculated of the reflected electric field magnitude are summarized in Table 1 along with the theoretical values. They are in good agreement. Plotted in Figure 6 are the phase changes based on the case where the perfect conducting plane is motionless. Once more they reveal the vibration characteristics of the object, frequency and form.

5. CONCLUSION

The method of characteristics has been shown to accurately predict the Doppler effects on the reflected fields from vibrating perfect conducting plane in one dimension. It is also shown that the application of the combined boundary conditions, the relativistic and characteristic variable boundary conditions, is appropriate. The goal of the future research is to develop the existing code for two- and three-dimensional problems.

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TABLE 1: Doppler shifts: reflected electric field magnitude.

Velocities b_v	Theoretical	Zigzag (Calculated)	Sinusoidal (Calculated)
– 0.2	1.5000	1.5009	1.5009
– 0.1	1.2222	1.2224	1.2228
+ 0.1	0.8182	0.8179	0.8178
+ 0.2	0.6667	0.6665	0.6665

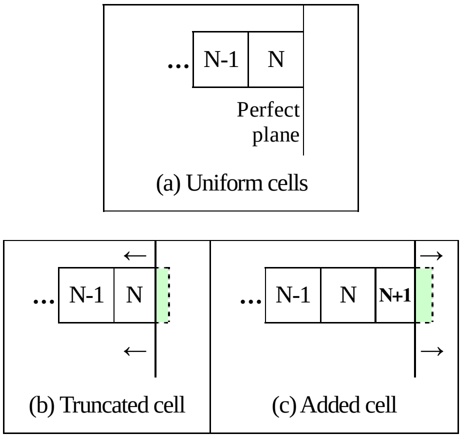

Fig. 1: Computational cell indexing: (a) Stationary grid system, (b) The Nth cell is partially truncated, (c) The (N+1)th fractional cell is introduced.

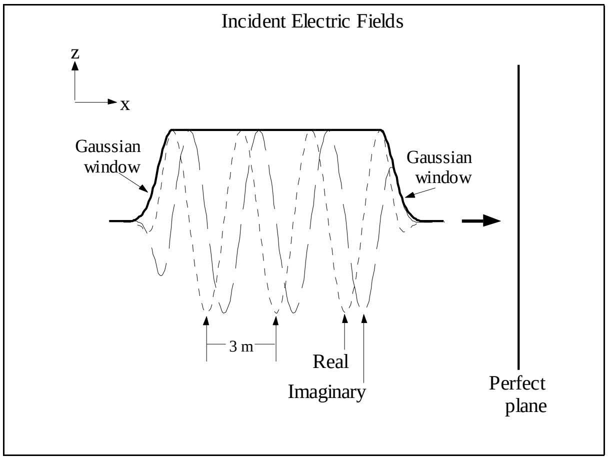

Fig. 2: Incident EM wave train: electric field intensity.

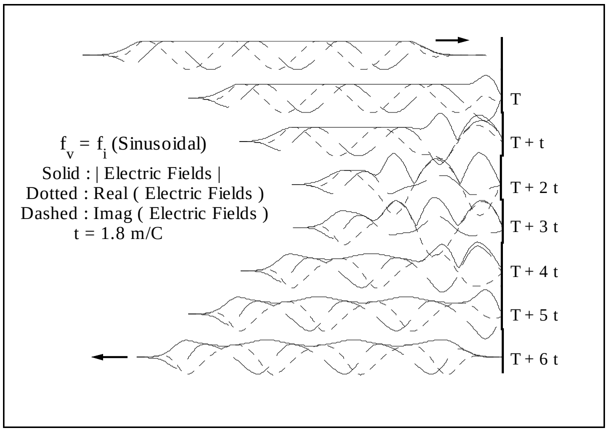

Fig. 3: Electromagnetic waves interact with the vibrating perfect conducting planes ($f_v = f_i$; Sinusoidal).

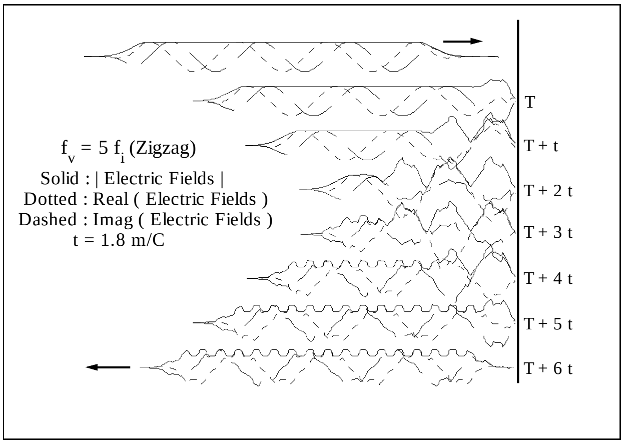

Fig. 4: Electromagnetic waves interact with the vibrating perfect conducting planes ($f_v = 5\ f_i$; Zigzag).

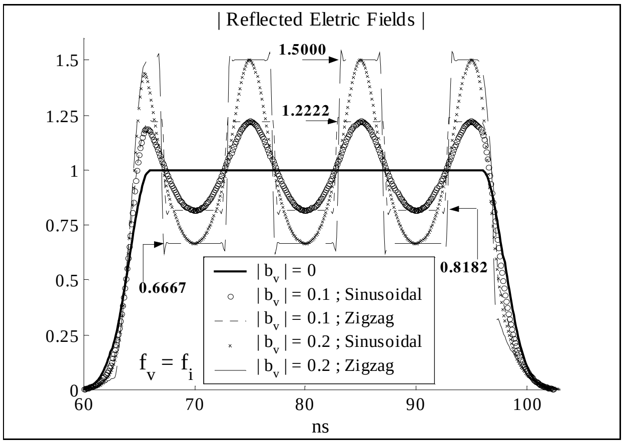

Fig. 5: Reflected electric fields from vibrating perfect conducting planes with various velocities.

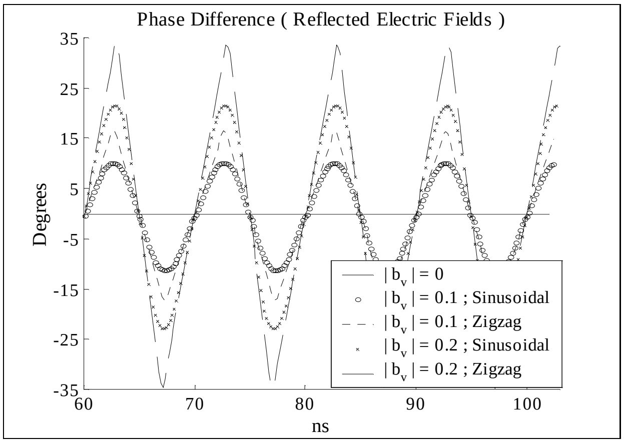

Fig. 6: Phase differences based on the | b_v | = 0 case.