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An Optimal Shape Design Method based on FDTD and Design Sensitivity Analysis with B-Spline Parameterization

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1. Introduction

In this paper, we propose a new shape design methodology to find an optimal shape working in a wide band from a given shape. The proposed algorithm is based on the finite difference time domain (FDTD) analysis and the design sensitivity (DSA) analysis to evaluate the gradient information of the objective function which represents the performance of the design.

Recently, the authors proposed the optimal shape design algorithm based on FDTD and DSA [1][2]. In [1][2], however, the design sensitivity was not directly evaluated because the adjoint variable equation in the FDTD could not be derived in a straightforward way. In order to solve this problem, we employed an adjoint variable equation that was derived from the finite element time domain (FETD) formulation. This adjoint variable equation was, then, transformed to the coupled Maxwellian differential equations, which were solved by the general Yee's algorithm. In the previous algorithm, a mesh generator is needed at each design iteration to generate new meshes because the updated shape is different from the old one and therefore the old mesh is not suited for the updated shape.

By using the conformal FDTD (CFDTD) algorithm [3][4], we propose the new shape optimization algorithm which does not need the use of the mesh generator at each design iteration. Also, since the adjoint variable equation can be derived directly using the CFDTD, we can calculate the gradient information easily.

Use of polynomial and spline representations for shape parameterization can obviously reduce the total number of design variables. A polynomial or spline can describe a curve in a very compact form with a small set of design variables. In addition, one can avoid unrealistic design when the shape variables are the point-based coordinates. In this paper, the design variables are parameterized using the Bezier spline (B-spline) curve. In order to verify the proposed algorithm, we apply it to the design of a tapered slot array antenna, of which the object is to find a new shape to reduce the return loss in wideband than that of the initial shape.

2. Conformal FDTD and Design Sensitivity Analysis

A general finite difference form of the time-domain Maxwell equations can be represented as

$$\begin{pmatrix} \dot{E} \\ \dot{H} \end{pmatrix} = \begin{bmatrix} \Sigma_2 & L_2 \\ L_1 & \Sigma_1 \end{bmatrix} \begin{pmatrix} E \\ H \end{pmatrix} + \begin{pmatrix} J/\varepsilon \\ M/\mu \end{pmatrix}, \text{ subject to } E(t=0)=0,\ H(t=0)=0 \quad (1)$$

where upper dot denotes the derivative with respect to the time variable. J and M are the electric and magnetic current density, respectively. And the diagonal terms are $\Sigma_2 = \sigma/\varepsilon$ and $\Sigma_1 = \sigma^*/\mu$, where σ and σ^* mean the electric and magnetic conductivity, respectively. And the off-diagonal terms, L_1 and L_2, are related to the finite difference coefficients for the curl operators. If the time derivatives of (1) are replaced with the central difference relations, (1) becomes the conventional Yee's equations.

In case of an arbitrary shaped model, the conventional FDTD algorithm may introduce significantly numerical errors due to the staircasing approximation, unless a relatively fine mesh is used. The conformal FDTD (CFDTD) technique can be used to obviate this problem and to improve the accuracy of modeling the curved boundaries such as PECs and dielectrics. In the CFDTD, the electric field update algorithm remains unchanged from that in the conventional Yee's scheme, but the

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magnetic field is updated differently, according to the deformed mesh. In Fig. 1, for the H_z component, we use

$$H_z\Big|_{i,j,k}^{n+1/2} = H_z\Big|_{i,j,k}^{n-1/2} + \frac{\Delta t}{\mu}\left(\frac{\Delta y E_y\big|_{i,j,k}^{n} - \delta y E_y\big|_{i+1,j,k}^{n}}{\Delta x \Delta y} + \frac{\delta x E_x\big|_{i,j+1,k}^{n} - \Delta x E_x\big|_{i,j,k}^{n}}{\Delta x \Delta y}\right) \quad (2)$$

where δx and δy denotes the cell length along the x- and y-directions, respectively, outside the PEC. Therefore, for an undistorted cell, the H-field update equation is the same as that of the conventional scheme.

In order to testify the optimization performance, we introduce a cost or objective function as

$$F(p) = \int_{\Omega_m}\int_0^{T_f} G\big(E(p,t), H(p,t)\big)\,dt\,d\Omega \quad (3)$$

where T_f is a fixed final time and G an arbitrary differentiable function of the electromagnetic fields E, H, and the design variable vector p. Ω_m is an observation domain where the objective function is evaluated. In this problem, the design variable vector p means the shape of the model which is to be designed. Note that the electromagnetic fields E and H are dependent of the design variable vector p.

Design sensitivity is the gradient of the objective function with respect to design variable vector p, which indicates that how the variation of design variables will affect the electromagnetic performance or the objective function. Using the gradient information, one can find the maximum or minimum of the objective function easily.

The total derivative of F with respect to p is represented as

$$\frac{dF}{dp} = \int_0^{T_f}\left(\frac{\partial G}{\partial E}\frac{\partial E}{\partial p} + \frac{\partial G}{\partial H}\frac{\partial H}{\partial p}\right)dt = \int_0^{T_f}\begin{pmatrix} G_E \\ G_H \end{pmatrix}^T \begin{pmatrix} E_P \\ H_p \end{pmatrix} dt \quad (4)$$

where $G_E \equiv \frac{\partial G}{\partial E}$, $G_H \equiv \frac{\partial G}{\partial H}$, $E_p \equiv \frac{\partial E}{\partial p}$, and $H_p \equiv \frac{\partial H}{\partial p}$.

Differentiating (1) with respect to p, we can obtain E_p and H_p, then

$$\begin{pmatrix} \dot{E}_p \\ \dot{H}_p \end{pmatrix} = \begin{bmatrix} \Sigma_2 & L_2 \\ L_1 & \Sigma_1 \end{bmatrix}\begin{pmatrix} E_p \\ H_p \end{pmatrix} + \begin{pmatrix} J_p \\ M_p \end{pmatrix} \quad (5)$$

where $J_p = \frac{\partial \Sigma_2}{\partial p}\tilde{E} + \frac{\partial L_2}{\partial p}\tilde{H} \equiv \Sigma_{2,p}\tilde{E} + L_{2,p}\tilde{H}$ and $M_p = \frac{\partial \Sigma_1}{\partial p}\tilde{H} + \frac{\partial L_1}{\partial p}\tilde{E} \equiv \Sigma_{1,p}\tilde{H} + L_{1,p}\tilde{E}$. The notation ~ denotes that the variable is held constant for the derivative process with respect to design variable vector p. In (5), in order to evaluate E_p and H_p, one should solve (5) at each variation of p, which requires the solving time as many as times of the number of the design variables. In this work, we introduce the adjoint variable method [5] to reduce the computational burden to calculate (5).

Multiplying (5) by the transpose of the adjoint variable vector $(\lambda(t), \gamma(t))$, and integrating over the time interval $\left[0, T_f\right]$, we have

$$\int_0^{T_f}\begin{pmatrix} \lambda \\ \gamma \end{pmatrix}^T \left\{\begin{pmatrix} \dot{E}_p \\ \dot{H}_p \end{pmatrix} - \begin{bmatrix} \Sigma_2 & L_2 \\ L_1 & \Sigma_1 \end{bmatrix}\begin{pmatrix} E_p \\ H_p \end{pmatrix} - \begin{pmatrix} J_p \\ M_p \end{pmatrix}\right\} dt = 0\,. \quad (6)$$

Applying the integration by parts to (6) and using $E_p(t=0)=0$ and $H_p(t=0)=0$, then we have

$$\begin{pmatrix} \lambda \\ \gamma \end{pmatrix}^T \begin{pmatrix} E_p \\ H_p \end{pmatrix}\Bigg|_{t=T_f} - \int_0^{T_f}\left\{\begin{pmatrix} \dot{\lambda} \\ \dot{\gamma} \end{pmatrix}^T + \begin{pmatrix} \lambda \\ \gamma \end{pmatrix}^T \begin{bmatrix} \Sigma_2 & L_2 \\ L_1 & \Sigma_1 \end{bmatrix}\right\}\begin{pmatrix} E_p \\ H_p \end{pmatrix} dt - \int_0^{T_f}\begin{pmatrix} \lambda \\ \gamma \end{pmatrix}^T \begin{pmatrix} J_p \\ M_p \end{pmatrix} dt = 0\,. \quad (7)$$

Since $(\lambda(t), \gamma(t))$ is an arbitrary vector, we can introduce the terminal condition $\lambda(T_f) = \gamma(T_f) = 0$ to eliminate the first term of (7), then we have

$$-\int_0^{T_f}\begin{pmatrix} \lambda \\ \gamma \end{pmatrix}^T \begin{pmatrix} J_p \\ M_p \end{pmatrix} dt = \int_0^{T_f}\left\{\begin{pmatrix} \dot{\lambda} \\ \dot{\gamma} \end{pmatrix}^T + \begin{pmatrix} \lambda \\ \gamma \end{pmatrix}^T \begin{bmatrix} \Sigma_2 & L_2 \\ L_1 & \Sigma_1 \end{bmatrix}\right\}\begin{pmatrix} E_p \\ H_p \end{pmatrix} dt\,. \quad (8)$$

From (4) and (8), we obtain the adjoint variable equation

$$\begin{pmatrix}\dot{\lambda}\\ \dot{\gamma}\end{pmatrix} = -\begin{bmatrix}\Sigma_2^T & L_1^T\\ L_2^T & \Sigma_1^T\end{bmatrix}\begin{pmatrix}\lambda\\ \gamma\end{pmatrix} + \begin{pmatrix}G_E\\ G_H\end{pmatrix}, \text{ subject to } \lambda(T_f) = \gamma(T_f) = 0 . \tag{9}$$

The above equation is a terminal value problem. To handle this, we introduce the backward time scheme $\tau \equiv T_f - t$ to (9). Then, we have

$$\begin{pmatrix}\dot{\bar{\lambda}}(\tau)\\ \dot{\bar{\gamma}}(\tau)\end{pmatrix} = \begin{bmatrix}\Sigma_2^T & L_1^T\\ L_2^T & \Sigma_1^T\end{bmatrix}\begin{pmatrix}\bar{\lambda}(\tau)\\ \bar{\gamma}(\tau)\end{pmatrix} - \begin{pmatrix}\bar{G}_E(\tau)\\ \bar{G}_H(\tau)\end{pmatrix} \text{ where } \bar{\lambda}(\tau) = \lambda(T_f - t) \text{ and } \bar{\gamma}(\tau) = \gamma(T_f - t). \tag{10}$$

Note that $d/dt \equiv -d/d\tau$. Using (1) and (10), we obtain the design sensitivity as

$$\frac{dF}{dp} = -\int_0^{T_f}\begin{pmatrix}\lambda\\ \gamma\end{pmatrix}^T\begin{pmatrix}J_p\\ M_p\end{pmatrix}dt = -\int_0^{T_f}\{\lambda J_p + \gamma M_p\}dt . \tag{11}$$

In the conformal FDTD scheme, the only L_1 is dependent on the shape design variable vector p. Therefore, the design sensitivity becomes

$$\frac{dF}{dp} = -\int_0^{T_f}(\gamma)\left(\frac{\partial L_1}{\partial p}\right)(\tilde{E})dt . \tag{12}$$

Equation (12) is the point-based gradient information. In this paper, to avoid a roughly designed shape, we use a parametric shape based on Bezier spline (B-spline) functions. The B-spline curves are generated by the control points which act as the design variables in this paper. During the optimization procedure, only the control points are changed so that their corresponding nodal points lie on a smooth B-spline curve. The design variables and the control points are related by the parameterization using the B-spline curve and it is represented as

$$p(\alpha) = \sum_{i=0}^{NC-1} B_i(\alpha)\varsigma_i \text{ , where } B_i(\alpha) = {}_N C_i \alpha^i (1-\alpha)^{N-i} \text{ and } 0 \le \alpha \le 1 . \tag{13}$$

From (12) and (13), the derivative of F with respect to the control point is given by

$$\frac{dF}{d\varsigma_i} = \sum_{k=1}^{NP}\frac{dF}{dp_k}\frac{dp_k}{d\varsigma_i} = \sum_{k=1}^{NP}\frac{dF}{dp_k}B_i(\alpha_k) = -\sum_{k=1}^{NP}B_i(\alpha_k)\int_0^{T_f}(\gamma)\left(\frac{\partial L_1}{\partial p_k}\right)(\tilde{E})dt \tag{14}$$

where NP is the number of the nodal points. Using the sensitivity information of (14), we can perform the shape optimization of a given geometry. Since the objective function F is related to the design variables in an implicit manner, an iterative optimization technique is preferred to find the optimum value. In this work, we use the steepest descent algorithm.

3. Numerical Examples

The proposed shape optimization method is applied to the taper shape design of an E-plane tapered slot array antenna (E-TSA) as shown in Fig. 2. Width of the unit element is 10mm and length of taper region is 20mm. The design object is to minimize the return loss in wideband for a given linear tapered shape. If we assume that the shape of E-TSA and the excitation of each element are symmetric in E-plane, we can model the E-TSA as a single element with proper boundary conditions such as PEC and absorbing boundary conditions (ABCs). Fig. 3 shows the initial shape of the linear tapered antenna and the design variables which are the control points. During the optimization procedure, we assume that the control points are allowed to move along the only *y*-direction holding the symmetric conditions. In this problem, the number of design variables or control points is 6. But the four empty circles are only movable and the two filled circles are fixed during the optimization procedure. Fig. 4 shows the optimized shape of the tapered antenna after the 50 design iterations. Fig. 5 represents the values of the objective function which are normalized by the initial value as the optimization iteration proceeds. And Fig. 6 represents the values of S_{11} in 6~18 GHz of the linear tapered shape and the optimized shape by the proposed algorithm.

4. Conclusions

In this paper, we proposed a new algorithm for shape optimal design using the conformal FDTD and the design sensitivity analysis to improve the performance of microwave device in broadband. The previous algorithm needs to call a mesh generator at each design iteration to segment the computational region into general quadrilaterals, but the new algorithm needs not to call it since the new algorithm can use the initial rectangular mesh during the design procedure. Also, in order to guarantee a smoothly designed shape, we introduce the parametric design variables instead of the point-based design variables. To verify the proposed algorithm, we applied it to the reduction of the return loss for a tapered slot antenna array in a wideband.

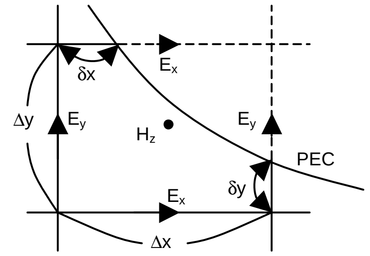

Fig. 1: Intersection between FDTD mesh and PEC

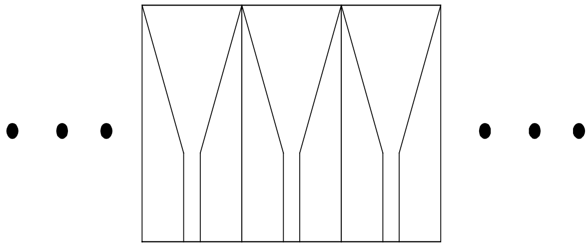

Fig. 2: E-TSA

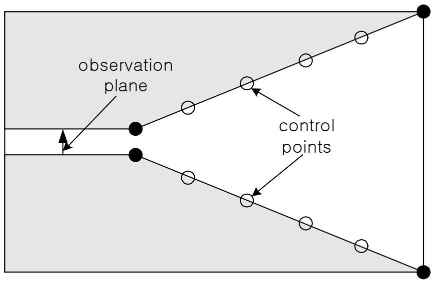

Fig. 3: Initial linear taper shape.

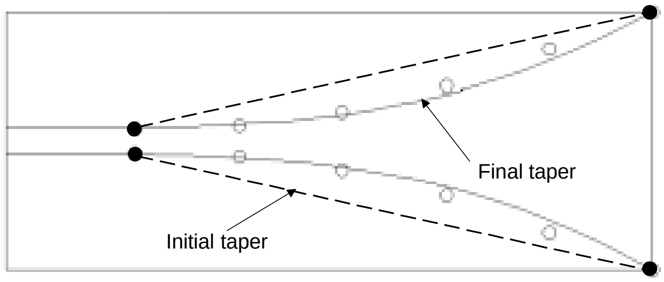

Fig. 4: Taper shape variation

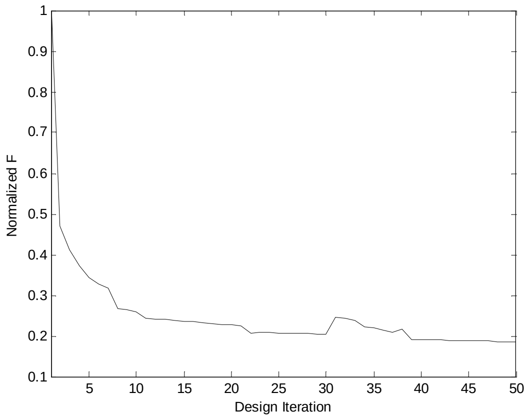

Fig. 5: Objective Function vs. design iteration

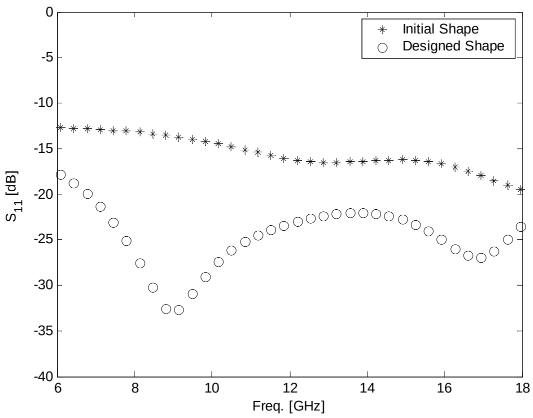

Fig. 6: Comparison of S_{11}

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