# Effective MPIE formulation for LTCC electrically small antennas design at Ka band

Min Zhang, Ban-Leong Ooi, Le-Wei Li, Mook-Seng Leong Department of Electrical and Computer Engineering National University of Singapore, 10 Kent Ridge Crescent, Singapore, 119620 <u>elezm@nus.edu.sg</u>

**Abstract:** An effective mixed potential integral equation (MPIE) formulation in multilayered media is presented in this paper. In terms of the discrete complex image method (DCIM) with higher order Sommerfeld identity, three-dimensional spatial-domain Green's functions are calculated in closed-form, and the numerical differentiation of the curl operator in MPIE is avoided properly in spatial domain. Numerical results of the Green's functions and return loss of the low temperature co-fired ceramics (LTCC) electrically small antennas at Ka band are shown to demonstrate the efficiency and accuracy of the method.

Key words: LTCC, MPIE, Green's functions, DCIM, Multilayered media

## **1** Introduction

LTCC multilayer technologies [1] are widely gaining attraction over other substrate technologies. These include three dimensional (3-D) highly integration capabilities that result in size reduction and low-cost design, especially suited to high frequency operation, and concurrently allowing an arbitrary number of layers with flexibility connection using vertical viaholes or field coupled transitions, thus leading to good shielding between the layers. It is a well-known fact that with an increase in frequency and a decrease in the antennas size through the use of high permittivity but thin substrate with vertical components, the simulation and design effort would be tremendous. In this paper, we present a set of effective mixed potential integral equation formulation in multilayered media. Using Chew-Aksun's dipole source method [2] and the Michalski-Zheng's C-formulation [3], three-dimensional spatial-domain Green's functions are calculated fast in closed-form in terms of DCIM with higher order Sommerfeld identity. The numerical differentiation of the curl operator in MPIE is also avoided properly in spatial domain. Numerical results of the Green's functions and return loss of the LTCC electrically small antennas at Ka band are shown to demonstrate the efficiency and accuracy of the method.

### **2** Formulation

Consider an arbitrarily shaped object embedded in a planar multilayered media, and excited by arbitrary currents distribution  $(\vec{J}, \vec{M})$ , as shown in Fig.1. The fields due to these sources may be express as mixed potential forms,

$$\vec{E} = -jw\mu_0 < \vec{G}^{AJ}, \vec{J} > + \frac{1}{jw\varepsilon_0} \nabla < G^{VJ}, \nabla' \cdot \vec{J} > + < \vec{G}^{EM}, \vec{M} > , \qquad (1)$$

$$\vec{H} = -jw\varepsilon_0 < \vec{\overline{G}}^{AM}, \vec{M} > + \frac{1}{jw\mu_0} \nabla < G^{VM}, \nabla' \cdot \vec{M} > + < \vec{\overline{G}}^{HJ}, \vec{J} > .$$
<sup>(2)</sup>

where,  $\overline{G}^{EAJ/AM}$  are the dyadic Green's functions for magnetic and electric vector potentials as discussion in [3].  $G^{VJ/VM}$  are the Green's functions for corresponding electric and magnetic scalar potentials and  $\overline{G}^{EM/HJ}$  are the dyadic Green's functions for coupled field, which can be obtained by the curl of the  $\overline{G}^{AJ/AM}$ , respectively.



Fig 1. An object embedded in a planar multilayered media with excited current  $\vec{J}, \vec{M}$ 

The derivation of the spectral domain Green's functions in multilayered media can be accomplished in terms of Chew-Aksun's dipole source method [2]. The Green's functions in spectral can be concisely expressed as,

$$\widetilde{\widetilde{G}}^{IJ} = \begin{bmatrix}
\mu_m F_n^{TE} & 0 & -jk_x \mu_m V_n^{TM} \\
0 & \mu_m F_n^{TE} & -jk_y \mu_m V_n^{TM} \\
jk_x \mu_n V_n^{TE} & jk_y \mu_n V_n^{TE} & \frac{\mu_m \varepsilon_m}{\varepsilon_n} F_n^{TM} - \mu_n \frac{\partial}{\partial z'} V_n^{TE}
\end{bmatrix},$$

$$\widetilde{G}^{VJ} = \frac{1}{\varepsilon_m} \left( F_n^{TE} + \frac{\partial}{\partial z'} V_n^{TM} \right),$$
(4)

$$\overline{\widetilde{\widetilde{G}}}^{HJ} = \begin{bmatrix} -k_x k_y V_n^{TE} & -\frac{\mu_m}{\mu_n} \frac{\partial}{\partial z} F_n^{TE} - k_y^2 V_n^{TE} & j k_y F_n^{TM} \\ \frac{\mu_m}{\mu_n} \frac{\partial}{\partial z} F_n^{TE} + k_x^2 V_n^{TE} & k_x k_y V_n^{TE} & j k_x F_n^{TM} \\ -\frac{\mu_m}{\mu_n} j k_y F_n^{TE} & \frac{\mu_m}{\mu_n} j k_y F_n^{TE} & 0 \end{bmatrix} .$$
(5)

where,  $\mu_m, \varepsilon_m$  and  $\mu_n, \varepsilon_n$  are relative permeability and permittivity in source and field layer, respectively.  $F_n^{TE}$  and  $F_n^{TM}$  can be described by the basically vector potentials to which related to the horizontal electric and magnetic dipole in [2].

Correspondingly, the  $V_n^{TE/TM}$  can be derived as follow,

$$V_n^{TE/TM} = -\frac{1}{k_{\rho}^2} [c \cdot \partial_z (F_n^{TE/TM} - b) + \partial_{z'} (F_n^{TM/TE} - b)].$$
(6)  
Here,  $c = \begin{cases} \mu_m / \mu_n & \text{for } V_n^{TE}, \\ \varepsilon_m / \varepsilon_n & \text{for } V_n^{TM}. \end{cases}$  and  $b = \begin{cases} \frac{1}{2jk_{mz}} e^{-jk_{mz}|z-z'|}, & \text{when } m = n, \\ 0, & \text{when } m \neq n. \end{cases}$ 

In terms of the duality theorem, we can get the  $\overline{G}^{AM}$ ,  $\widetilde{G}^{VM}$  and  $\overline{G}^{EM}$  by changing  $\varepsilon \Leftrightarrow \mu$ ,  $TE \Leftrightarrow TM$  in equation (3-5). The generalized pencil of function method (GPOF) is next applied to extract the spectral kernel by a sum of complex exponentials. To solve the aperture coupled problem in an actual design circumstance, we start with the boundary conditions such that the magnetic field is continuous through the aperture and the total electric fields vanish on the electric conductors. The three MPIEs are obtained according equations (1) and (2). One of the advantages in this paper is that the differentiation of the curl operator is avoided directly by our

new representation of G in spatial domain in equation (5).

#### **3** Numerical results

All components of the closed-form Green's functions in spatial domain can be simulated easily. We use a four-layered structure as our example to demonstrate the efficiency and accuracy of the algorithm when applied to multilayered media. The dielectric constants and thinknesses of the layers are  $\varepsilon_{r1} = 2.1$ ,  $\varepsilon_{r2} = 12.5$ ,  $\varepsilon_{r3} = 9.8$ ,  $\varepsilon_{r4} = 8.6$  and  $d_1 = 0.7mm$ ,  $d_2 = 0.3mm$ ,  $d_3 = 0.5mm$ ,  $d_4 = 0.3mm$ , the frequency is 30 GHz. We compute the Green's functions in spatial domain with the source point and field point in the same layer (z = z' = 1.3mm, i.e. m=n=4) and also in different layer (z = 0.4mm, z' = 1.3mm, i.e. m=4 and n=1). The results are compared with reference [4]. A very good agreement has been obtained.

Fig.2 shows the magnitude of magnetic victor potential Green's function components of  $G_{xx}^{AJ}$ ,  $G_{zx}^{AJ}$ ,  $G_{zx}^{AJ}$ ,  $G_{zz}^{AJ}$  and electromagnetic scalar potential Green's function  $G^{VJ}$ , and compare with the magnitude of electric victor potential Green's function components of  $G_{xx}^{AM}$ ,  $G_{zx}^{AM}$ ,  $G_{zz}^{AM}$ ,  $G_{zz}^{AM}$  and magnetic scalar potential Green's function  $G^{VM}$  when the source and field points in the same layer.



Fig.2 The magnitude of  $G_{xx}^{AJ}$ ,  $G_{zx}^{AJ}$ ,  $G_{zz}^{AJ}$ ,  $G^{VJ}$  and  $G_{xx}^{AM}$ ,  $G_{zx}^{AM}$ ,  $G_{zz}^{AM}$ ,  $G^{VM}$ .

The components of  $\overline{G}^{EM/HJ}$  are obtained without the numerical differentiation. Fig.3 shows the magnitude of the Green's functions components  $G_{zy}^{EM}$  and  $G_{yx}^{HJ}$ , which are usually related to the electromagnetic coupled fields. We further consider the return loss from the LTCC aperture coupled electrically small antennas at high frequency. Fig.4 shows the results of  $S_{11}$  versus different frequency for the LTCC antennas, the parameters are: number of layers is eight, each layer's thickness is 3.7mils, aperture dimensions are 8 mils × 40 mils, feeding line dimensions are 200 mils × 8 mils,  $L_s = 40$  mils,  $\varepsilon_r = 5.9$ , and other dimensions are shown in Fig.4. It is found that the bandwidth of the LTCC antennas is around 6%, and good VSWR characteristics are achieved.







Fig.4 Magnitude of  $S_{11}$  versus frequency for the LTCC antennas

# **4** Conclusion

In this paper, the effective mixed potential integral equation (MPIE) formulation in multilayered media is presented. In terms of the discrete complex image method (DCIM) with higher order Sommerfeld identity, 3-D spatial-domain Green's functions are calculated fast in closed-form, and the numerical differentiation of the curl operator in MPIE is avoided properly in spatial domain. These new techniques are very useful for the efficient spatial-domain analysis in a multilayered media.

# **5** References

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