

## FDTD APPLIED TO LOSSY ANISOTROPIC MEDIUM AND ITS PARALLEL COMPUTING

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**Abstract:** A three-dimensional anisotropic medium FDTD (ANI-FDTD) of dyadic constitutive parameters  $\boldsymbol{\varepsilon}$ ,  $\boldsymbol{\mu}$ ,  $\boldsymbol{\sigma}$  and  $\boldsymbol{\sigma}_m$  is discussed. A 28 field component grid model is given showing the relation of computation grid of a field component in ANI-FDTD with its neighboring grids. Based on this model, an FDTD parallel computing algorithm for lossy anisotropic medium is proposed. Some numerical results are given to exemplify the feasibility of presented algorithm.

### 1. Introduction

Research on the electromagnetic scattering by anisotropic objects has been growing recently because of the extensive and potential applications in microwave technology and radar absorbing material (RAM). An algorithm for applying FDTD to analysis of an anisotropic object problem was first proposed by Schneider and Hudson (1993). However, only one-dimensional examples are discussed in their paper. Two-dimensional cases are then considered after their publications. The three-dimensional problem becomes more complicated because of the coupling of each component in constitutive relations, such as  $D_x$  is related with three components of  $E_x$ ,  $E_y$  and  $E_z$ . It conducts to a far more lengthy expression in anisotropic medium FDTD (ANI-FDTD) formulation, because the sampling grids of each field component, such as  $E_x$ ,  $E_y$  and  $E_z$  are not set in the designated spatial sampling point in Yee's cell. In this paper we present a detailed analysis of FDTD formulation for anisotropic object, in particular for the medium with dyadic permeability and permittivity. A 28 grid model concerning the lossy anisotropic medium is discussed. Based on this model, an FDTD parallel computing algorithm for anisotropic medium is proposed. Finally some numerical results are given to exemplify the feasibility of presented algorithm.

### 2. ANI-FDTD formulation for lossy anisotropic medium

The Maxwell equations for lossy anisotropic media are

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t - \mathbf{J}_m = -\boldsymbol{\mu} \cdot \partial \mathbf{H} / \partial t - \boldsymbol{\sigma}_m \cdot \mathbf{H} \quad (1)$$

$$\nabla \times \mathbf{H} = \partial \mathbf{D} / \partial t + \mathbf{J} = \boldsymbol{\varepsilon} \cdot \partial \mathbf{E} / \partial t + \boldsymbol{\sigma} \cdot \mathbf{E} \quad (2)$$

in which  $\boldsymbol{\mu}$ ,  $\boldsymbol{\varepsilon}$ ,  $\boldsymbol{\sigma}$  and  $\boldsymbol{\sigma}_m$  are constitutive parameters. If  $\mathbf{E}$  and  $\mathbf{H}$  are sampled alternatively in time domain as it does in conventional FDTD, and the temporal derivative is replaced by its finite-difference approximation, we may obtain

$$\mathbf{E}^{n+1} = \mathbf{v} \cdot \mathbf{E}^n + \boldsymbol{\kappa} \cdot (\nabla \times \mathbf{H})^{n+1/2} \quad (3)$$

$$\mathbf{H}^{n+1/2} = \mathbf{v}_m \cdot \mathbf{H}^{n-1/2} + \boldsymbol{\kappa}_m \cdot (\nabla \times \mathbf{E})^n \quad (4)$$

where

$$\begin{aligned} \mathbf{v} &= (\boldsymbol{\varepsilon}/\Delta t + \boldsymbol{\sigma}/2)^{-1} \cdot (\boldsymbol{\varepsilon}/\Delta t - \boldsymbol{\sigma}/2), \quad \boldsymbol{\kappa} = (\boldsymbol{\varepsilon}/\Delta t + \boldsymbol{\sigma}/2)^{-1} \\ \mathbf{v}_m &= (\boldsymbol{\mu}/\Delta t + \boldsymbol{\sigma}_m/2)^{-1} \cdot (\boldsymbol{\mu}/\Delta t - \boldsymbol{\sigma}_m/2), \quad \boldsymbol{\kappa}_m = (\boldsymbol{\mu}/\Delta t + \boldsymbol{\sigma}_m/2)^{-1} \end{aligned} \quad (5)$$

If the spatial domain is discretized by Yee's cell, the Eq.(3) then becomes

$$\begin{aligned} E_z^{n+1} \Big|_{i,j,k+0.5} &= \left[ \kappa_{31} \left( \frac{\partial H_z}{\partial y} \Big|_{i,j,k+0.5}^{n+0.5} - \frac{\partial H_y}{\partial z} \Big|_{i,j,k+0.5}^{n+0.5} \right) + \kappa_{32} \left( \frac{\partial H_x}{\partial z} \Big|_{i,j,k+0.5}^{n+0.5} - \frac{\partial H_z}{\partial x} \Big|_{i,j,k+0.5}^{n+0.5} \right) \right. \\ &\quad \left. + \kappa_{33} \left( \frac{\partial H_y}{\partial x} \Big|_{i,j,k+0.5}^{n+0.5} - \frac{\partial H_x}{\partial y} \Big|_{i,j,k+0.5}^{n+0.5} \right) \right] + \left[ \nu_{11} E_x \Big|_{i,j,k+0.5}^n + \nu_{12} E_y \Big|_{i,j,k+0.5}^n + \nu_{13} E_z \Big|_{i,j,k+0.5}^n \right] \end{aligned} \quad (6)$$

for the z-component of electric field. While the spatial derivatives on the RHS of above equation are replaced by its finite difference version, it is found that an average approach has to be invoked to make all electric and magnetic field component sampling points to coincidence with the arrangement of Yee's cell. The complete expressions of ANI-FDTD are too lengthy to include in this report. However its configuration can be shown in Fig.1, in which there are totally 28 field component grid points involved for computation of the electric component such as  $E_z(i, j, k+0.5)$ . The same configuration can be obtained for the magnetic field component by using the duality principle. As a comparison, there are only 4 field component grid points surrounding the computation point, such as  $E_z(i, j, k+0.5)$  in the conventional FDTD. Obviously the 28 grid point configuration in ANI-FDTD, as shown in Fig.1, is quite different from the one in conventional FDTD.

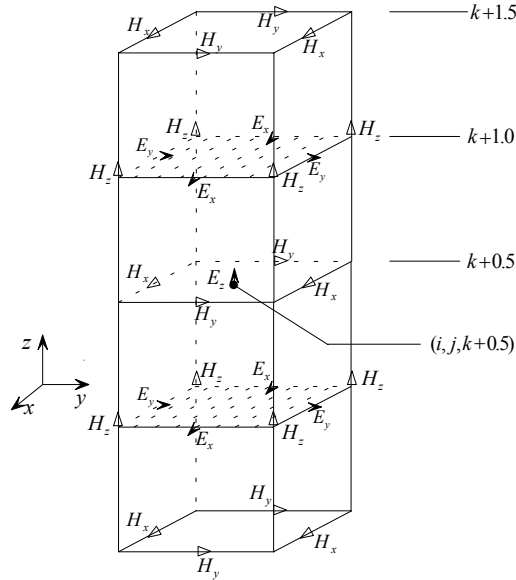


Fig.1 The 28 related field component sampling points for computing  $E_z(i, j, k+0.5)$

### 3. ANI-FDTD in parallel computing

For scattering problem it is common to divide the FDTD region into total field region and scattered field region. Suppose the scatterer is anisotropic and the surrounding medium is isotropic. The ANI-FDTD is then applied only within the total field region. The computation of the rest of FDTD region is still by using the conventional FDTD. In parallel computing the FDTD region is partitioned into several sub-regions, such as  $1 \times 2$ ,  $2 \times 2$  or  $2 \times 2 \times 2$  sub-regions. In doing the partitioning, the electric field components are always set to be

tangential to the interface of sub-regions. In addition a half grid overlapping in adjacent sub-regions must be introduced in order to make FDTD computation advancing in time domain successfully.

Suppose the interface of sub-regions is perpendicular to the y-axis. At the interface there are three field components  $E_x, E_z$  and  $H_y$ , which are tangential and perpendicular to the interface, respectively. Assume these three field components at the interface belong to the right sub-region. By inspection of Fig.1, it is found that the advancing computation of field components located at or near the interface will involve with the field grids located within the left sub-region. Therefore it is necessary to have data communication between sub-regions. We consider the following cases:

- (1) The tangential field components  $E_x, E_z$  at the interface. Based on Fig.1 their computation needs the grid points located in the left sub-region within the distance of  $0.5 \Delta y$  from interface.
- (2) The perpendicular field component  $H_y$  at the interface. Referring to Fig.1 and using the duality principle and recurrence of subscription x, y and z, it is derived that the computation of  $H_y$  needs the grid points located in the left sub-region within the distance of  $\Delta y$  from interface.
- (3) The field component  $E_y$  in the left sub-region but  $0.5 \Delta y$  away from the interface. Referring to Fig.1 and using the recurrence of subscription x, y and z, it is found that the computation of this  $E_y$  needs the grid points located in the right sub-region within the distance of  $0.5 \Delta y$  from interface.
- (4) The field components  $H_x, H_z$  in the left sub-region but  $0.5 \Delta y$  away from the interface. Referring to Fig.1 and using the duality principle, it is found that the computation of  $H_z$  (or  $H_x$ ) needs the grid points located at the interface, which belongs to the right sub-region.

According to the above analysis we may conclude that the field component data in the left sub-region and  $\Delta y$  away from interface must be transferred to the right sub-region; and the field component data in the right sub-region and  $0.5 \Delta y$  away from interface must be transferred to the left sub-region in each time step of FDTD advancing in order to keep the parallel computing successfully.

#### 4. Numerical examples

To exemplify the availability of the ANI-FDTD in parallel computing, consider the scattering of an anisotropic plate backed by a perfectly conducting (PEC) plate. The two plates are assumed to be  $12cm \times 15cm \times 1.5cm$  and  $12cm \times 15cm \times 3.0cm$ , respectively. The computation domain is partitioned into  $2 \times 2 \times 1$  sub-regions. The plane wave of Gaussian pulse is incident perpendicularly to the plate. In the ANI-FDTD, we take  $\delta = 0.15cm$ ,  $dt = \delta/2c$ , and the total time step is 800. The dyadic permittivity and permeability of the anisotropic plate is

$$\boldsymbol{\varepsilon} = \varepsilon_0 \begin{bmatrix} 3.250 & 0.217 & 0.375 \\ 0.217 & 2.438 & 0.758 \\ 0.375 & 0.758 & 3.313 \end{bmatrix} \quad \boldsymbol{\mu} = \mu_0 \begin{bmatrix} 1.250 & 0.217 & 0.375 \\ 0.217 & 1.438 & 0.758 \\ 0.375 & 0.758 & 1.313 \end{bmatrix}$$

The backscattered RCS for co-polarization are given in Fig.2, where  $\triangle$  stands for the result

computed by the ANI-FDTD parallel computing code, and the solid line for sequential ANI-FDTD code. They are in good agreement. Furthermore if the dyadic electric and magnetic conductivity are also introduced, for instance

$$\boldsymbol{\sigma} = \begin{bmatrix} 0.3250 & 0.0217 & 0.0375 \\ 0.0217 & 0.2438 & 0.0758 \\ 0.0375 & 0.0758 & 0.3313 \end{bmatrix} s/m \quad \boldsymbol{\sigma}_m = \begin{bmatrix} 0.1250 & 0.0217 & 0.0375 \\ 0.0217 & 0.1438 & 0.0758 \\ 0.0375 & 0.0758 & 0.1313 \end{bmatrix} \Omega/m$$

for a lossy anisotropic material, the corresponding RCS computed result is depicted in Fig. 2 as well, showing the reduction of RCS by material loss.

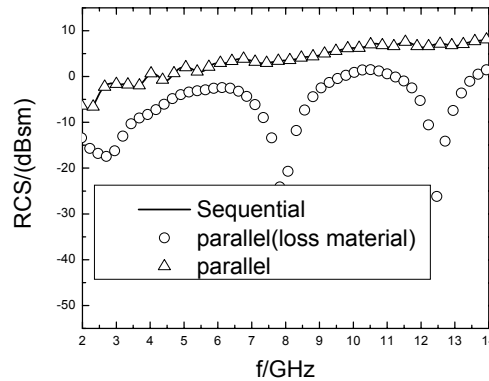


Fig. 2 Backscattered RCS for an anisotropic plate backed by PEC

## 5. Conclusions

A three dimensional ANI-FDTD is analyzed and its parallel computing on a distributed network by using the message passing module is fulfilled. The complexity of ANI-FDTD is demonstrated by its 28 related grid point configuration for the computation of a field component grid while advancing in time domain. This configuration, as shown in Fig.1 also help us to understand the data communication between the adjacent sub-regions while the parallel computing is performed in partitioned FDTD region. The availability of presented scheme is exemplified. The developed parallel algorithm may be applied to the scattering problem by an anisotropic object of large scale.

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