A Study on Accuracy of FDTD Method Using Fixed-Point Arithmetic

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Abstract Although the Finite Difference Time Domain (FDTD) method is extensively used as an electromagnetic field analysis technique, it spends considerable time in calculating electromagnetic fields one after the other for each cell. Supercomputers and PC clusters are put to practical use to solve this problem. Furthermore, implementing the FDTD method into a Field Programmable Gate Array (FPGA) is examined as an alternative technique to solve this problem. Although this technique is put to practical use in a simple model such as 2D-FDTD and Mur's absorption boundary conditions, for complicated models such as 3D-FDTD and the perfectly matched layer (PML) absorption boundary conditions it is not practical. Usually, the FDTD method is calculated using a floating-point arithmetic. The arithmetic in the FPGA, however, is a fixed-point arithmetic in many cases. Therefore, it is important to clarify the accuracy of the fixed-point arithmetic when implementing it into an FPGA for the FDTD method. In this paper, analysis based on a half-wavelength dipole antenna and lossy material with a half-wavelength dipole antenna is performed and the operation accuracy is clarified. The results show that the operation accuracy of 0.01 or less is achieved when the bit length is 28 or more.

Keyword FDTD method, fixed-point arithmetic, operation accuracy

1. INTRODUCTION

Recently, simulation techniques such as the method of moment (MoM), finite element method, and Finite Difference Time Domain (FDTD) method, have been used extensively. The FDTD method is a technique that divides the operation domain into a minute cube cell, and calculates electric and magnetic fields one after the other for each cell. This causes it to incur a very long processing time. Supercomputers and PC clusters are practically applied to solve this problem. However, using supercomputers and PC clusters also generates problems such as the need for large-scale equipment. On the other hand, a Field Programmable Gate Array (FPGA), which excels in parallel processing of simple calculations and is comparatively small, has attracted much attention [1]-[3]. The main conventional uses of the FPGA are specific such as examining the composition of hardware and the effects of high-speed algorithms, or examining a simple model that is used for 2D-FDTD and Mur's absorption boundary conditions when the FDTD method is implemented in the FPGA. We plan to apply this technique practically to antenna analysis using 3D-FDTD. Usually the FDTD method is calculated using a floating-point arithmetic to enable calculation on a PC. However, it is necessary to implement the FDTD method using a fixed-point arithmetic in order for the FPGA to perform calculations using a fixed-point arithmetic in many cases. In this case, it is important to clarify the relationship between the length of the operation bits and the operation accuracy before implementing the FDTD method into the FPGA. In this paper, we employ a half-wavelength dipole antenna as the basic model for antenna analysis. We examine the operation accuracy when applying 16 to 32 bits as the length of the operation bits. Based on the results, the minimum length of the operation bits in FPGA is determined.

2. EVALUATION MODEL AND METHOD

Two models were prepared. In the first, the half-wavelength dipole antenna is considered not to have any thickness in free space, and in the second, the half-wavelength dipole antenna near the lossy material is used for the evaluation model. The operation accuracy is examined using the impedance characteristic calculated from the current and voltage distribution in the half-wavelength dipole antenna in a simple substance model. On the other hand, the observation point was set inside the lossy material, and the operation accuracy is estimated using the half-wavelength dipole antenna with the lossy material model because there may be a decrease in the operation accuracy that originates from a loss in the lossy material. The analytical parameters are shown in Table I. The analysis space comprises 100x100x100 cells, and the cell size is set to 5 mm^3

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(fixed). Moreover, the perfectly matched layer (PML) (seven layers) formulation was applied as the absorption boundary conditions. The half-wavelength dipole antenna is arranged at the center of the analysis space, or near the lossy material, and it excites a gauss pulse or sine wave using a gap electric supply. The supplied electric voltage is 1V and the transmission frequency is 2.0 GHz. Furthermore, the convergence conditions were deemed to be satisfied when the average of one cycle of the total of the electric field and the total of the magnetic field in the analysis domain has a difference of 1% or less compared to the previous cycle.

2.1. Fixed-Point Arithmetic

In this examination, C language was used in order to enable bit operation processing and bit shift processing. The electric field, the magnetic field, and each coefficient of each operation were established using the 32-bit integer with a mark. The arrangement of the operation bits is shown in Fig. 1(a). All parameters must not exceed one because the decimal point position was set between the mark bit and the following bit as shown in the figure. Therefore, the variable, which assumes the greatest value in the operation process of this analysis, was the electric field value, and it is calculated by scaling so that the maximum may become one or less. Furthermore, when evaluating the operation accuracy of less than 32 bits, zero was substituted in below bits the application bit using the bit shift. The bit arrangement of 26 bit operations is shown in Fig. 1(b) as an example.

2.2. Evaluation Method of Operation Accuracy

The operation accuracy is evaluated based on the error of the fixed-point arithmetic value over a floating-point arithmetic value (refer to Formulae (1) and (2)). Formula (1) shows the error of the impedance characteristics, and Formula (2) shows the average error in an observation point or an observation plane. Here, it is shown that the error occurred once compared to the maximum electric field of a floating-point arithmetic when this error is set to one.

$$\mathrm{Error} = \left| \frac{\mathrm{IMP}^{\mathrm{float}} - \mathrm{IMP}^{\mathrm{fixed}}}{\mathrm{IMP}^{\mathrm{float}}} \right| \quad \cdots (1)$$

Here, $\mathrm{IMP}^{\mathrm{float}}$ represents the impedance characteristic of the floating-point operation, and $\mathrm{IMP}^{\mathrm{fixed}}$ represents the impedance characteristic of the fixed-point arithmetic.

$$\mathrm{Error} = \frac{1}{N \cdot E|_{\mathrm{MAX}}} \sqrt{\sum_{t=1}^{N} \left| E^{\mathrm{float}} - E^{\mathrm{fixed}} \right|^2} \quad \cdots (2)$$

Here, N represents the amount of data for one cycle, $E|_{MAX}$ represents the maximum electric field of one cycle at the observation point, E^{fixed} represents the electric field value in a fixed-point arithmetic, and E^{float} represents the value in a floating-point arithmetic.

2.3. Half-Wavelength Dipole Antenna Model

An analytical model is shown in Fig. 2. The half-wavelength dipole antenna was arranged at the center of the analysis space. A gap electric supply was used for this antenna, and a gauss pulse was generated. The Fourier transform of current I (t) calculated from the circumference integration of the magnetic field and voltage V(t) a given to this point supplying electric power are calculated, and V (ω)/I (ω) (V (ω) and I (ω) are the voltage and the current after the Fourier transform, respectively) is calculated. Moreover, the impedance characteristic of 2.0 GHz, which is the excitation frequency, was substituted for Formula (1), and the error was calculated. The x-y plane distribution of the average error in a one-round period after convergence determination was observed when the electric power was supplied as a sine wave. This average error is calculated by substituting Formula (2) for the electric field value on the observation plane.

2.4. Dipole Antenna and Lossy Material Model

The analytical model is shown in Fig. 3. The cubical lossy material (εr=41.5, σ=1.3) is arranged at the center of the analysis space. The half-wavelength dipole antenna is embedded 5 mm from the surface of the lossy material, and the supplied electric power is a sine wave. Furthermore, while calculating the average error at the observation point, the distribution of the average error in the x-y plane was observed.

3. CALCULATION RESULTS

3.1. Half-Wavelength Dipole Antenna Model

The current characteristics of the point where the electric power is supplied are shown in Fig. 4 when a gauss pulse is excited. From the figure, when the length of the fixed-point arithmetic bits is 28, the current characteristics of the fixed-point arithmetic bits agree fairly well with those of the floating-point arithmetic bits. The impedance characteristic calculated by the Fourier transform of this current characteristic and the electric supply voltage are shown in Fig. 5. The figure shows that the results of the floating point arithmetic and those of the 28-bit fixed-point arithmetic are in good agreement. On the other hand, it is apparent that the results of the floating point arithmetic and those of the 26-bit fixed-point arithmetic cannot be correctly calculated. Moreover, the operation accuracy of the impedance characteristic at 2.0 GHz is shown in Fig. 6. As for the operation accuracy, the figure shows that there is no significant change; however, 0.01 or less in 27 to 32 bits is achieved. On the other hand, it is apparent that the accuracy degrades greatly in 26 operation bits or less, and the operation accuracy becomes approximately one.

The distribution of the operation accuracy in the x-y plane is shown in Fig. 7. Figure 7(a) shows the case of a 28-bit fixed-point arithmetic, and Fig. 7(b) shows the case of a 26-bit fixed-point arithmetic. These figures clarify that the operation accuracy can be analyzed with sufficient accuracy in all domains (0.0001 or less error in all domains) for 28 bits. On the other hand, considering the entire analysis domain surface the operation itself failed (error is about one) in the case of 26 bits.

3.2. Dipole Antenna and Lossy Material Model

The operation accuracy is estimated by using the half-wavelength dipole antenna with the cubical lossy material model because there is concern that the operation accuracy will decrease due to a loss inside the lossy material. The error of the z component of the electric field at the observation point (inside the lossy material) in Fig. 3 is shown in Fig. 8. As for the operation accuracy, the figure shows that there is no significant change; however, 0.00001 or less in 28 to 32 bits is achieved. On the other hand, it is apparent that the accuracy degrades greatly in 26 operation bits or less, and the operation accuracy becomes approximately one. The length of the operation bits using the dipole antenna with lossy material model when operation accuracy yields good results, and the operation bit length is one bit longer than that when dipole antenna simple substance model is used. This is based on the decrease in the electromagnetic field value corresponding to the loss in the lossy material.

The error distribution in the x-y plane is shown in Fig. 9. Figure 9(a) shows the average error for 28- bit operation, and Fig. 9(b) shows the average error for 26-bit operation. The figures show that the operation accuracy (0.01 or less error in all domains) is sufficient in all domains in the case of 28 bits; however, considering the entire analysis domain surface the operation itself failed (error is about 0.1-1) in the case of 26-bit operation. These results are almost the same as those of a half-wavelength dipole antenna using a simple substance model.

4. CONCLUSION

To achieve FDTD high-speed operation using FPGAs, the operation error pertaining to the bit length of a fixed-point arithmetic required for mounting was examined. The evaluation model used a half-wavelength dipole antenna. Consequently, the operation error was calculated using the impedance characteristic of the half-wavelength dipole antenna. Lossy material with a half-wavelength dipole antenna achieved a level of accuracy of 0.01 or less using 28 bits or more. When 26 bits or less were employed, the error deteriorates to approximately one. The FDTD method is mounted in the FPGA, which employed a 32-bit fixed-point arithmetic, and based on the results improvement in the speed of the FDTD method operation was achieved.

Reference

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Table I. Parameters

Cell size	5.0 mm
Number of cells	100×100×100
Absorption boundary	PML 7 layers
Operational bit length	16 to 32 bits
Excitation wave	Pulse/sine wave
Frequency	2.0 GHz
Convergence conditions	1% or less

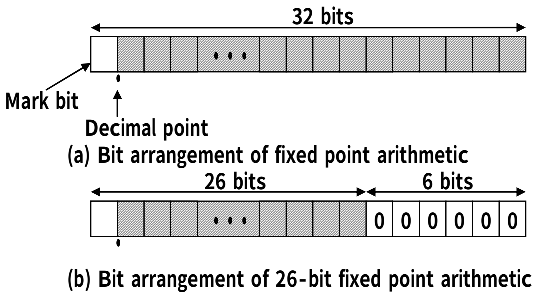


Fig.1 Bit arrangement

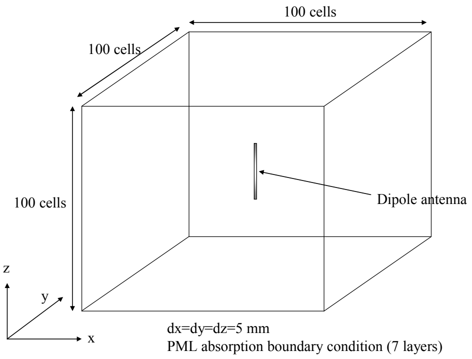


Fig.2 Analytical model for dipole antenna

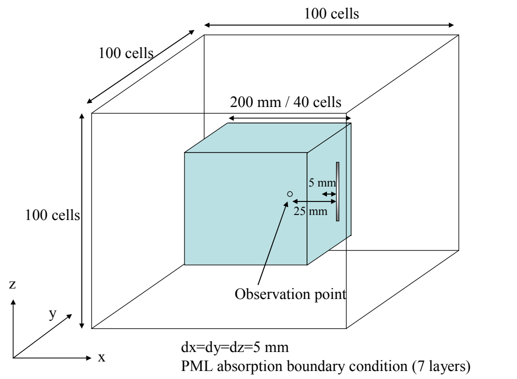


Fig.3 Analytical model for dipole antenna with lossy material

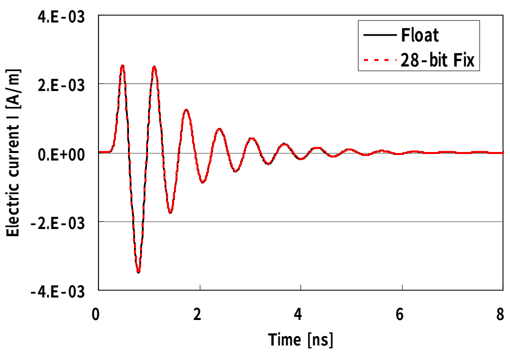


Fig.4 Electric current characteristics

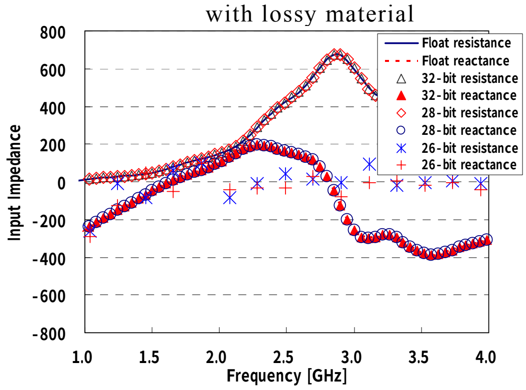


Fig.5 Impedance characteristics

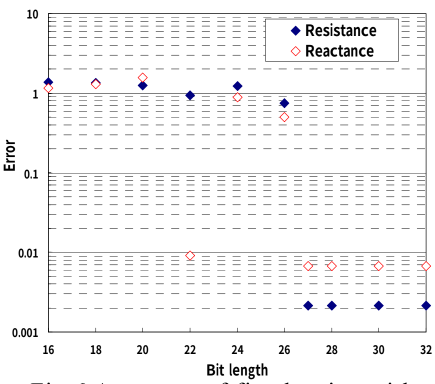
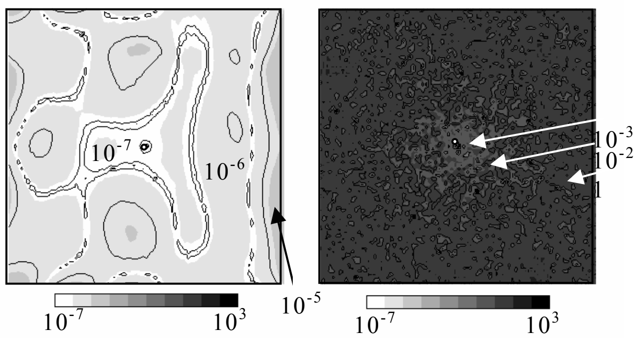


Fig.6 Accuracy of fixed-point arithmetic



(a) 28-bit arithmetic (b) 26-bit arithmetic

Fig.7 Accuracy distribution (x-y plane)

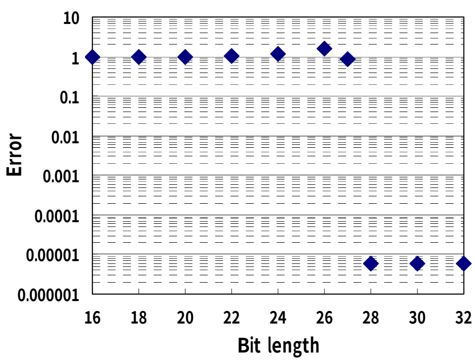
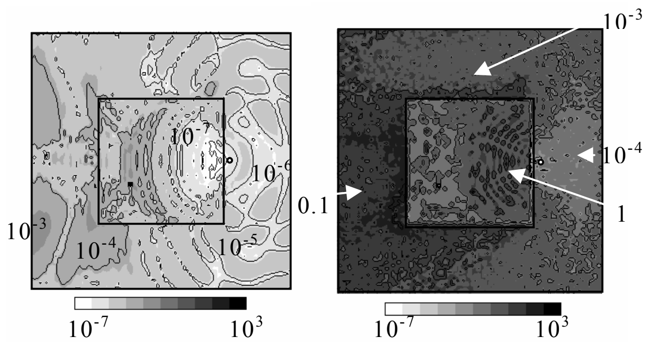


Fig.8 Accuracy of fixed-point arithmetic



(a) 28-bit arithmetic (b) 26-bit arithmetic

Fig.9 Accuracy distribution (x-y plane)