Radar Cross Section of a Rotating Rectangular Metal Plate
by High Frequency Methods

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Abstract - In this paper, we use the physical optics (PO) method and the method of equivalent currents (MEC) to calculate the radar cross section (RCS) of a rotating rectangular metal plate from various observation directions. It is shown that for small offset angles, the MEC gives almost the same RCS as that predicted by the PO method. When the offset angle becomes significant, the MEC gives far more accurate results than the PO method and predicts two peak values of RCS which are the same as those predicted by geometric theory of diffraction(GTD).

I. INTRODUCTION

Many methods such as, physical optics (PO), geometric theory of diffraction (GTD), method of equivalent currents (MEC) etc, for computing the radar cross section (RCS) of an object have been proposed. For specular scattering from plates, PO provides a useful high frequency (HF) approximation. GTD is also a HF approximation but it fails when the observation point is not in the direction of the Keller cone [1] and predicts infinite fields at caustics. In order to overcome the latter drawback of GTD, the method of equivalent currents (MEC) was proposed. Unfortunately, the expressions of the equivalent currents, which depend on the direction of observation, have been derived only for directions lying on the Keller cone of the diffracted rays [2], for a given point on the edge.   In 1984 and 1986, Arie Michaeli proposed a new MEC method which is valid for arbitrary aspect angles of observation [2,3]. Michaeli reduced the asymptotic surface integral to an edge integral. Since the surface integral involves the total currents near the edge, Michaeli's edge current contains two components, the fringe component and the physical optics (PO) component. The fring component is due to the presence of the discontinuous surface and is added to correct the PO current. Thus, for a metal plate, we can obtain the backscattering field at any observation point by integrating the current alone the plate's edge. That is, if the direction of observation is fixed at the direction of the incidence (i.e. backscattering), no matter what position the plate rotates to, we can obtain the backscattering field by using Mechaeli's MEC edge integal. In this paper, the RCS of a rotating rectangular metal plate is calculated as a function of rotating angle using Mechaeli's method. It is shown that the function has a period corresponding to half a cycle of the rotation period and two peaks which occur when the edges are perpendiculer to the direction of the incidence (i.e., the angle of the Keller cone is 90 degrees). The RCS of the rotating plate obtained based on the PO approximation is included in the paper for a comparison.

II. HIGH FREQUENCY METHODS

In this section, we derived the scattered fields from a rectangular metal plate using PO method and MEC. The plate is assumed to rotate in time, and the scattered field at any given time is calculated by assuming that the

instantaneous position of the plate is "frozen in its track". we assume that (1) all material velocities are much less than the velocity of light, and (2) the angular frequency of the rotation of the body is very small compared to the angular frequency of the incident electromagnetic waves.

Fig. 1 illustrates a rectangular plate of dimensions $L_1 \times L_2$, at time t=0, located in the x-z plane with its axis A-A' aligned along the z-axis of a rectangular coordinate system. The axis A-A' of the plate rotates in the x-z plane with angular frequency w. The configuration at time t_0 is as shown in Fig. 2, where $\theta_s = wt_0$.
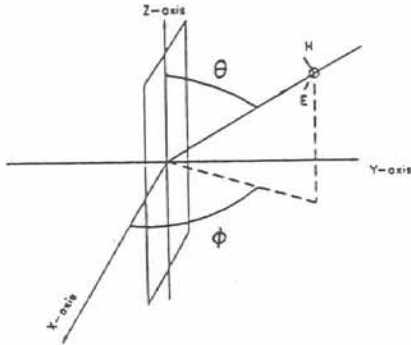
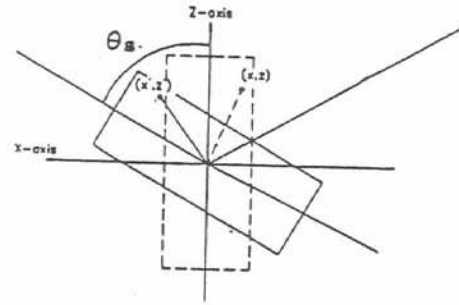Fig. 1 Configuration of a rotating rectangular metal plate illuminated by an incident EM wave at t=o.

Fig. 2 Configuration of a rotating rectangular metal plate illuminated by an incident EM wave at t=t₀.

A plane wave polarized in the x-y plane is incident on the plate from direction (θ, ϕ) as shown in Fig. 1. The incident fields E_i, H_i are given by

$$\overline{E}^i = \hat{a}_E \, E_0 \, \exp\{jk(x \sin\theta \cos\phi + y \sin\theta \sin\phi + z \cos\theta)\} \qquad (1a)$$

$$\overline{H}^i = \hat{a}_H \, H_0 \, \exp\{jk(x \sin\theta \cos\phi + y \sin\theta \sin\phi + z \cos\theta)\} \qquad (1b)$$

where E_0, H_0 are the complex amplitude of the incident plane wave at the origin, k is the wavenumber of the incident wave and

$$\hat{a}_E = \sin \phi \, \hat{x} - \cos \phi \, \hat{y} \qquad (2a)$$

$$\hat{a}_H = -\cos \theta \cos \phi \, \hat{x} - \cos \theta \sin \phi \, \hat{y} + \sin \theta \, \hat{z} \qquad (2b)$$

A.) Physical Optics (PO) Method:

According to physical optics, the surface current density J_s at a point r(x,z) on the illuminated side S of the plate is given by

$$\overline{J}_s(r) = 2(\hat{n} \times \overline{H}^i|_s)$$

$$= (\cos \theta \cos \phi \, \hat{z} + \sin \theta \, \hat{x}) \cdot 2H_0 \cdot$$
$$\exp \{jk(x' \sin\theta \cos\phi + y' \sin\theta \sin\phi + z' \cos\theta)\} \qquad (3)$$

From Fig. 2, we obtain :
$$x' = x \cos \theta_s + z \sin \theta_s \qquad (4a)$$
$$y' = y = 0 \qquad (4b)$$
$$z' = -x \sin \theta_s + z \cos \theta_s \qquad (4c)$$
Then, the scattered fields E^s and H^s in the far field are given by

$$\overline{E}^s = -Z \; \widehat{R} \times (\; -jk/4\pi \;)\widehat{R} \times \iint_S \overline{J}_s \; G(\overline{r},\overline{R}) \cdot \widehat{y} \; dxdy \qquad (5)$$

with
$$G(\overline{r},\overline{R}) = \exp(-jk|\overline{R} - \overline{r}|)/ \; 4\pi \; |\overline{R} - \overline{r}| \qquad (6)$$
$$\sim [\exp(-jkR)/4\pi R]\exp(jk\overline{R}\cdot\overline{r})$$
$$\sim [\exp(-jkR)/4\pi R]\exp(jk(x'\sin\theta\cos\phi+y'\sin\theta\sin\phi+'z\cos\theta))$$

where $\overline{R} = R \; \widehat{R} = (R, \theta, \phi)$ is the observation point in the far field, $\overline{r}$ is a point on S and Z is the wave impedance. And the coordinate transformation is given by Eqs.(4a),(4b) and (4c).

B.) Method of Equivalent Currents:

By Michaeli's equivalent current method, in the far-field the scattered field E^s at an observation point which is not necessary at the Keller cone or at the specular reflection point is approximated by

$$\overline{E}^s \sim jk \int_c [\; ZI \; \widehat{s}\times(\widehat{s}\times\widehat{t}) + M \; \widehat{s}\times\widehat{t} \;] \; G(\overline{r},\overline{R}) \; dl \qquad (7)$$

where $\widehat{s}$ is the direction of the incident wave, $\widehat{t}$ is the direction of the tangent to the edge of the rectangular plate, I is the electric equivalent current and M is the magnetic equivalent current. And the Green function is given by Eq.(6).

In the case of backscattering, $\widehat{s}=-\widehat{R}=-(1,\theta,\phi)$ and

$$I = I^{po} + I^f \text{ (PO component + fringe component)}$$
$$= E_t^i \; \frac{jY}{K \sin^2\beta} \; \{ \; \frac{\sin\delta \; \sin\phi}{\sin\delta \; \cos\phi - \cos\delta \; \cot\beta}$$
$$+ \; \frac{\sqrt{2} \sin(\phi / 2)}{\cos\phi - \cot^2\beta} \; [\; \sqrt{1-u} - \sqrt{2} \cos(\theta/2) \;] \; \}$$
$$+ H_t^i \; \frac{j}{k\sin^2\beta} \; \frac{2 \sqrt{2} \cos(\phi/2) \cot\beta}{\sqrt{1-u}} \qquad (8)$$

$$M = M^{po} + M^f \text{ (PO component + fringe component)}$$
$$= H_t^i \; \frac{jZ \sin\phi}{k \sin^2\beta} \; \{ \; \frac{\sin\delta}{-\sin\delta \; \cos\phi + \cos\delta \; \cot\beta}$$
$$+ \; \frac{1}{\cos\phi - \cot^2\beta} [\; 1 - \frac{\sqrt{2} \cos(\phi/2)}{\sqrt{1-u}} \;] \; \} \qquad (9)$$

where E_t^i, H_t^i are the components of the incident wave tangent to the edge of the plate, $Y=1/Z$ is the wave admittance, $\beta = \cos^{-1} (\widehat{t}\cdot\widehat{s})$ is the angle between the direction of incidence $\widehat{s}$ and edge $\widehat{t}$, δ is the skew angle of the "ray coordinate" [5] and $u = \cos\phi - 2\cot^2\beta$. The scattered field E^s is then derived by integrating the 4-edge currents of the rectangular metal plate according to Eq.(7).

III. SIMULATION RESULT & CONCLUSION

In the simulation, a rectangular metal plate of size $L_1=L_2=5$ inches is used and the frequency of the incident wave is 9 GHz. The RCS is obtained as a function of rotation angle θ_s for three different incident directions shown in Figs. 3 and 4 respectively. In Figs. 3 and 4 the abscissa is angle θ_s and the coordinate is the RCS (in dB). In each figure, two curves are included. They are (1) P.O method with dash line and (2) M.E.C. method with solid line.
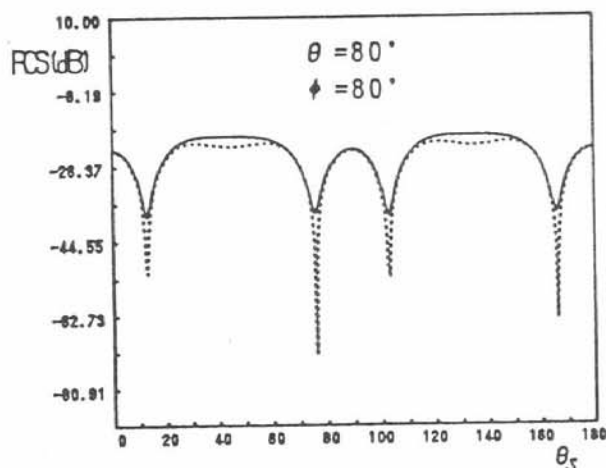

Fig. 3 the variation of RCS as a function of θ_s for two different method when $\theta=80^0$ $\phi=80^0$.
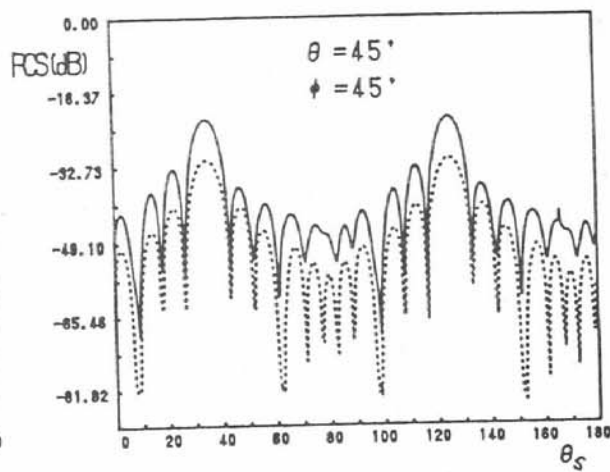
Fig. 4 the variation of RCS as a function of θ_s for two different method when $\theta=45^0$ $\phi=45^0$.

When $\theta=\phi=90^0$, both MEC and PO method predict the same flat line of RCS=4 (not shown) which is independent of the rotation angle. This is because the scattering due to diffraction is far less than that due to reflection when the incident direction is normal. In Fig. 3, the curve obtained by using MEC are almost the same as that obtained the PO method when $\theta=\phi=80^0$. In Fig. 4, the offset angle becomes significant and the MEC method is far more accurate than the PO approximation [4]. In Fig. 4, we see that 2 peak values occur which correspond to the cases when the plate rotates to the position where the edge of the plate is perpendicular to the direction of the incidence. From GTD, this is when both the incident and reflection "cone angles" are 90 degrees, which is corresponding to the backscattering case and, therefore, results the maximum RCS [4]. From Fig. 4, we read $\theta_s=35.26^0$ and 125.26^0 when the peak values occur. The values of RCS at the above two points are the same as those predicted by GTD [4].

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[2] A. Michaeli, "Equivalent edge currents for arbitrary aspects of observation," IEEE Trans. Antennas Propagat., vol. AP-32, pp. 252-258, 1984 (correction, vol. AP-33, pp.227, Feb. 1985).

[3] ---, "Elimination of infinities in equivalent edge currents, part I: fringe current components," IEEE Trans. Antennas Propagat., vol. AP-34, pp. 912-918, July 1986 And PP. 1034-1037, Aug. 1986.

[4] R. A. Ross, "Radar Cross Section of Rectangular Flat Plates as a Function of Aspect Angle," IEEE Tran. Antennas Propagat., vol. AP-14, pp. 320-335, May 1966.