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COMPUTER SIMULATION OF A CERENKOV TYPE WAVE-PARTICLE INTERACTION BETWEEN OBLIQUELY PROPAGATING WHISTLER MODE WAVES AND AN ELECTRON BEAM

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There have been many studies including computer simulations on whistler mode waves propagating parallel to the external magnetic field in a magnetized plasma. Less attention, however has been paid to a problem in whistler mode waves propagating obliquely to the external magnetic field [1]. In this report we describe a method for computer simulation of obliquely propagating whistler mode waves, and simulation results of the Cerenkov type wave-particle interaction between these waves and an electron beam streaming along the external magnetic field.

SIMULATION METHOD AND MODEL

In the present simulation only electron dynamics are considered in a homogeneous magnetized plasma. Ions are assumed to form a stationary positive background. The electron plasma is assumed to be composed of two components. One is a cold background plasma, while the other is a hot electron beam.

Basic equations which describe plasma dynamics are Maxwell's equations and the equations of motions of plasma particles. In this study we have tried to simulate obliquely propagating one-dimensional plane waves. Only waves propagating with a finite angle to the external magnetic field $\mathbb{B}_0$ are dealt with. The basic equations to be solved in the present simulation are

$$\mathbb{e}_z \frac{\partial}{\partial z} \times \mathbb{E}w = - \frac{\partial \mathbb{B}w}{\partial t} , \qquad (1)$$

$$\mathbb{e}_z \frac{\partial}{\partial z} \times \mathbb{B}w = \mu_0 \mathbb{J} + \frac{1}{c^2} \frac{\partial \mathbb{E}w}{\partial t} , \qquad (2)$$

$$\mathbb{J} = - \sum_{s=b,e} n_s \, e \, \mathbb{v}_s , \qquad (3)$$

$$\frac{d \mathbb{v}_s}{dt} = - \frac{e}{m} [\mathbb{E}w + \mathbb{v}_s \times (\mathbb{B}w + \mathbb{B}_0)] . \qquad (4)$$

Where m,-e, $\mathbb{v}_s$, $\mathbb{E}w$, $\mathbb{B}w$, $\mathbb{J}$, μ_0, c, and n_s are the electronic mass, charge, velocity, wave electric field intensity, wave magnetic field intensity, conduction current density, permeability of free space, light speed, and number density of the s-th electrons(beam and background electrons), respectively. A unit vector in the direction of z-axis is denoted by $\mathbb{e}_z$, and z-axis is fixed to the direction of the wave normal vector.

In order to reduce numerical errors, all equations are solved by a complete central difference method (Morse and Nielson Particle in Cell method) [2] for both time and space by dividing time and space into mesh and half-mesh points. The computer simulation code for the cyclotron instability of longitudinally propagating whistler mode wave [3] is used for the present simulation with a minor modification.

The simulation model is shown in Fig. 1. The direction of the wave-normal vector is along the z axis, and $\mathbb{B}_0$ is in the x-z plane. The plasma is assumed to be spatially periodic with a periodic length L in the z direction which is divided into 1024 meshes. All equations (1)∿(4) are computed for three dimensional components in velocity-space and real-space, but the spatial change is assumed to be one-dimensional only in the z axis.

In this simulation, 1024 cold super-electrons and 2048 hot beam super-electrons are traced self-consistently. The particle adopted in this simulation is assumed to have a square distribution of charge which spreads over one mesh spacing. The conduction current due to each particle motion is divided into the current at the nearest two mesh points in such a rule that the mesh point current is inversely proportional to the distance from the center of particle position to the mesh point. This situation is shown schematically in Fig. 2.

Initially, one cold background particle and two hot beam particles are loaded at each mesh point. The velocity distribution function of the hot beam particles is a shifted Maxwellian in the direction parallel to $\mathbb{B}_0$, and the perpendicular velocity is assumed to be zero. The spatial distribution of the hot particles is made so uniform that the particle velocities should satisfy a given velocity distribution function even in any local region.

In the first case of this study, no wave is assumed to exist in the system initially. Although waves may be excited by the electron beam in all directions with respect to $\mathbb{B}_0$, in this simulation model only a wave propagating in a particular direction making an angle Θ with $\mathbb{B}_0$ is taken into account. In the second case of our simulation, an initial existence of a finite amplitude obliquely propagating whistler mode wave is assumed.

SIMULATION RESULTS

Linear dispersion relation of beam-plasma system with the parameters same as those initially used for our simulation is shown in Fig. 3, where the ratio of the plasma frequency, Π_e to the cyclotron frequency, Ω_e is 0.5, the wave propagating angle, Θ is 30°, the electron beam velocity, U_0 is 0.289c, the beam temperature in the direction to $\mathbb{B}_0$, $T_{\parallel}$ is 10^4 K, and the density ratio of the beam to the background plasma, n_0 is 0.01. In Fig. 3, thin lines represent the dispersion relation for the case of no beam (n_0 = 0). A branch, $\omega/\Omega_e \sim 0.4$ is a whistler mode wave. Thick solid lines,dashed lines, and dotted lines represent real solutions and real and imaginary parts of complex solutions of ω for given real k in the case of n_0=0.01, respectively. Two instability regions are seen including a whistler mode instability.

The time evolution of the wave number spectrum of longitudinal electric field E_z obtained from the simulation of the first case is shown in Fig. 4, where $\tilde{k}$ is a wavenumber normarized by 1/L, and is same as that shown in the upper abscissa in Fig. 3. Up to $20T_H$ (T_H : cyclotron period), the time evolution of the spectrum is consistent with the linear theory. Nonlinear effects become dominant after $20T_H$.

Wave frequencies can be determined from a phase difference in a short time interval for each component in the k-spectrum. By this procedure, ω-k diagrams are constructed for 6 different times, which are shown in Fig. 5 with the normarized k as the abscissa and the normarized ω as the ordinate as shown in Fig. 3. Initially, waves are excited on such a line in the ω-k diagram on which the Cerenkov condition with the electron beam is satisfied. For the time interval between $10T_H$ and $20T_H$, the ω-k relationship of growing components lie on the growing branch of the ω-k diagram shown in Fig. 3. After $30T_H$, the dispersion relations begin to change by nonlinear effects. Finally, the dispersion relations tend to those for the no beam case. This final stage may be due to a thermalization of the beam distribution function.

In the second case, a finite amplitude whistler mode wave with a k giving the maximum growth rate in the linear dispersion relation as shown in Fig.3 is assumed to propagate initially. It turns out that the amplitude of the wave grows up until a particular saturation amplitude which is almost identical with that of the first case, then an amplitude oscillation starts. Time evolutions of longitudinal electric field with different typical $\tilde{k}$ are shown in Fig.6. The finite amplitude wave with $\tilde{k}$ = 16 shows an amplitude oscillation after a linear growth due to an amplification by the beam.

The wave-particle interaction treated in this paper is considered as a kind of coherent emissions of the Cerenkov radiation in a magnetized plasma, and this result is helpful in investigating the generation mechanism of VLF hiss,spontaneously occurring in the earth's magnetosphere [4][5][6].

Computations are done at both the Data Processing Center of Kyoto University and the Computer Center of Institute for Plasma Physics Research, Nagoya University.

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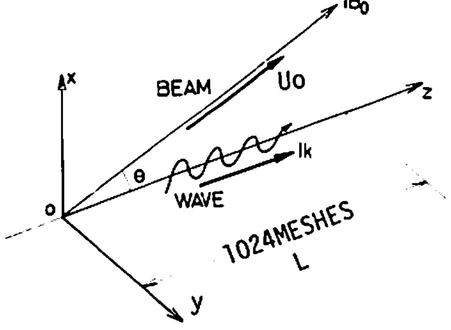

Fig. 1 Simulation model

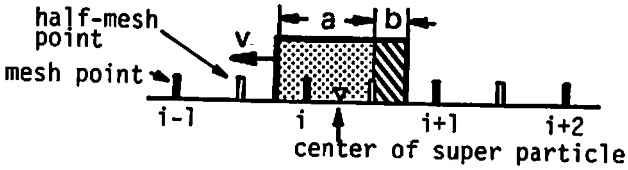

Fig. 2 A finite sized super particle and a distributing method of its conduction current onto adjacent mesh points. The current at i-th and (i+1)-th mesh points are computed by

$$J_i = -\frac{a}{a+b}\,env, \quad J_{i+1} = -\frac{b}{a+b}\,env.$$

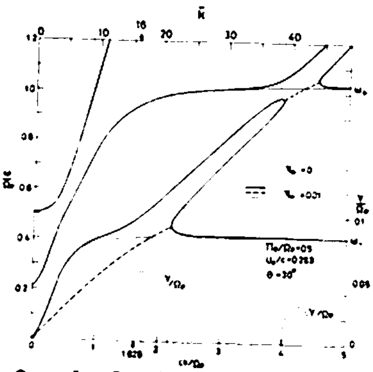

Fig. 3 Analytically calculated linear dispersion relation for beam-plasma system.

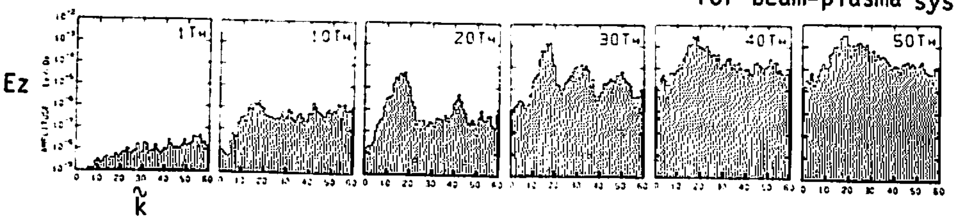

Fig. 4 Time evolution of the k-spectrum for the first case.

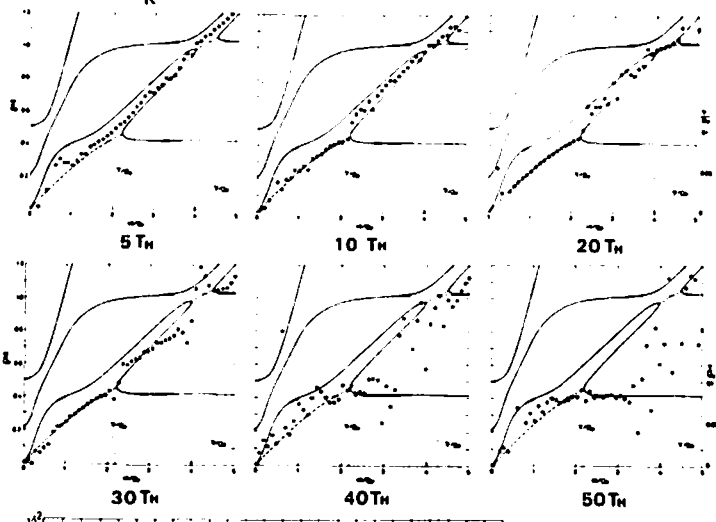

Fig. 5 Time evolution of the dispersion relations obtained from the simulation for the first case.

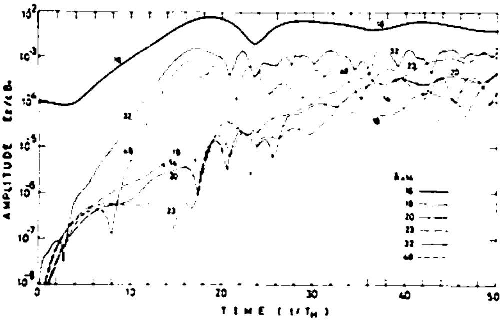

Fig. 6 Simulation results showing time evolution of k-components of Ez for the second case.