SCATTERING OF A THREE-DIMENSIONAL HERMITE-GAUSSIAN BEAM BY PARALLEL CONDUCTING CIRCULAR CYLINDERS AT OBLIQUELY INCIDENCE

Mitsuhiro Yokota and Otozo Fukumitsu
Department of Computer Science and Communication Engineering
Kyushu University 36, Fukuoka 812, Japan

INTRODUCTION

It is well known that a field of radiation from a waveguide horn or from a laser cavity is well approximated by a linear superposition of paraxial beams. So it is important to study the propagation and scattering of beams. The scattering of circular cylinders which is dealt with the two dimensional problem has been studied by many workers.

Recently, Langlois *et al.* [1] investigated the diffraction problem of a transversal three-dimensional Gaussian beam at grazing incidence. The authors [2] analyzed the scattering of the three-dimensional lowest-order Gaussian beam at obliquely incidence upon the parallel circular cylinders.

In this paper, we consider the scattering of a three-dimensional Hermite-Gaussian beam at obliquely incidence upon the parallel conducting circular cylinders by using the complex-source-point method [3] and the dyadic Green's functions [4]. As numerical examples, the fields scattered by an array of eight cylinders are calculated and examine the effects of the tilt of the incident beam axis on the scattered fields. The case of an E-polarized wave incidence is discussed and the time factor $\exp(j\omega t)$ is suppressed.

EXPRESSION OF THE INCIDENT BEAM

In this section, we consider the expression of an incident Hermite-Gaussian beam in terms of the complex-source-point multipole fields. The radiated field from the infinitesimal current, which is located at the complex source point $(0,\ 0,\ -jb)$ in the coordinate frame $(X,\ Y,\ Z)$ and polarized in the X direction, is represented by using the vector potential $\mathbf{A}$ as follows:

$$\mathbf{E} = -j\omega\left(\mathbf{A}+\frac{1}{k^2}\nabla\nabla\cdot\mathbf{A}\right),\quad \mathbf{H} = \frac{1}{\mu_0}\nabla\times\mathbf{A},\quad \mathbf{A} = \frac{\mu_0}{4\pi}\frac{\exp[-jkR]}{R}\mathbf{i}_X\ , \tag{1}$$

where $k(=2\pi/\lambda)$ is the wave number in free space, $R(=\sqrt{X^2+Y^2+(Z+jb)^2})$ is the complex distance from the source point to the field point $(X,\ Y,\ Z)$, b is related with the smallest spot size $w_0(=\sqrt{2b/k})$ and $\mathbf{i}_X$ is the unit vector in the X direction. In this paper, we choose the branch of R such that its real part is positive in order to satisfy the radiation condition.

We define the vector field excited by a multipole as

$$\mathbf{G}_{mn} = \left(\frac{\partial}{\partial X}\right)^m\left(\frac{\partial}{\partial Y}\right)^n\mathbf{E}\ . \tag{2}$$

If the field is far from the branch line $(|kR|\gg 1)$ and in the paraxial region $(X^2+Y^2\ll Z^2+b^2)$, the multipole field is approximated by

$$\mathbf{G}_{mn} \sim -\frac{\omega\mu_0}{4\pi}\frac{\exp(kb)}{b}\left(\frac{-1}{w_0}\right)^{m+n}\widetilde{\psi}_{mn}\mathbf{i}_X\ , \tag{3}$$

where $\widetilde{\psi}_{mn}$ is the complex beam proposed by Siegman [5]. When we use the relation between the conventional and complex beam [6], the conventional Hermite-Gaussian beam ψ_{mn} which is mainly polarized in the X direction is expressed as follows:

$$\psi_{mn}\mathbf{i}_X \sim \frac{-2kw_0\sqrt{\pi\, m!\, n!}}{\omega\mu_0}\exp(-kb)$$

$$\cdot\sum_{p=0}^{[m/2]}\sum_{q=0}^{[n/2]}\frac{2^{-(p+q)}(-w_0)^{m+n-2(p+q)}}{p!\,q!\,(m-2p)!\,(n-2q)!}\mathbf{G}_{m-2p\ n-2q} \quad . \tag{4}$$

SCATTERING BY CYLINDERS

Consider an arbitrary configuration of parallel conducting circular cylinders as shown in Fig. 1. For the following analysis, we introduce the coordinate frame (x_i, y_i, z_i) associated with the ith cylinder and the reference coordinate frame (x, y, z). The axis of the ith cylinder with radius a_i coincides with the z_i axis and the x_i axis is parallel to each other.

The coordinates X, Y and Z are related to the coordinates x_i, y_i and z_i by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} l_{1i} & m_{1i} & n_{1i} \\ l_{2i} & m_{2i} & n_{2i} \\ l_{3i} & m_{3i} & n_{3i} \end{pmatrix}\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \qquad i=1,2,\cdots,N \ , \tag{5}$$

where N is the number of cylinders and l_{1i}, l_{2i} and l_{3i} are the direction cosines of x_i axis with respect to the X, Y and Z axes, and so on.

In this paper, the dyadic representation of fields is used in order to investigate the scattering of a beam whose propagation axis is tilted in any direction. By utilizing the electric dyadic field for the infinitesimal current [4], the electric dyadic field for a multipole is given by

$$\overline{\mathbf{G}}_{mn}(\mathbf{R}_i, \mathbf{R}_i^s) = -\frac{j}{8\pi}(-1)^{m+n}\int_{-\infty}^{\infty} dh \sum_{l=-\infty}^{\infty}\frac{(-1)^l}{\eta^2}$$

$$\begin{cases} [\mathbf{M}_l^{(2)}(\mathbf{R}_i, \eta, h)\,\boldsymbol{\alpha}_l^{(1)}(m,n,i;\,h) + \mathbf{N}_l^{(2)}(\mathbf{R}_i, \eta, h)\,\boldsymbol{\beta}_l^{(1)}(m,n,i;\,h)], \ r_i > |r_i^s| & (6) \\ [\mathbf{M}_l^{(1)}(\mathbf{R}_i, \eta, h)\,\boldsymbol{\alpha}_l^{(2)}(m,n,i;\,h) + \mathbf{N}_l^{(1)}(\mathbf{R}_i, \eta, h)\,\boldsymbol{\beta}_l^{(2)}(m,n,i;\,h)], \ r_i < |r_i^s| \ , & (7) \end{cases}$$

where

$$\boldsymbol{\alpha}_l^{(\nu)}(m,n,i;\,h) = \sum_{p=0}^{m}\sum_{q=0}^{p} P(m,p,q)\sum_{p'=0}^{n}\sum_{q'=0}^{p'} Q(n,p',q')$$

$$\cdot\mathbf{M}^{(\nu)}_{-l+(p+p')-2(q+q')}(\mathbf{R}_i^s, \eta, -h) \quad (\nu=1,2) \quad , \tag{8}$$

$$\boldsymbol{\beta}_l^{(\nu)}(m,n,i;\,h) = \sum_{p=0}^{m}\sum_{q=0}^{p} P(m,p,q)\sum_{p'=0}^{n}\sum_{q'=0}^{p'} Q(n,p',q')$$

$$\cdot\mathbf{N}^{(\nu)}_{-l+(p+p')-2(q+q')}(\mathbf{R}_i^s, \eta, -h) \quad , \tag{9}$$

$$P(m,p,q)=\frac{m!}{(m-p)!\,(p-q)!\,q!}\,P_0^{\,m-p}\,P_{-1}^{\,p-q}\,P_1^{\,q} \quad ,$$

$$P_{-1}=-\frac{\eta}{2}\,(l_{1i}+jm_{1i}),\ P_0=jhn_{1i},\ P_1=\frac{\eta}{2}\,(l_{1i}-jm_{1i}) \quad , \tag{10}$$

$$Q(n,p',q')=\frac{n!}{(n-p')!\,(p'-q')!\,q'!}\,Q_0^{\,n-p'}\,Q_{-1}^{\,p'-q'}\,Q_1^{\,q'} \quad ,$$

$$Q_{-1}=-\frac{\eta}{2}\,(l_{2i}+jm_{2i}),\ Q_0=jhn_{2i},\ Q_1=\frac{\eta}{2}\,(l_{2i}-jm_{2i}) \quad . \tag{11}$$

$\mathbf{R}_i$ and $\mathbf{R}_i^s$ are the position vectors of the observation point and the source point, respectively, in the coordinate frame of the ith cylinder and $\eta=\sqrt{k^2-h^2}$. The cylindrical vector wave functions $\mathbf{M}_l^{(\nu)}$ and $\mathbf{N}_l^{(\nu)}$ are defined in the cylindrical coordinate frame (r_i, ϕ_i, z_i) of the ith cylinder [4].

The scattered electric dyadic field by the ith cylinder which ensures the radiation condition is expressed as follows:

$$\overline{\mathbf{G}}_{mn}^{\,s}(i)=-\frac{j}{8\pi}(-1)^{m+n}\int_{-\infty}^{\infty}dh\sum_{l=-\infty}^{\infty}\frac{(-1)^l}{\eta^2}$$

$$\cdot[\,\mathbf{M}_l^{(2)}(\mathbf{R}_i,\eta,h)\,\mathbf{A}_l(m,n,i;\,h)+\mathbf{N}_l^{(2)}(\mathbf{R}_i,\eta,h)\,\mathbf{B}_l(m,n,i;\,h)\,] \quad . \tag{12}$$

Applying the boundary conditions at r_i=a_i after expressing the scattered electric field by the ith cylinder in the coordinate frame of the i'th cylinder [7], we obtain the equations which determine the unknown coefficients $\mathbf{A}_l$ and $\mathbf{B}_l$.

Since a higher-order beam is expressed in terms of the multipole fields, the total scattered field is given by

$$\psi_{mn}^{\,s}\,\mathbf{i}_X\sim-\frac{2\sqrt{\pi\,m!\,n!}\;kw_0}{\omega\mu_0}\exp(-kb)$$

$$\cdot\sum_{i=1}^{N}\sum_{p=0}^{[m/2]}\sum_{q=0}^{[n/2]}\frac{2^{-(p+q)}(-w_0)^{m+n-2(p+q)}}{p!\,q!\,(m-2p)!\,(n-2q)!}\,\overline{\mathbf{G}}_{m-2p\ n-2q}^{\,s}(i)\cdot\mathbf{i}_X \quad . \tag{13}$$

The evaluation of the integral in Eq. (12) by the method of steepest descent, after using the asymptotic formulas for the Hankel functions, leads to the far scattered field.

Numerical calculations are carried out for the scattering pattern of the lowest-order beam (m=n=0) with w_0=2.5λ incident upon the eight same cylinders(N=8) with a=0.1λ and the separation d=λ located symmetrical on the y axis. It is assumed that the propagation axis of the incident beam makes an angle of θ_0(=70°) with z axis and ϕ_0(=30°) with x axis and the distance between the reference origin and the position of the beam waist $\overline{oO}$=5λ. Figure 2 shows the normalized scattering pattern for the θ component in the azimuthal plane(θ=110°) where the scattered field has the maximum value. The right and left semicircles in Fig. 3 correspond to the meridional plane ϕ=30° and ϕ=210°, respectively. The scattering pattern in the azimuthal plane is different from that at normal incidence because the condition for the angle of the space harmonics of order -1(Bragg angle) at obliquely incidence deviates from that at normal incidence. The scattering pattern in the meridional plane is approximately identical with the far field pattern of the incident beam.

CONCLUSIONS

The scattering of a three-dimensional Hermite-Gaussian beam at obliquely incidence upon the parallel conducting circular cylinders has been analyzed by using the complex-source-point method. Numerical calculations have been carried out for the scattering pattern of the lowest-order beam. The results indicate that the scattering pattern in the meridional plane is almost the same as the far field pattern of the incident beam and in the azimuthal plane the directions of peaks of the scattered beams at obliquely incidence are different from those at normal incidence because of the effect of the tilt of the incident beam axis. The method in this paper can be extended to the scattering of parallel dielectric circular cylinders.

REFERENCES

[1] P. Langlois *et al.*, J. Opt. Soc. Am. A, **2**, pp.858-862(1985).
[2] T. Takenaka *et al.*, Proc. of ICAP, pp.31-34(1987).
[3] L. B. Felsen, Symposia Matematica, **18**, pp.319-359(1976).
[4] C. T. Tai, "Dyadic Green's Functions in Electromagnetic Theory", Scraton. PA:Intext Educational Publishers(1971).
[5] A. E. Siegman, J. Opt. Soc. Am. **63**, pp.1093-1094(1973).
[6] M. Yokota *et al.*, J. Opt. Soc. Am. A, **3**, pp.580-586(1986).
[7] W. M. Leach and D. T. Paris, IEEE Trans. Antennas & Propag., **AP-21**, pp.435-445(1973).

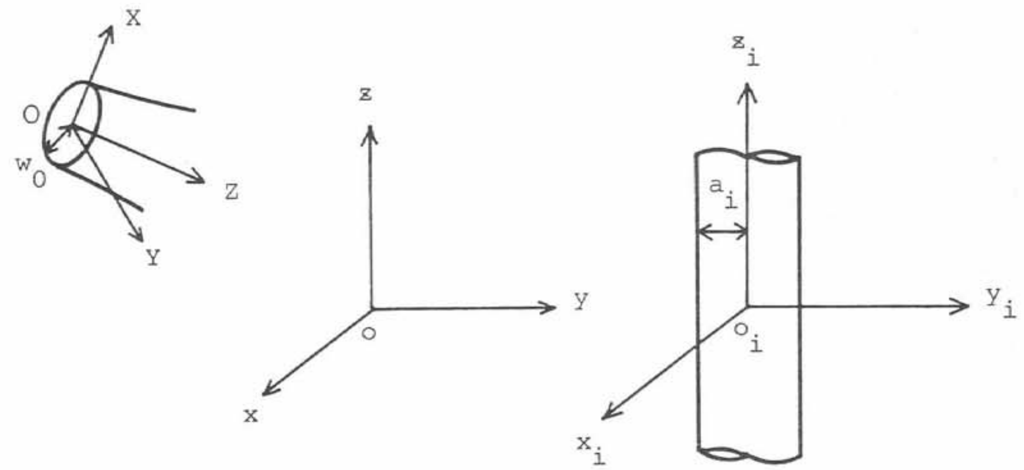

Fig. 1. Geometry of the problem.

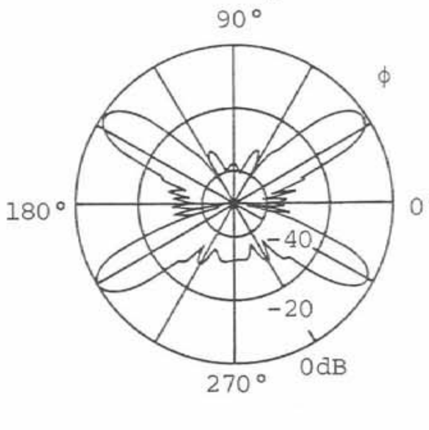

Fig. 2. Scattering pattern in the azimuthal plane θ=110°.

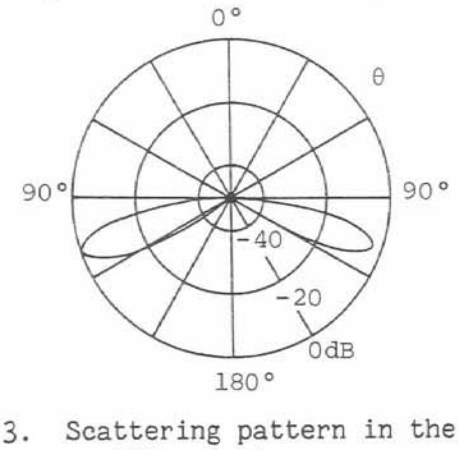

Fig. 3. Scattering pattern in the meridional plane ϕ=30° (right semicircle), ϕ=210° (left semicircle).