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REFLECTION AND TRANSMISSION OF ELECTROMAGNETIC WAVES BY AN ACCELERATED DIELECTRIC SLAB

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ABSTRACT

The problem of reflection and transmission of plane electromagnetic waves by a linearly accelerated dielectric slab is investigated theoretically. The reflected and transmitted waves are obtained by using summention method. It is found that there exists small Doppler shift in frequency for the transmitted wave which depends on the acceleration, the index of refraction and the width of the slab and vanishes for the case of the constant velocity motion.

INTRODUCTION

The problem of reflection and transmission of electromagnetic waves by a moving medium is one of the basic problems in electrodynamics of moving media, and has been investigated by many authors. For the case where the medium moves uniformly with constant velocity, the problem can be treated by Maxwell-Minkowski's theory.

Recentry, several attempts have been made to extend Maxwell-Minkowski's theory to the case where observer or/and material media are performing accelerated motion with respect to inertial frames [1],[2]. The problem of reflection and transmission of electromagnetic waves by a linearly accelerated dielectric slab is a basic and interesting application of the theory. The purpose of this paper is to present the solution to this problem. The results show that there exists Doppler shift in frequency for the transmitted wave due to the accelerated motion of the slab and it vanishes for the case of the constant velocity motion.

RELATIVISTIC MODEL OF THE LINEARLY ACCELERATED SLAB

The geometry of the problem is shown in Fig.1. It is assumed

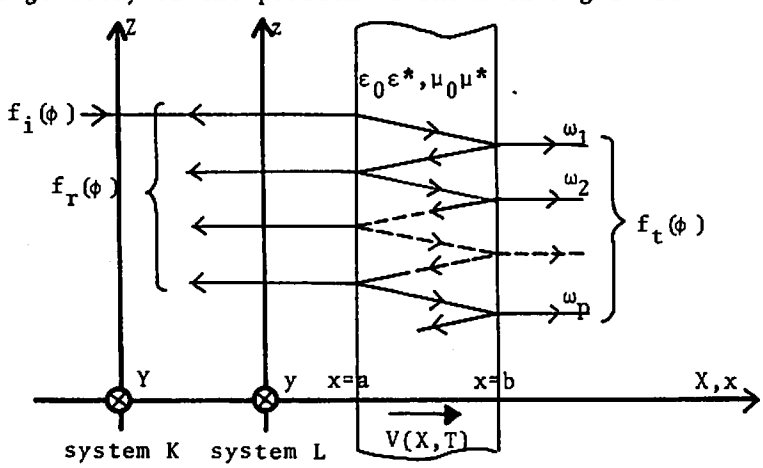

Fig.1. Geometry of the problem.

that the dielectric slab whose permittivity is $\varepsilon_0\varepsilon^*$ and permeability is $\mu_0\mu^*$ performs an accelerated motion perpendicular to the interface in the X direction in the observer's inertial system K(X,Y,Z, T). Consider the plane electromagnetic wave normally incident to the dielectric slab as shown in Fig.1. In the system K, the incident wave E_i, the reflected wave E_r and the transmitted wave E_t can be written as

$$E_i=f_i(\frac{X}{c} - T), \quad E_r=f_r(-\frac{X}{c} - T), \quad E_t=f_t(\frac{X}{c} - T). \tag{1}$$

where $c=(\varepsilon_0\mu_0)^{-1/2}$ is the light velocity in free space and $f_i(\phi)$, $f_r(\phi)$ and $f_t(\phi)$ are the arbitrary incident, reflected and transmitted wave functions, respectively.

Let us consider the simultaneous moving system L(x,y,z,t) which is stationary with respect to the accelerated slab. It is assumed that the accelerated slab is a solid dielectric and it preserves the Born's condition for a rigid body [1]. The motion of the origin of the system L is assumed to be a hyperbolic motion with constant acceleration g. Under the above assumptions, the description of the coordinate transformations which relate the accelerated system L to the inertial system K can be written as follows [3]:

$$X=\frac{c^2}{g}(1+\frac{gx}{c^2})\cosh(\theta+\theta_0)-\frac{c^2}{g}\cosh\theta_0, \quad Y=y,$$
$$Z=z, \qquad T=\frac{c}{g}(1+\frac{gx}{c^2})\sinh(\theta+\theta_0)-\frac{c}{g}\sinh\theta_0 \tag{2}$$

where $\theta=gt/c$, $\sinh\theta_0=\beta(1-\beta^2)^{-1/2}$ and $\beta=v/c$. Since the relative velocity has been adjusted to v at T=t=0, equations (2) reduce to the description of the Lorentz transformations of velocity v by putting g=0. If we put v=0 and neglect terms of orders higher than the first in gx/c^2 and gt/c, equations (2) reduce to the description of the Newton transformations given by

$$X=gt^2/2, \quad Y=y, \quad Z=z, \; T=t. \tag{3}$$

It is assumed that the two interfaces of the slab exist at x=a and x=b in the system L. The velocity of a given point x(a<x<b) of the slab in the inertial system K is given by $V(X,T)=V(t)=ct\tanh(\theta+\theta_0)$ from equations (2).

On the assumption that the macroscopic properties of the slab are not changed by the acceleration acting on it, the constitutive equations for the accelerated dielectric slab are given by the following constitutive equations of the Minkowski type for the moving medium in the system K:

$$\mathbb{D}+\frac{1}{c^2}\{\mathbb{V}(X,T)\times\mathbb{H}\}=\varepsilon_0\varepsilon^*\{\mathbb{E}+\mathbb{V}(X,T)\times\mathbb{B}\}, \quad \mathbb{B}-\frac{1}{c^2}\{\mathbb{V}(X,T)\times\mathbb{E}\}=\mu_0\mu^*\{\mathbb{H}-\mathbb{V}(X,T)\times\mathbb{D}\}. \tag{4}$$

THE REFLECTED AND TRANSMITTED WAVES

In the accelerated system L, the expressions for the incident wave $E_i(x,t)$, the reflected wave $E_r(x,t)$ and the transmitted wave $E_t(x,t)$ can be obtained by using the field transformation formulas for arbitrary coordinate transformations [1],[3]. The results are given by

$$E_i(x,t)=(1+\frac{gx}{c^2})\exp(-\theta-\theta_0)f_i\{\frac{c}{g}(1+\frac{gx}{c^2})\exp(-\theta-\theta_0)-\frac{c}{g}\exp(-\theta_0)\}$$
$$E_r(x,t)=(1+\frac{gx}{c^2})\exp(\theta+\theta_0)f_r\{-\frac{c}{g}(1+\frac{gx}{c^2})\exp(\theta+\theta_0)+\frac{c}{g}\exp(\theta_0)\}$$
$$E_t(x,t)=(1+\frac{gx}{c^2})\exp(-\theta-\theta_0)f_t\{\frac{c}{g}(1+\frac{gx}{c^2})\exp(-\theta-\theta_0)-\frac{c}{g}\exp(-\theta_0)\}. \quad (5)$$

The constitutive equations for the dielectric slab in the accelerated system L can be obtained by transforming equations (4) to the system L according to the coordinate transformations (2). The results are given by

$$\varepsilon_0\varepsilon^*\mathbb{E}=(1+\frac{gx}{c^2})\mathbb{D}, \qquad \mu_0\mu^*\mathbb{H}=(1+\frac{gx}{c^2})\mathbb{B} \quad (6)$$

Since Maxwell's equations are covariant under arbitrary coordinate transformations, they take the same form in the system L as in the system K. The wave equation for the electric field in the system L can be obtained from Maxwell's equations and the constitutive equations (6) as

$$(1+\frac{gx}{c^2})^2\frac{\partial^2 E}{\partial x^2} + \frac{g}{c^2}(1+\frac{gx}{c^2})\frac{\partial E}{\partial x} = \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} \quad (7)$$

where $n=(\varepsilon^*\mu^*)^{1/2}$ is the index of refraction of the slab. The rigorous solutions of the equation (7) can be obtained by making use of the covariant properties of Maxwell's equations under the coordinate transformations (2). The two independent solutions $E_1(x,t)$ and $E_2(x,t)$ can be expressed in the form [4]:

$$E_1(x,t)=1/n\cdot(1+\frac{gx}{c^2})\exp(-\psi-\theta_0)f_1\{\frac{c}{g}(1+\frac{gx}{c^2})\exp(-\psi-\theta_0)-\frac{c}{g}\exp(-\theta_0)\} \quad (8a)$$

$$E_2(x,t)=1/n\cdot(1+\frac{gx}{c^2})\exp(\psi+\theta_0)f_2\{-\frac{c}{g}(1+\frac{gx}{c^2})\exp(\psi+\theta_0)+\frac{c}{g}\exp(\theta_0)\} \quad (8b)$$

where $\psi=t/cn$ and $f_1(\phi)$ and $f_2(\phi)$ are arbitrary wave functions. Equations (8a) and (8b) express waves traveling in the positive and negative x direction, respectively. The magnetic fields corresponding to the electric fields (5) and (8) in the system L can be obtained from Maxwell's equations and the constitutive equations (6). The fields obtained must satisfy the boundary conditions that electric and magnetic fields be continuous across the boundaries x=a and x=b in the system L for arbitrary time t.

Once the two independent traveling waves in the dielectric slab are obtained, we can solve this boundary value problem by using summation method [6]. The reflected and transmitted waves are composed of a sequence of component waves which have been reflected and transmitted at x=a and x=b as shown in Fig.1. Calculating the difference in phase and amplitude of the various component waves, we can express the reflected wave function $f_r(\phi)$ and the transmitted wave function $f_t(\phi)$ in terms of the incident wave function $f_i(\phi)$. The results for them can be written as follows:

$$f_r(\zeta)=-R(1+\frac{ga}{c^2})^2\{\exp(\theta_0)-\frac{g}{c}\zeta\}^{-2}f_i\{\phi_1(\zeta)\}$$
$$+T_1T_2\{\exp(\theta_0)-\frac{g}{c}\zeta\}^{-2}\sum_{p=1}^{\infty}R^{2p-1}(1+\frac{gb}{c^2})^{2pn}(1+\frac{ga}{c^2})^{2-2pn}f_i\{\phi_2(\zeta,p)\}, \quad (9)$$

$$f_t(\xi)=T_1T_2\sum_{p=1}^{\infty}R^{2(p-1)}\{(1+\frac{gb}{c^2})/(1+\frac{ga}{c^2})\}^{2pn-(n+1)}f_i\{\phi_3(\xi,p)\}, \quad (10)$$

where $\phi_1(\zeta)=\frac{c}{g}(1+\frac{ga}{c^2})^2\{\exp(\theta_0)-\frac{g}{c}\zeta\}^{-1}-\frac{c}{g}\exp(-\theta_0)$,

$$\phi_2(\zeta,p)=\frac{c}{g}\{(1+\frac{gb}{c^2})/(1+\frac{ga}{c^2})\}^{2pn}\{\exp(\theta_0)-\frac{g}{c}\zeta\}^{-1}-\frac{c}{g}\exp(-\theta_0),$$

$$\phi_3(\xi,p)=\frac{c}{g}\{\frac{g}{c}\xi+\exp(-\theta_0)\}\{(1+\frac{gb}{c^2})/(1+\frac{ga}{c^2})\}^{2pn-(n+1)}-\frac{c}{g}\exp(-\theta_0), \quad (11)$$

and $R=(n-1)/(n+1)$, $T_1=2/(n+1)$, $T_2=2n/(n+1)$, $\zeta=-\frac{X}{c}-T$, $\xi=\frac{X}{c}-T$. (12)

The expression for the amplitude of the reflected wave obtained depends explicitly on X and T because the results depend on the spatial position of the incident wave and the accelerated slab at initial condition T=t=0. If we put g=0, equations (9) and (10) reduce to the reflected and transmitted waves for the case of the constant velocity motion. The amplitude and instantaneous angular frequency of the reflected wave vary continuously according to the Doppler effect formula [4]. It is known that there exists no Doppler shift in frequency for the transmitted wave due to the movement of the slab for the case of the constant velocity motion [5]. The instantaneous angular frequency ω_p of the pth order transmitted component wave can be obtained as

$$\omega_p=\frac{\partial\phi_3(\xi,p)}{\partial T}=\{(1+\frac{gb}{c^2})/(1+\frac{ga}{c^2})\}^{(2p-1)n-1}\omega_0, \quad (13)$$

and

$$\omega_p\simeq\omega_0+\frac{g}{c^2}(b-a)\{(2p-1)n-1\}\omega_0, \quad (14)$$

for small values of ga/c^2 and gb/c^2. In equations (13) and (14), ω_0 is the angular frequency of the incident wave. The results show that there exists Doppler shift in frequency for the transmitted wave. Though this effect is small, it is interesting that it depends on the acceleration g, the index of refraction n and the width of the slab (b-a) and it is independent of the velocity of the slab V(X,T).

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