# Analysis of the Relations between Nearfield Tolerances and Inaccuracies in Calculated Farfield Patterns

Hisao IWASAKI and Yoshihiko MIKUNI
TOSHIBA Research and Development Center, Kawasaki, Japan

## 1. Introduction

Shaped beam antennas and multiple beam antennas for satellite communication use are desired to have low sidelobe level and low cross polarization characteristic to decrease interference.

A precise measurement method to evaluate antenna performances accurately is required to develop such antennas.

Planar near field antenna measurement is one of the most adequate methods. However, acceptable tolerances, which involve probe position error and instrumentation error, for test facility design have not been clear by an analytical method for the various antennas which have been tested.

These tolerances have been investigated experimentally or using by computer simulations.(1),(2) This paper clarifies the acceptable near field tolerances required to insure a given far field accuracy by an analytical method.

This paper describes the analysis of the relations between near field tolerances and accuracies in calculated far field patterns. Quantitative evaluations are carried out to prove this analysis under the condition that near field tolerances have some statistical property. A more detailed explanation will be presented in the following sections.

## 2. Analysis

### 2-1 Probe position tolerance

Figure 1 shows the coordinate system which is used in this analysis.

The desired measurement plane is considered at $z = 0$.

In general, errors in the probe drive mechanism will contain both systematic and randam probe position tolerances.

A difference exists between the desired near field at point $r_t$ and the field actually measured at point $r_m$, due to probe position tolerance.

Electric field $\mathbf{E}_m$ at point $\mathbf{r}_m$ is given by the following equation:

$$\mathbf{E}_m(\mathbf{r}_m) = \frac{1}{(2\pi)^3} \int \mathbf{A}_t(\mathbf{k})\exp(-j\,{}^t\mathbf{r}_m\cdot\mathbf{k})d\mathbf{k} \quad , \tag{1}$$

$$\mathbf{r}_m = T\mathbf{r}_s + \mathbf{r}_o + T\mathbf{e}_s \quad . \tag{2}$$

$$\mathbf{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z \quad . \tag{3}$$

$\mathbf{A}_t(\mathbf{k})$ is the true plane wave spectrum which is radiated test antenna and $\mathbf{k}$ is the wavenumber vector. t denotes transpose and T denotes a linear transformation matrix which transforms vectors in $O_u$ coordinate into vectors in $O_d$ coordinate.

Replacing the vector as following equation:

$$\mathbf{V} = {}^tT \cdot \mathbf{k} \tag{4}$$

and substituting Eqs. (2) and (4) into Eq. (1), the following equation is obtained:

$$\mathbf{E}_m(\mathbf{r}_m) = \frac{1}{(2\pi)^3} \int \mathbf{A}_t(T\mathbf{V})\exp(-j\,{}^t\mathbf{r}_o\cdot T\mathbf{V})\cdot\exp(-j\,{}^t\mathbf{r}_s\cdot\mathbf{V})\cdot\exp(-j\,{}^t\mathbf{e}_s\cdot\mathbf{V})d\mathbf{V} \quad . \tag{5}$$

On the other hand, plane wave spectrum $\mathbb{A}(\mathbb{K})$ which contains error spectrum, calculated from the actually measured fields $\mathbb{E}_m(\mathbb{r}_m)$, is given by the following equation:

$$\mathbb{A}(\mathbb{K}) = \int \mathbb{E}_m(r_m)\cdot\exp(j^t\mathbb{r}_t\cdot\mathbb{K})d\mathbb{K} \quad , \qquad (6)$$
$$\mathbb{K} = \hat{a}_x k_x + \hat{a}_y k_y \quad .$$

Here, the elements of the position vector $\mathbf{r}_s$ and $\mathbb{r}_t$ are equal and those for $\mathbf{e}_s$ and $\mathbf{e}_t$ are equal.

Furthermore, position error $e_t$ is very small, in comparison with wavelength in a practical system, so the following approximation may be derived:

$$\exp(-j^t e_t\cdot\mathbb{V}) \fallingdotseq 1 - j^t e_t V \quad . \qquad (7)$$

Substituting Eq. (5) into (6), far field pattern error $\varepsilon(\mathbb{K})$, which is caused by probe position tolerance, is expressed by the following equation:

$$\varepsilon(\mathbb{K}) = \mathbb{A}(\mathbb{K}) - \frac{1}{2\pi}\int A_t(Tk)\exp(-j^t\mathbb{r}_o Tk)dk_z$$
$$= \int \exp(j^t\mathbb{r}_t\cdot\mathbb{k})\cdot\{\frac{1}{(2\pi)^3}\int A_t(T\mathbb{V})\exp(-j^t\mathbb{r}_o\cdot T\mathbb{V})$$
$$\cdot\exp(-j^t\mathbb{r}_t\cdot\mathbb{V})\cdot{}^t\mathbf{e}_t\cdot\mathbb{V}\cdot d\mathbb{V}\}d\mathbb{r}_t \quad . \qquad (8)$$

Now, let us consider the upper band of $\varepsilon(\mathbb{K})$ by an analytical method.

Using the Schwarz and Minkowski inequality equations and normalizing Eq. (8) by all radiated power, the normalized error spectrum $\varepsilon_1$ is expressed by the following equation:

$$|\varepsilon_1(\mathbb{K})|^2 < [\{\int|\mathbb{B}_x(\mathbb{K}-\mathbb{P}|^2\cdot|P_x|^2 d\mathbb{P}\}^{1/2} + \{\int|\mathbb{B}_y(\mathbb{K}-\mathbb{P})|^2\cdot|P_y|^2 d\mathbb{P}\}^{1/2}$$
$$+ \{\int|\mathbb{B}_z(\mathbb{K}-\mathbb{P})|^2\cdot|P_o - P_x - P_y|^2 d\mathbb{P}\}^{1/2}]^2 \quad , \qquad (9)$$

$$P_o = 2\pi/\lambda, \quad \mathbb{P} = \hat{a}_x P_x + \hat{a}_y P_y \quad ,$$
$${}^t\mathbb{B}(\mathbb{P}) = \int {}^t e_t \exp(j^t\mathbb{r}_t\cdot\mathbb{P})d\mathbb{r}_t \quad . \qquad (10)$$

Equation (9) represents the relationship between probe position tolerance and the inaccuracies in a calculated far field pattern.

## 2-2 Instrumentation error

The errors in the near field data, caused by the inaccuracies in the receiver or instrumentation used to measure the probe output, produce error in the calculated far field.

An analysis of the relation between the error in the near field and that in the calculated far field is accomplished by the same method.

In this analysis, the actually measured field $E_m$ is expressed as the following equation:

$$\mathbb{E}_m(X) = E(\mathbb{X})\{1+\Delta I(\mathbb{X})\}\exp\{-j(\phi(\mathbb{X})+\Delta\phi(\mathbb{X}))\} \quad . \qquad (11)$$

$E(\mathbb{X})$ and $\phi(\mathbb{X})$ represent the true amplitude and ture phase and $\Delta I(\mathbb{X})$ and $\Delta\phi(X)$ represent the amplitude error and the phase error.

The normalized error spectrum $\varepsilon_2$ is given by the following equation:

$$|\varepsilon_2(\mathbb{K})|^2 < \frac{1}{(2\pi)^4}[\{\int|\mathbb{G}(\mathbb{K}-\mathbb{P})|^2 d\mathbb{P}\}^{1/2} + \{\int|\mathbb{Q}(\mathbb{K}-\mathbb{P})|^2 d\mathbb{P}\}^{1/2}$$
$$+ \{\int|\mathbb{S}(\mathbb{K}-\mathbb{P})|^2 d\mathbb{P}\}^{1/2}]^2 \quad , \qquad (12)$$

$$\mathbb{G}(\mathbb{K}) = \int\Delta I(X)\exp(j\mathbb{K}\cdot\mathbb{X})dX \quad , \qquad (13)$$
$$\mathbb{Q}(\mathbb{K}) = \int\Delta\phi(X)\exp(j\mathbb{K}\cdot\mathbb{X})dX \quad , \qquad (14)$$
$$\mathbb{S}(\mathbb{K}) = \int\Delta I(X)\cdot\Delta\phi(\mathbb{X})\exp(j\mathbb{K}\cdot X)dX \quad , \qquad (15)$$
$$\mathbb{X} = \hat{a}_x x + \hat{a}_y y \quad .$$

## 3. Quantitative evaluation

Consider the case whose probe position and instrumentation error have some statistical properties, to evaluate the upper band of the normalized error spectrum $|\varepsilon_1|$ and $|\varepsilon_2|$.

In planar near field measurement, the above error sources can be expressed as the summation of the systematic and randam error components.

Therefore, in this paper, statistical properties for probe, amplitude and phase errors are modeled so as to be expressed by the linear Markoff process. Auto correlation with their distribution is expressed by the following equation:

$$R(\tau) = \rho_1{}^2\, e^{-f_1|\tau|} - \rho_2{}^2\, e^{-f_2|\tau|} \quad . \qquad (16)$$

The relationship between probe position error and the normalized error spectrum is shown in Fig. 2.

Also, the relation between amplitude and phase errors and the normalized error spectrum is shown in Fig. 3.

In this calculations, the values of auto correlation are chosen in consideration of the practical planar near field test facility.

Figures 2 and 3 indicate that the z axis direction probe position and phase error are critical tolerances in insuring a given far field accuracy.

Relations between near field tolerances and accuracies in calculated far field patterns are calculated by using Figs. 2 and 3.

## 4. Conclusion

It has been clarified that acceptable near field tolerances are required to insure a given far field accuracy, using the analytical method.

These expressions are written in a form that can be used to stipulate design criteria for the construction of planar near field scanning facilities.

## 5. References


(1) Kerns. D. M., "Correction of Near-Field Antenna Measurement Made with an Arbitrary but known Measuring Antenna", Electrics Letters, 6, 11, 29, May, 1970.

(2) Newell, A. C. and M. L. Crowford, "Planar Near Field Measurements on High Performance Array Antennas" NBSIR 74-380, July, 1974.


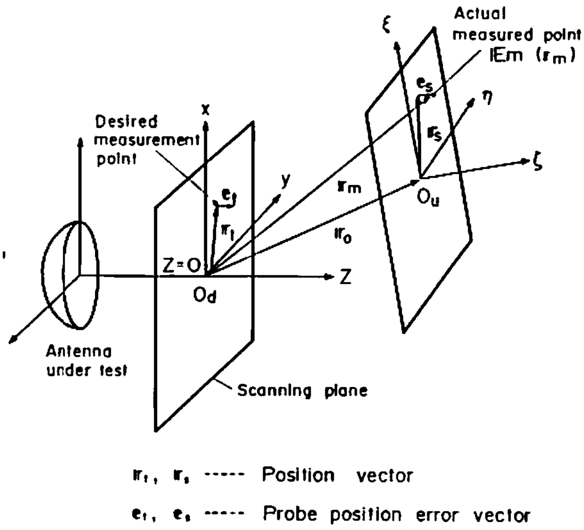


Figure 1. Coordinate system used in this analysis.

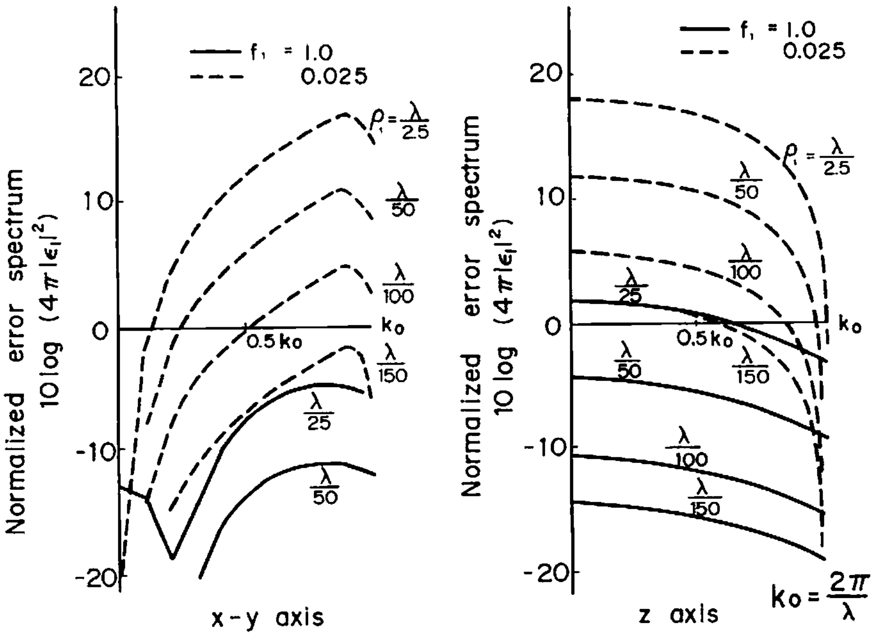


Figure 2. Relationship between probe position error and normalized error spectrum.

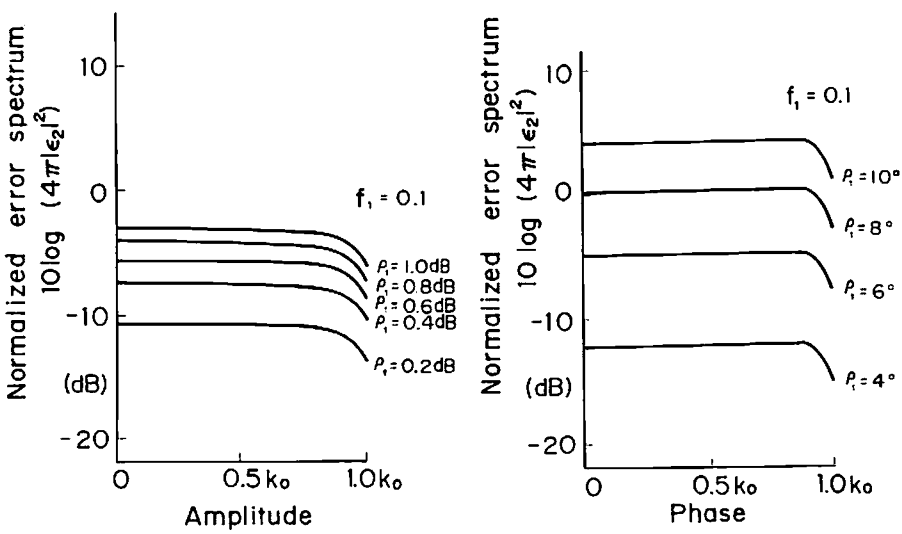


Figure 3. Relation between amplitude and phase errors and the normalized error