1-III B2

DIFFRACTION LOSSES IN A TOROIDAL OPEN-CAVITY RESONATOR

by

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An investigation is carried out to estimate the diffraction losses suffered by an optical wave that might be trapped in a toroidal open-cavity resonator. The resonator and its mode structure was reported in an earlier paper.[1] In brief, the resonator is a toroidal shaped gas medium which is inhomogeneous in three dimensions, and the refractive index is assumed to be decreasing monotonically in the radial as well as axial direction. The media are formed in high-pressure gas nozzling systems and thus have practical realizations. It was shown previously that existence of realizable eigenvalues and eigenfunctions suggest the existence of bound electromagnetic wave modes in such a resonator. It is, however, essential in contemplating the use of such systems as active laser media to evaluate the associated diffraction losses.

Mathematical Formulation

To evaluate diffraction loss we use a heuristic picture of a laser beam propagating around inside the inhomogeneous media, which has just the right variation so that the beam makes complete loops. We assume that the radius of the curvature of the beam path is large compared to the cross section of the laser and that the wave form is reproduced after it has travelled one loop, except for possibly a complex constant, σ, which is associated with loss per transit.

Since a direct approach to this problem is difficult, we seek an equivalent media representation. We approximate the system by a Fabry-Perot open resonator with an inhomogeneous media in between two reflectors.

Further, we consider that the laser starts out with a cross section of ($2v \times 2w$) and the portion of the wave that is outside ($2v \times 2w$) after each transit is considered as diffraction loss. Admittedly there are some ambiguities in selecting the cross section ($2v \times 2w$) for the equivalent end mirror, and the diffraction loss will depend on the cross section picked. It may be possible to agree upon the beam width of the lowest order mode as the defining cross section, or in the radial direction, the turning point may be considered as one side of the mirror. The value of the loss function D calculated from the model will be larger compared to that of the actual physical system since the portion of the wave that is considered as diffracted from the end of the equivalent mirror that is closer to $r = 0$ will still be stored inside the actual physical system.

The diffraction field at (z_2,ρ_2) due to the field at (z_1,ρ_1) is approximated as

$$E(z_2,\rho_2) \doteq \int_{z_1} \int_{\rho_1} \frac{iK(\rho,z)}{2\pi\ell} e^{-iKR} F(z_1)G(\rho_1)dz_1 d\rho_1 \qquad (1)$$

We examine the KR of the phase variations, by assuming a wave-number variation of the form

$$K^2(z,\rho) = k^2[1 - a^2(\frac{z_1+z_2}{2})^2 - b^2(\frac{\rho_1+\rho_2}{2})^2] \qquad (2)$$

With the assumptions used earlier, and by using binomial expansion, we have shown that the phase term is similar to that given by Fox and Li's paper.[2] We see that the propagation in the inhomogeneous gas resonator, with the above approximations, is equivalent to that of a Fabry-Perot resonator with homogeneous media having new medium constants. In fact each of the z and ρ variations can be represented by a non-confocal Fabry-Perot resonator with homogeneous new wave numbers. This is not totally unexpected from intuitive considerations.

The equivalent end mirrors will have radii of curvature,

$$R_z = \ell/(1-q_1),\ R_\rho = \ell/(1-q_2)$$

in z and ρ direction, respectively.

$$q_1 = \frac{4-a^2\ell^2}{4+a^2\ell^2},\ q_2 = \frac{4-b^2\ell^2}{4+b^2\ell^2},\ \ell = \text{path length.}$$

By equating q_1 and q_2 to zero we obtain the confocal-confocal conditions of

$$a = b = 2/\sqrt{\ell}, \qquad (3)$$

which has minimum diffraction loss. The general form of equation (1) may be solved analytically[3] to find the diffraction losses, mode patterns, loss regions, etc. We separate equation 1 as

$$\sigma_z F(z_2) = \left(\frac{ike^{-ik_0}}{2\pi\ell}\right)^{1/2} \int_{-v}^{v} F(z_1)$$

$$\exp.\{\frac{ik_1}{\ell}[z_1 z_2 - \frac{q_1}{2}(z_1^2+z_2^2)]\}\ dz_1 \qquad (4)$$

where σ_z arises because of the limits we placed on the integration. The constants σ_z and σ_ρ are unity as v and w tend to infinity, corresponding to no diffraction loss. The total loss is $D = 1 - |\sigma_z \sigma_\rho|^2$. By assigning the same value of v and w for different gas profiles, D may be used as a basis for comparing how much the laser beam will be spread in the media.

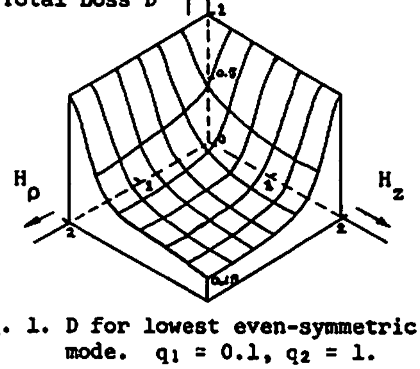

Fig. 1. D for lowest even-symmetric mode. $q_1 = 0.1$, $q_2 = 1$.

The loss function σ_z, the total loss function D and the wave function F(z) have been computed, plotted and studied for different modes. An isotropic view of D is shown in figure 1 for a particular set of parameters.

Discussion and Conclusions

It is found that, in general, the loss functions for the even-symmetric modes are small compared to that of the corresponding odd-symmetric ones. The losses decrease rapidly and approach zero as we increase the Fresnel number H_1, which for our case is a measure of the beam size, the path length and the equivalent wavelength. Furthermore, for small values of the Fresnel numbers the loss function for different modes are well separated, whereas for large Fresnel numbers, they are close to one another. Based on loss consideration the even-symmetric modes are preferable to the odd symmetric functions.

In this paper we have shown an approximate method for obtaining the diffraction losses associated with electromagnetic waves in a toroidally shaped inhomogeneous resonator; the approach was in terms of the classical Fabry-Perot open-resonator, the properties of which are fairly well known. It can be shown that the diffraction losses of a toroidal resonator are at least as small as that of a corresponding Fabry-Perot resonator. However, the main advantage of the former is its ability to store high intensity light beams, since no mirrors are utilized to bend the beam and thus there will be essentially no limit to its use below breakdown levels of the gas medium.

References

1. Paper presented by the authors at Joint URSI/GAP Sym. on Ant. and Prop., Columbus, Ohio (1970). (Accepted for publication of the J. Opt. Soc. Am.)

2. A. G. Fox and T. Li, Proc. of IEEE 51, 80 (1963).

3. H. Schachter and L. Bergstein, Proc. of Sym. on Optical Masers, (Interscience Publisher, New York, 1963).