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TRANSIENT FIELDS DUE TO A LINE SOURCE IN A SLAB MEDIUM

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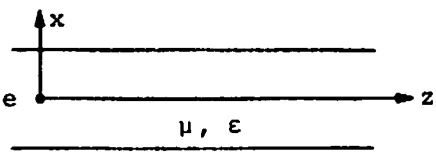
In this paper we study transient fields due to an infinite line source located in a slab medium by using a modified modal approach via double deformation. Consider a line source situated at the center of a layer of dielectric medium with thickness $2a$. We choose the coordinate origin to coincide with the line source [Fig. 1].

Figure 1

line source



Let the source be initially at rest and have the excitation function

$$x(t) = e^{-\alpha t} \sin \omega_o t \; u(t) \tag{1}$$

where $u(t)$ is 1 for $t \geq 0$ and 0 for $t < 0$. The Fourier spectrum of the source excitation function is then

$$X(\omega) = \int_0^\infty dt \; e^{i\omega t} x(t) = \frac{\omega_o}{(\alpha - i\omega)^2 + \omega_o^2} . \tag{2}$$

For an observation point at $x = 0$ and a distance z away from the source, the total transient response is

$$\phi(z, t) = -\frac{1}{2\pi^2} \text{Real} \left\{ \int_0^\infty d\omega \; e^{-i\omega t} X(\omega) \int_{SIP} dk_z \; e^{ik_z z} \frac{g(\omega, k_z)}{f(\omega, k_z)} \right\} \tag{3}$$

where

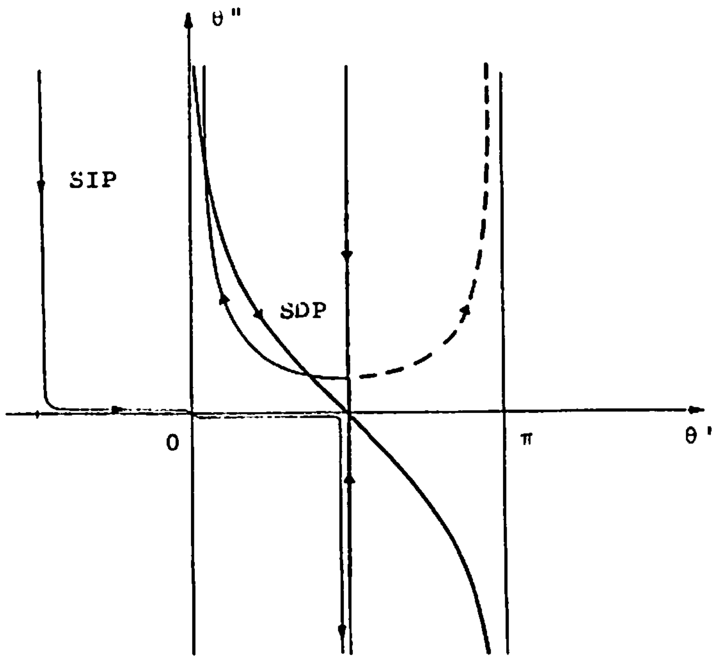
$$\frac{g(\omega,\ k_z)}{f(\omega,\ k_z)} = \frac{1 + R_{01}\ e^{i2k_x a}}{ik_x(1 - R_{01}\ e^{i2k_x a})}\ , \tag{4}$$

$$k_x = (\omega^2\mu\varepsilon - k_z{}^2)^{1/2}, \tag{5}$$

and R_{01} is the Fresnel reflection coefficient. The integration for k_z follows the Sommerfeld integration path (SIP) which is slightly above the negative real k_z' axis for $k_z' \leq 0$ and slightly below the positive real k_z' axis for $k_z' \geq 0$.

Time harmonic fields in layer media have been extensively studied [1-6] with the use of the Sommerfeld integration representations by appropriate deformation in the complex wave number plane. The classical modal approach to time harmonic excitations yield normal modes pertaining to the structure represented by poles situated between the Sommerfeld integration path (SIP) and the steepest descent path (SDP) on the complex wave number plane. Let us focus our attention on one particular mode ℓ. As frequency changes, the location of the pole representing the ℓth modes moves on the complex angle plane [Fig. 2]. Depending on the frequency, the mode ℓ can exist as a guided mode [2-6] or a leaky mode. Over certain ranges, the mode is unexcited because it lies outside the Sommerfeld integration path and the steepest descent path. Over these frequency bands, the mode amplitude is zero. Integrating over all frequencies, the Paley-Wiener criterion is violated [7] and the mode in time-domain becomes noncausal.

Figure 2



In this paper we develop time domain modes that are causal by employing the technique of double deformation. The procedure consists of first a familiar deformation from the SIP to the SDP on the complex wavenumber plane and then a second deformation from the real frequency axis to the steepest descent path on the complex frequency plane. In the process of the first deformation, residue contributions due to the poles of guided modes and the leaky modes are included. We have

$$\phi \sim \int_0^\infty d\omega \int_{SIP} dk_z = \int_0^\infty d\omega \text{ (Residues)} + \int_0^\infty d\omega \int_{SDP} dk_z. \tag{6}$$

The second deformation follows an interchange of the order of integration in the double integral. Residues of poles that are encountered during the deformation are taken into account and we find

$$\phi \sim \int_0^\infty d\omega \text{ (Residues)} + \int_{SDP} dk_z \text{ (Residues)} + \int_{SDP} dk_z \int_{SDP} d\omega. \tag{7}$$

The modified modal approach as outlined in the above is useful in obtaining complete causal transient response at all times.

It is known that the first term in (6) and in (7) gives rise to noncausal modal response. By examining the location of poles in the second term in (7) as a function of k_z on SDP, we show that for time less than either the head wave arrival time or the direct wave arrival time, which ever is earlier, the first and the second terms in (7) exactly cancel each other and the third term is also identically equal to zero. In Fig. 3 we illustrate the complete transient response for the excitation function $x(t)$. The head arrival time is denoted as t_h and the direct wave arrival time as t_d. For the case considered, $t_h < t_d$, causality is seen to be preserved.

The results presented in Fig. 3 are also checked with the method of explicit inversion and by direct numerical integration. It is noted that while the explicit inversion method can be applied to our present problem it has severe inherent restriction. The reflection coefficient R_{01} after transformation to the complex plane by $k_z = k \sin \theta$ must be ω-independent. In cylindrical geometry, for instance, R_{01} will be ω-dependent after the transformation. The explicit inversion approach is also not applicable to dispersive media, say with the permittivity $\varepsilon = \varepsilon' + i\,\sigma/\omega$. Such restrictions, on the other hand, do not appear in the modified modal theory as developed in this paper.

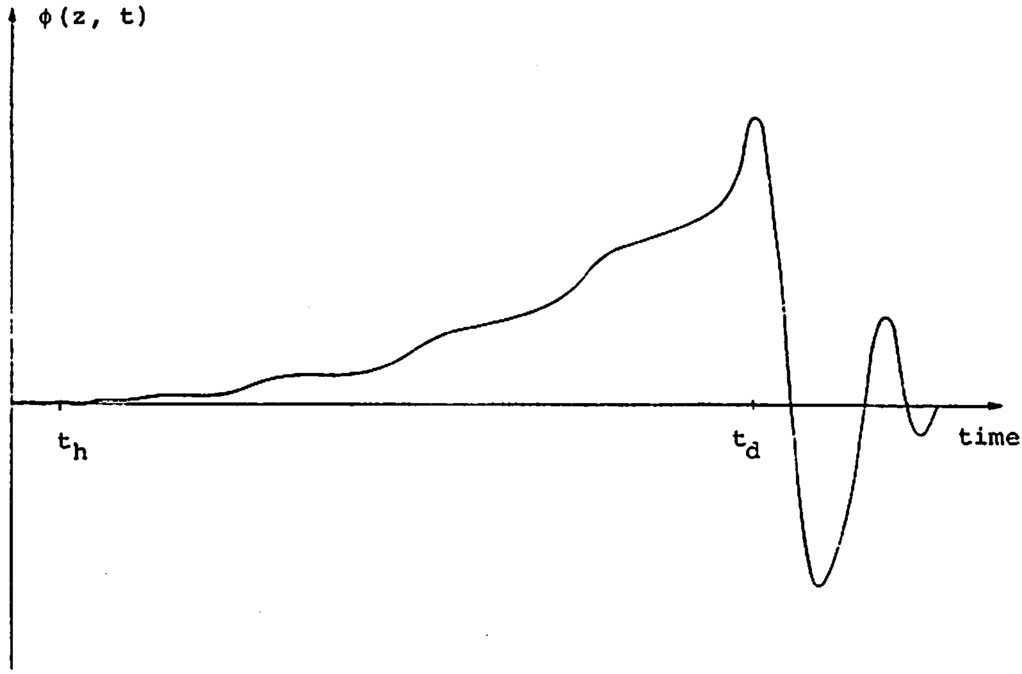


Figure 3

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