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PULSE WAVE PROPAGATION IN A RANDOMLY PERTURBED LIGHT-FOCUSING BEAM WAVEGUIDE

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1. Introduction

The light waveguide with a parabolic or rectangular distribution of refractive index is suitable for optical communication over long distances[1,2]. However, imperfections of the refractive index distribution, due to manufacturing processes and unintentional bends will produce deformation of the configuration of the signal mode. Each waveguide has significantly different characteristics with respect to the mode properties in actual situation. In this paper, the electromagnetic field in a beam waveguide with a parabolic distribution of refractive index is studied. The statistical properties of the mode conversion and pulse wave distortion due to fluctuations of the dielectric constant and the irregularities of bends of the beam waveguide are investigated by the stochastic process theory of the electromagnetic wave propagation. A new conformal mapping technique and the associated vector integral equation are derived in a form suitable for application to the randomly perturbed waveguide[3,4].

2. The electromagnetic field in a randomly perturbed light beam waveguide

This section considers the optical field in the beam wavegude with randomly distributed refractive index $\varepsilon = \varepsilon^{(1)} + \varepsilon^{(2)} = \varepsilon'\{1-(l_x x)^2-(l_y y)^2\} + \varepsilon^{(2)}(x,y,z)$ and the random radius of curvature $R(z)$, where $\varepsilon^{(2)}(x,y,z)$ and $R(z)$ are the random functions. By use of the conformal mapping technique and Green's theorem[3]

$$E_{toll} = E_{inc} + \int \Big[\{-(E_{toll}-E_{inc})\nabla\nabla\cdot + \omega^2(\varepsilon^{(1)}-\varepsilon^{(inc)})\mu E_{inc} + (\omega^2\varepsilon^{(2)}\mu E_{toll} - F(E_{toll})\}\Gamma - \omega^2\varepsilon^{(1)}\mu(E_{toll}-E_{inc})(\Gamma-\Gamma_0)\Big] dv \quad (1)$$

The vector function $F(E)$ is determined by the metrical functions of bends. Γ is the Green's dyadic for the ideal beam waveguide. Using the technique of the functional analysis and the theory of stochastic process, the statistical average and the second moment of the total field are determined in the form of an iterative series related to the multiple process of the mode conversions.

We discuss a general case where the incident wave E_{inc} is a $(\underline{m},\underline{n})$ mode with a amplitude $E_0 \dot{i}_z$. The output field of the randomly perturbed waveguide with multiple process of mode conversion and reconversion is given by

$$\langle E_{toll}\rangle = E_{inc} + \sum_{m,n} \dot{i}_z e^{-j\sqrt{\quad}\,l} e^{-a_r^2 r^2} H_m(a_r x) H_n(a_r y) \frac{E_0}{4\pi^2} \frac{\beta^2}{2^{n+m} n!\,m!} \sum \frac{l_r^2}{2^{\underline{n}+\underline{m}}\underline{n}!\,\underline{m}!} \Big[\langle \gamma_{\underline{m}\underline{n}:mn}\gamma_{\underline{m}\underline{n}:mn}\rangle \langle \delta\varepsilon(z)\rangle G_z \quad (2)$$

$$+ 4K_x(1,m,\underline{m})K_x(1,\underline{m},m)K_y(0,n,\underline{n})K_y(0,\underline{n},n)\langle \delta\tfrac{1}{R}\rangle G_R\Big]$$

where

$$\sqrt{(\ )(\ )} = \sqrt{\beta^2-(2\underline{m}+1)l_x\beta-(2\underline{n}+1)l_y\beta}, \quad a_r^2 = a_x^2 = a_y^2 = \beta l_r = \omega\sqrt{\varepsilon'\mu}\, l_r, \quad l_r = l_x = l_y$$

and phase factor differences between two modes $\Delta\beta = \sqrt{\quad} - \sqrt{\quad}$, $\Delta\beta^{(2)} = \sqrt{(\ )(\ )} - \sqrt{\quad}$

$$G_\zeta = l_r^2 (j\Delta\beta'' - \tfrac{1}{\gamma_\zeta})^{-1} (j\Delta\beta' - \tfrac{1}{\gamma_\zeta})^{-1} \Big[e^{j\Delta\beta' l} - \tfrac{l}{\gamma_\zeta} + \tfrac{1}{j\Delta\beta}\{ (j\sqrt{\quad} - j\sqrt{\quad}\, e^{j\Delta\beta l}) + (\sqrt{\quad} + \tfrac{1}{\gamma_\zeta})(e^{j\Delta\beta l} - 1)\}\Big]$$

$$K_t(l,m,\underline{m})a_t = \tfrac{1}{2}K_t(l-1,m+1,\underline{m}) + mK_t(l-1,m-1,\underline{m})$$

$$K_t(0,m,\underline{m}) = \tfrac{1}{a_t} 2^{\underline{m}}\underline{m}!\sqrt{\pi}\ (m=\underline{m}),\ 0\ (m\neq\underline{m}),\ (t=x,y)$$

for example

$$\langle \gamma_{\underline{m}\underline{n}:00}\gamma_{\underline{m}\underline{n}:00}\rangle = \langle \delta\varepsilon(x,y)\rangle \pi \prod_{\substack{u=\underline{m},\underline{n}\\ t=x,y}} \frac{\gamma_t 2^u (2u-1)!!}{a_t(\gamma_t^2 a_t^2+2)^{u/2+1/2}} \quad (\zeta = z, R)$$

l is the length of the guide line.

In above equation, the autocorrelation of the fluctuations are assumed to be exponential,

$$\langle \varepsilon^{(2)}(z'')\varepsilon^{(2)}(z''+z')\rangle = \langle \delta\varepsilon^{(2)}(z)\rangle e^{-\frac{z'}{\gamma_z}}$$

$$\langle \frac{1}{R(z'')}\frac{1}{R(z''+z')}\rangle = \langle \delta\frac{1}{R}\rangle e^{-\frac{z'}{\gamma_R}}$$

and gaussian

$$\langle \varepsilon^{(2)}(x',y')\varepsilon^{(2)}(x'',y'')\rangle = \langle \delta\varepsilon^{(2)}(x,y)\rangle e^{-\frac{(x'-x'')^2}{\gamma_x^2}-\frac{(y'-y'')^2}{\gamma_y^2}}$$

3. Pulse wave propagation in a random beam waveguide

This section contains an investigation of the pulse deformation along the waveguide in case of the signal wave of the input $S(t) = e^{-(\frac{2t}{T})^2}\cos\omega t$.

By application of the asymptotic method for Fourier integral with regard to ω, to the preceding results, the output wave form of the signal can be shown as indicating the suffix 「o」 for the values at ω_0.