REFLECTOR ANTENNAS FOR ELECTRON CYCLOTRON RESONANCE HEATING OF FUSION PLASMA

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Abstract For electron cyclotron resonance heating (ECH) of fusion plasma, transformation is required of the millimeter wave output from a gyrotron, circular TE_{0n} mode, into a linearly polarized wave beam. It is easily realized by use of a parabolic cylinder reflector. Vlasov et al. proposed this type of reflector antenna which has a stair-cut aperture at an end of a circular waveguide[1]. On the other hand we proposed another type of antenna which also uses a parabolic cylinder and has an obliquely cut aperture[2]. In this paper, the transformation efficiencies of polarization and radiation fields of the two types of antennas are calculated by means of geometrical optics and the Kirchhoff-Huygens principle. Then we propose a mode converter, in which the output of this type of a parabolic reflector is led to a rectangular waveguide and transformed into TE_{10} mode. In addition, another reflector antenna is proposed which focuses the wave beam using an elliptic cylinder reflector and a parabolic one.

1 Geometrical-Optics Treatment

The longitudinal magnetic field of TE_{0n} mode from a gyrotron is given by

$$H_z = J_0(k_{cn} r) e^{-j\beta z} \qquad (1)$$

where $k_{cn} = \rho'_{0n}/a$, ρ'_{0n} is the root of $J_0'(\rho) = 0$, and a is the radius of the circular waveguide. By virtue of Hansen's Integral, Eq.(1) is written as

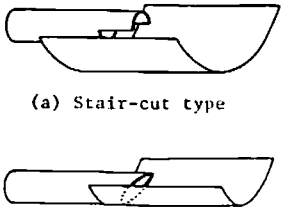
$$H_z(\mathbf{r}, z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\mathbf{k}_{cn}\cdot\mathbf{r}} e^{-j\beta z} d\phi \qquad (2)$$

where $\mathbf{k}_{cn}$ is a transverse wave vector and ϕ is the angle between $\mathbf{k}_{cn}$ and $\mathbf{r}$. According to Eq.(2), TE_{0n} mode is represented by a superposition of plane waves which propagate at an angle α with the waveguide axis, where

$$\alpha = \sin^{-1}(k_{cn}/k), \quad k = 2\pi/\lambda \qquad (3)$$

The electric field vectors are perpendicular to the waveguide axis.

Radiation from a highly oversized waveguide can be approximately treated by means of geometrical optics. From the aperture of the waveguide, each plane wave above-mentioned is radiated in free space at the same angle α. If the aperture is perpendicular to the waveguide axis, the radiation is axisymmetric. But when the lower half of the waveguide is cut away by a length $L(=2a\cot\alpha)$ as shown in Fig.1(a) [1], radiation occurs in the range of $\pi/2 < \varphi < 3\pi/2$ shown in Fig.2. If it is cut obliquely at the angle α as shown in



(a) Stair-cut type

(b) Obliquely-cut type

Fig.1 Parabolic reflector antennas

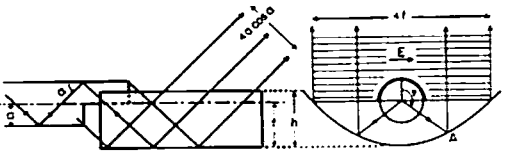


Fig. 2 Configuration of a stair-cut type

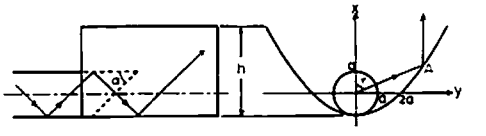


Fig.3 Configuration of an obliquely-cut type

Fig.1(b), most of the radiation is directed into the lower-half. After reflection from the parabolic cylinder reflector whose focal axis coinsides with the waveguide axis, the electric field of each plane wave is oriented in the direction of y-axis of Fig.s 2 and 3.

Treated by the geometrical optics, the beam width of the stair-cut type in the E-plane is 4f and that in the H-plane is $4a\cos\alpha$, where f is the focal distance of the parabolic cylinder in Fig.2. For all of the plane waves from the aperture to be reflected by the parabolic reflector, the height of the reflector h should be equal to f or higher. For the whole beam from the parabola not to be interrupted by the waveguide, f should be larger than 1.5a. The field distribution in the H-plane is uniform. The E-plane field distribution is obtained as follows: The radiation power in unit angle is constant and power in Δy at A of Fig.2 on the reflector ($y=2f\sin\varphi/(1-\cos\varphi)$) is proportional to $\Delta\varphi/\Delta y$, so that $|E(y)|^2 \propto 1/|dy/d\varphi|$ and

$$E(y)=E_0/\sqrt{f\,\{(y/2f)^2+1\}} \qquad (4)$$

The radiation from this antenna is regarded equivalent to that from a square plane wave source ($4f \times 2a\cos\alpha$) having the field distribution as Eq.(4)(see Fig.4).

In the case of the obliquely-cut type, cut at the angle α, f can be made as short as a. From this aperture, some of plane waves are radiated upwards, and the equivalent plane wave source is not square but spreads infinitely in width as shown in Fig.5. The field distribution is expressed by Eq.(4), most of the power being concentrated around the center of the reflector. So even a reflector with height h=f can catch about 82% of the power. For h=2f,3f,and 4f, the efficiency is as large as 91%, 94%, 96%, respectively (Fig.6). This efficiency η is derived from the ratio of the reflected power to the whole power. Using a variable $t=y/2f$, the reflected power from the parabolic reflector with height $h(=ft_0^2)$ is caluculated as

$$P(t_0)=\frac{1}{\zeta}\int_S \frac{E_0^2}{f\,\{(y/2f)^2+1\}}\,dS=\frac{8a\cos\alpha E_0^2}{\zeta}\left\{\frac{t_0}{t_0^2+1}+\tan^{-1}t_0\right\} \qquad (5)$$

where $\zeta=\sqrt{\mu_0/\varepsilon_0}$. Then η is given by

$$\eta=\frac{P(t_0)}{P(\infty)}=\frac{2}{\pi}\left\{\frac{t_0}{t_0^2+1}+\tan^{-1}t_0\right\} \qquad (6)$$

2 Radiation Fields of the Reflector Antennas

In this section we will calculate the radiation fields from the equivalent plane wave

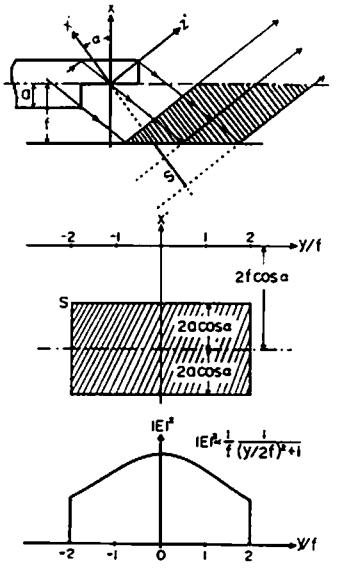


Fig.4 Secondary wave source in the stair-cut type

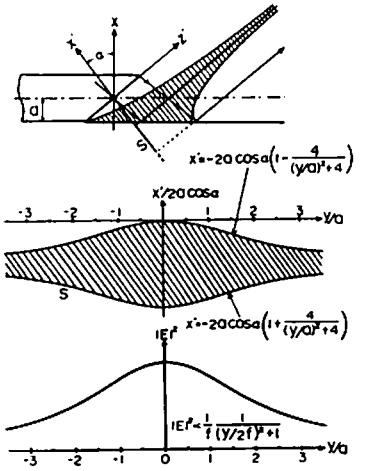


Fig.5 Secondary wave source in the obliquely-cut type

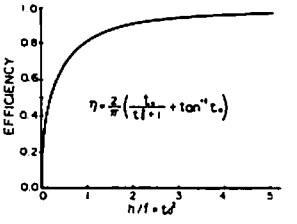


Fig.6 Efficiency of the obliquely-cut type antenna

source derived in the previous section.

From the equivalence theorem, the radiation from an aperture antenna is equivalent to that from electric and magnetic currents $\mathbb{J}$ and $\mathbb{M}$ on the aperture.

$$\mathbb{J} = \mathbb{n} \times \mathbb{H}_t, \quad \mathbb{M} = -\mathbb{n} \times \mathbb{E}_t \tag{7}$$

where $\mathbb{n}$ is an outward unit normal vector on the aperture S and the subscript t denotes tangential components of the field. The electromagnetic fields from the currents are expressed in terms of vector potentials $\mathbb{A}$ and $\mathbb{A}'$.

$$\begin{aligned} \mathbb{E} &= \frac{1}{j\omega\mu\varepsilon}(\nabla\nabla\cdot + k^2)\,\mathbb{A} - \frac{1}{\varepsilon}\nabla\times\mathbb{A}', & \mathbb{H} &= \frac{1}{\mu}\nabla\times\mathbb{A} + \frac{1}{j\omega\mu\varepsilon}(\nabla\nabla\cdot + k^2)\,\mathbb{A}' \\ \mathbb{A} &= \frac{\mu}{4\pi}\int_S \mathbb{J}\frac{e^{-jkr}}{r}\,dS, & \mathbb{A}' &= \frac{\varepsilon}{4\pi}\int_S \mathbb{M}\frac{e^{-jkr}}{r}\,dS \end{aligned} \tag{8}$$

From Eq.(8), the far fields are calculated in spherical coordinates as

$$\begin{aligned} E_\theta &= \zeta H_\varphi = j\omega(\zeta A'_\theta - A_\varphi), & E_\varphi &= -\zeta H_\theta = -j\omega(\zeta A'_\varphi + A_\theta) \\ E_r &= 0, & H_r &= 0 \end{aligned} \tag{9}$$

In the calculation of the far fields, we have used the normalized sizes f/a and h/f, and the normalized frequency $F(=k/k_{cn} = 2\pi a/\lambda p'_{on})$. F is proportional to frequency, F=1 corresponding to cut-off.

Fig.7 shows the radiation field pattern from the obliquely-cut type antenna. Incident mode is TE_{01} with F=1.5 (α=41.8°) and f/a=1. In the E-plane the beam is sharpened as the reflector is made deeper and η is improved. But in the H-plane the beam width does not change.

Fig.8 shows the radiation field pattern from the stair-cut type. In this case h/f=1 and η=100%. The greater the focal distance is, the wider the width of the source in the E-plane is and the sharper the beam. But the beam width in the H-plane does not change.

Compared in Fig.9 are (a) the stair-cut type (F=1.55, f/a=2.0, h/f=1.0) and (b) the obliquely-cut type (F=1.55, f/a=1.19, h/f=4.53). The aperture width in the E-plane is about (a) 8a and (b) 10a respectively (see Fig.10). Half-value angles in the E-plane are (a) 7° (6°), (b) 9° (9°), and those in the H-plane are (a) 18° (17°), (b) 20° (23°). The values in the parentheses are

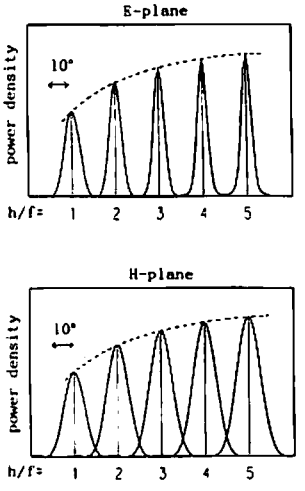


Fig.7 Far field patterns of the obliquely-cut type: F=1.5, f/a=1

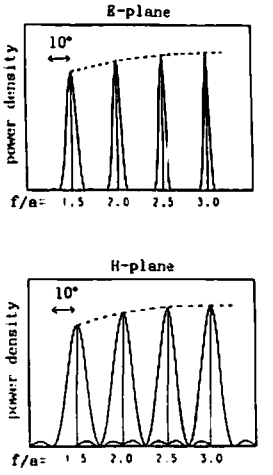


Fig.8 Far field patterns of the stair-cut type: F=1.5, h/f=1

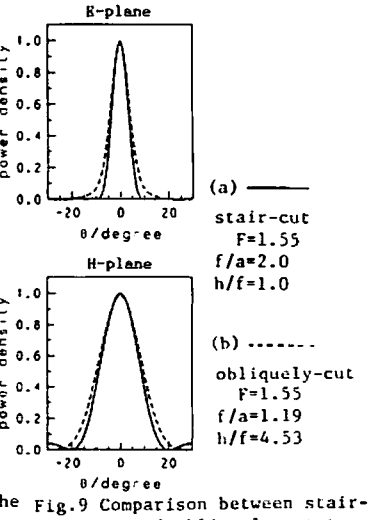


Fig.9 Comparison between stair-cut type and obliquely-cut type

measured half-value angles. Although the size of (a) is smaller than (b), the half-value angle is smaller. But side lobes of (a) are larger than those of (b).

When the incident wave is in higher mode than TE_{01}, the beam is sharper.

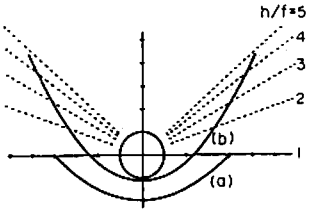


Fig.10 The size of parabolic cylinder
(a) Stair-cut type
(b) Obliquely-cut type

3 TE°_{0n}-$TE^{\square}_{10}$ Mode Converter

Fig.11 shows the configuration of a circular-TE_{0n} to rectangular-TE_{10} mode converter. The linearly polarized output wave beam ($h_0 \times 4f$) is deflected by a plane conductor and enter a rectangular waveguide ($a_1 \times b$). When the height of the waveguide (a_1) is appropriate, most of the power of the beam is converted to TE_{10} mode. Fig.12 shows numerical results for circular TE_{01} incidence. $\chi = a/\lambda$ is normalized frequency ($\chi = F\rho'_{0n}/2\pi$). The efficiencies have peaks near $a_1/h_0 = 1.3 \sim 1.4$.

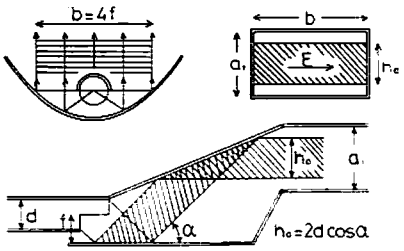


Fig.11 Circular-TE_{0n} to rectangular-TE_{10} mode converter

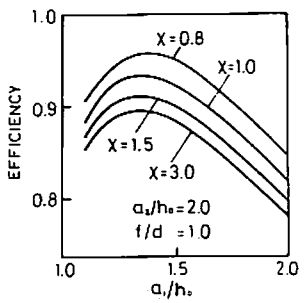


Fig.12 Mode conversion efficiency: circular TE_{01} incidence

4 Elliptic Cylinder Reflector Antenna

An elliptic cylinder can be used to focus wave beam in the transverse direction. To focus in the longitudinal direction, another parabolic cylinder reflector is used. Fig.13 shows the configuration of the reflector antenna. The beam can be concentrated at an arbitrary point by proper settings of the two reflectors.

5 Conclusions

Two types of parabolic reflector antennas have been compared, (a) the stair-cut type and (b) the obliquely-cut type. The latter antenna (b) can be made compact when h/f is equal to 2 or 3. If the sizes of reflectors are same, the beam of (a) is sharper than that of (b). The side lobe level of (b) is lower than that of (a). The obliquely-cut type antennas (b) have been used in Gamma-10 (Tsukuba Univ.) and other fusion plasma devices, and satisfactory results have been obtained[3].

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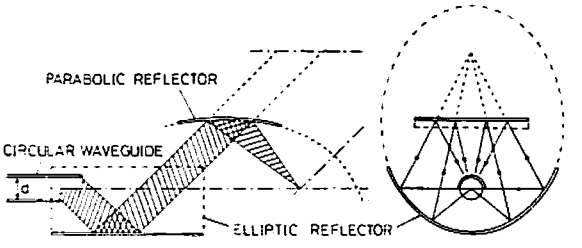

Fig.13 Elliptic cylinder reflector antenna

Reference

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[2] Nakajima,Wada:Report of the special research project on nuclear fusion, (1983),in Japanese
[3] Saito et al. :Conf. Digest 9th Int. Conf. on Infrared and Millimeter Waves, Osaka(1984)