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Optimum Design of Nonuniform Microstrip Couplers

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Abstract

In this paper, the idea of using optimum designed nonuniform microstrip couplers is proposed to design efficient wideband couplers. A nonuniform microstrip coupler is considered and the width and gap functions are written as two truncated Fourier series. Then, the coefficients of the series are determined optimally using a proposed least mean square method. The usefulness of the proposed structure is verified using a comprehensive example.

1. Introduction

A directional coupler is a four-port structure to sample a fraction of signals interring a specified port of it. A usual way to fabricate a coupler is utilizing two parallel-coupled uniform microstrip lines [1-2]. The uniform microstrip couplers are efficient (having constant arbitrary coupling and high directivity) only in a narrow frequency band. There is a significant interest to design structures for efficient microstrip couplers in a wide frequency band. Some of the introduced structures for this purpose is Lange coupler [3], shielded coupler [4], dielectric overlay coupler [5], lumped capacitors loaded coupler [6], planar combline coupler [7], multisection coupler [2], directional harmonic filter [8] and cosine shape coupler [9].

In this paper, optimum designed nonuniform microstrip coupler is proposed as a structure to achieve wideband efficient couplers. A nonuniform two-line microstrip coupler is considered and then optimally designed using a proposed least mean square method. Firstly, the width and gap functions are written as two truncated Fourier series. Then, the coefficients of the series are determined in an optimization process. The usefulness of the proposed structure is verified using a comprehensive example.

2. Analysis of Nonuniform Couplers

In this section the analysis of nonuniform couplers is presented. Figures 1-2 show the longitudinal view and the cross section of a nonuniform coupler, respectively, consisting of two parallel-coupled nonuniform lines. The lines have been loaded by resistors Z_0 at their two-ends and their length is d. The width of strips and the gap between them are $w(z)$ are $s(z)$, respectively. Also, the relative electric permittivity and the thickness of the substrate are ε_r and h, respectively.

To analyze the nonuniform couplers, they are usually subdivided into K uniform electrically short segments with length $\Delta z = d/K$. Here, we define voltage vector $\mathbf{V}(z) = [V_1(z) \quad V_2(z)]^T$, current vector $\mathbf{I}(z) = [I_1(z) \quad I_2(z)]^T$ and source vector $\mathbf{V}_S = [V_S \quad 0]^T$ in the angular frequency ω. There are the following terminal conditions
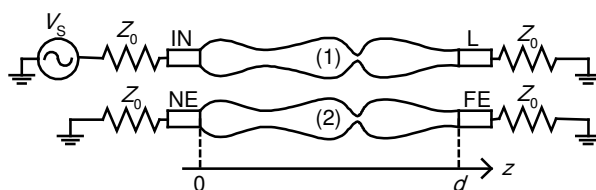

Figure 1. The longitudinal view of a nonuniform coupler consisting of two parallel-coupled nonuniform lines
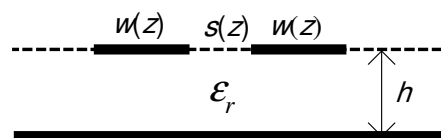

Figure 2. The cross section of the nonuniform microstrip coupler at point z

$$\mathbf{V}(0) = \mathbf{V}_S - Z_0\,\mathbf{I}(0) \tag{1}$$

$$\mathbf{V}(d) = Z_0\,\mathbf{I}(d) \tag{2}$$

Moreover, there is the following relation at two terminals

$$\begin{bmatrix} \mathbf{V}(d) \\ \mathbf{I}(d) \end{bmatrix} = \mathbf{\Phi} \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{I}(0) \end{bmatrix} \tag{3}$$

in which $\mathbf{\Phi}$ is the chain parameter matrix and is obtained as follows (in the case of neglecting the effect of discontinuities)

$$\mathbf{\Phi} = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} = \mathbf{\Phi}_K \mathbf{\Phi}_{K-1} \ldots \mathbf{\Phi}_2 \mathbf{\Phi}_1 \tag{4}$$

where the matrix $\mathbf{\Phi}_k$ ($k = 1, 2, \ldots, K$) is the chain parameter matrix of the k-th segment and has been written as follows [10]

$$\mathbf{\Phi}_k = \exp\left(-j\omega \frac{d}{K} \begin{bmatrix} \mathbf{0} & \mathbf{L}_k \\ \mathbf{C}_k & \mathbf{0} \end{bmatrix}\right) \tag{5}$$

where $\mathbf{L}_k$ and $\mathbf{C}_k$ are the per-unit-length inductance and capacitance matrices of the k-th segment, respectively. These matrices are related to the functions $w(z)$ and $s(z)$ at the center of the k-th segment and can be determined using some methods such as in [1] and [11]. Incorporating (1) - (3), leads us to the near and far-end terminal voltage vectors as follows

$$\mathbf{V}(0) = \mathbf{N}\mathbf{V}_S \tag{6}$$

$$\mathbf{V}(d) = \mathbf{F}\mathbf{V}_S \tag{7}$$

in which $\mathbf{N}$ and $\mathbf{F}$ are two 2 by 2 matrices as follows

$$\mathbf{N} = \mathbf{I}_d + Z_0(\varphi_{12} - Z_0\varphi_{11} - Z_0\varphi_{22} + Z_0^2\varphi_{21})^{-1}(\varphi_{11} - Z_0\varphi_{21}) \tag{8}$$

$$\mathbf{F} = Z_0\varphi_{21} - Z_0(\varphi_{22} - Z_0\varphi_{21})(\varphi_{12} - Z_0\varphi_{11} - Z_0\varphi_{22} + Z_0^2\varphi_{21})^{-1}(\varphi_{11} - Z_0\varphi_{21}) \tag{9}$$

in which $\mathbf{I}_d$ is an identity matrix. Usually, there are two factors to specify a coupler; the coupling factor and the directivity defined as follows, respectively, at frequency of f.

$$C(f) = 10\log\left(\frac{P_A(f)}{P_{NE}(f)}\right) = -10\log\left(4\,|\,\mathbf{N}(2,1)\,|^2\right); \ [dB] \tag{10}$$

$$D(f) = 10\log\left(\frac{P_{NE}(f)}{P_{FE}(f)}\right) = 10\log\left(\frac{|\,\mathbf{N}(2,1)\,|^2}{|\,\mathbf{F}(2,1)\,|^2}\right); \ [dB] \tag{11}$$

where three parameters P_A, P_{NE} and P_{FE} are defined as the available power, the power delivered to the near-end load and the power delivered to the far-end load, respectively. These powers have been written in (10)-(11) as follows.

$$P_A = \frac{V_S^2}{8Z_0} \tag{12}$$

$$P_{NE}(f) = \frac{V_S^2}{2Z_0}\,|\,\mathbf{N}(2,1)\,|^2 \tag{13}$$

$$P_{FE}(f) = \frac{V_S^2}{2Z_0}\,|\,\mathbf{F}(2,1)\,|^2 \tag{14}$$

3. Design of Nonuniform Couplers

In this section a general method is proposed to design optimally the nonuniform microstrip couplers. First, the width and gap functions are written as two following truncated Fourier series.

$$\ln\left(\frac{w(z)}{h}\right) = C_0 + \sum_{n=1}^{N}\left(C_n \cos(2\pi nz/d) + S_n \sin(2\pi nz/d)\right) \tag{15}$$

$$\ln\left(\frac{s(z)}{h}\right) = C_0' + \sum_{n=1}^{N}\left(C_n' \cos(2\pi nz/d) + S_n' \sin(2\pi nz/d)\right) \tag{16}$$

Utilizing truncated Fourier series expansion for the width and gap functions does not create any discontinuity in the resulted coupler. Knowing the width and gap functions, the per-unit-length

inductance and capacitance matrices and then the coupling factor and the directivity of the coupler can be obtained from (4)-(11).

Now, the coefficients of the series (15)-(16) are determined by minimization a suitable error function along with some constrained conditions. An optimum designed coupler has to has directivity D as large as possible and coupling factor C as close as to a desired coupling C_d, both in a defined large frequency range. Therefore, one may define the following error function and $2M$ constraints.

$$E = \sqrt{\frac{1}{2M}\left(\sum_{m=1}^{M}10^{-D(f_m)/10} + \sum_{m=1}^{M}|10^{C(f_m)/10} - 10^{C_d/10}|^2\right)}$$ (17)

$$|C(f) - C_d| \le R; \quad \forall f = f_1, f_2, ..., f_M$$ (18)

$$D(f) \ge D_{\min}; \quad \forall f = f_1, f_2, ..., f_M$$ (19)

where R is the maximum deviation from C_d and $D_{\min}$ is a minimum value for D. The variables used in this optimization method are the coefficients C_n, C_n', S_n, S_n' and the length of coupler d. Moreover, defined error function should be restricted by some other constraints to have matched lines at the ends of coupler and easy fabrication step.

$$w(0)/h = w(d)/h \Rightarrow Z_c(0) = Z_c(d) = Z_0$$ (20)

$$(w/h)_{\min} \le w(z)/h \le (w/h)_{\max}$$ (21)

$$(s/h)_{\min} \le s(z)/h \le (s/h)_{\max}$$ (22)

A suitable initial value for d may be given by

$$d_0 = \frac{\bar{\lambda}}{4} = \frac{1}{8}(\lambda_{\min} + \lambda_{\max})$$ (23)

in which $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum effective wavelength for a microstrip line, respectively, as follows

$$\lambda_{\min} = \frac{c}{f_M\sqrt{\varepsilon_r}}$$ (24)

$$\lambda_{\max} = \frac{c}{f_1\sqrt{(1+\varepsilon_r)/2}}$$ (25)

The optimum designed coupler will not necessarily become symmetric with respect to half of its length. However, one can consider $S_n = S_n' = 0$ to design a symmetric coupler. This condition causes reducing in the efficiency of designed coupler.

4. Example and Results

As an example, consider a microstrip coupler with $\varepsilon_r = 10$ and $Z_0 = 50$ Ω. We would like to design this coupler with $C_d = 10$ dB, $R = 0.1$ dB and $D_{\min} = 30$ dB in the frequency range of 1.0 to 4.0 GHz (two octave bandwidth). Using the proposed optimization method, considering $N = 5$, $(w/h)_{\min} = (s/h)_{\min} = 0.1$, $(w/h)_{\max} = (s/h)_{\max} = 10$ and $K = 135$, the optimum values of the parameters was obtained. Table 1 lists the values of the coefficients of the truncated Fourier series. Figures 3-4 illustrate the width and gap functions and also the top view of the optimally designed nonuniform microstrip coupler, whose optimum length is $d = 3.86$ cm. It is seen that the designed coupler is asymmetric. Figures 5-6 show the coupling factor and directivity of the coupler, respectively, versus frequency. One sees that the design conditions have been achieved successfully. It is obvious that the fabrication process of this nonuniform coupler is more difficult than that of a conventional uniform coupler.

Table 1. The values of the coefficients of the truncated Fourier series

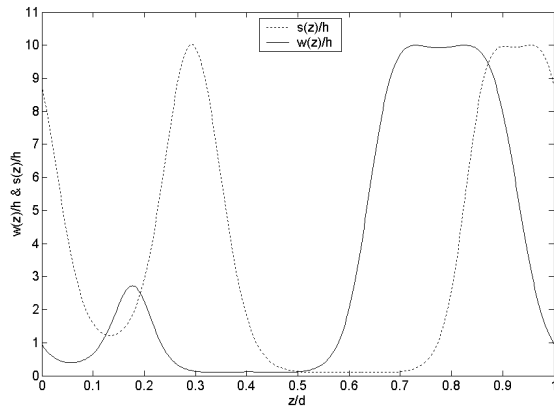| n	0	1	2	3	4	5
C_n	0.1476	1.0954	-0.9682	-0.2292	-0.2683	0.1821
S_n	---	-1.7186	0.3956	-0.3327	-0.4861	-0.0654
C_n'	0.3498	1.6626	-0.2671	0.2873	-0.0397	0.1773
S_n'	---	0.9644	-1.3746	-0.3606	0.2775	0.1073

Figure 3. The width and gap functions of optimally designed nonuniform microstrip coupler
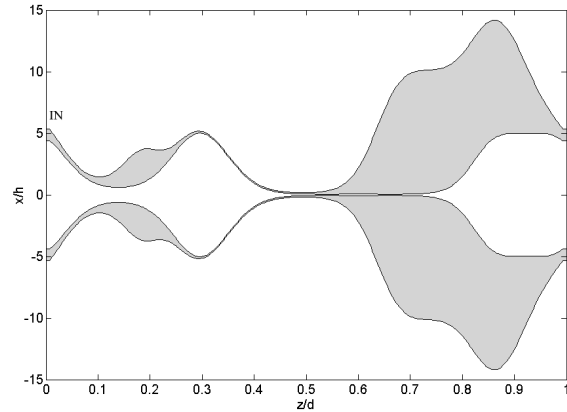

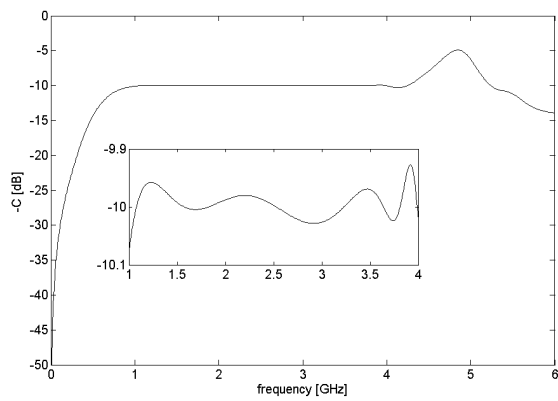Figure 4. The top view of optimally designed nonuniform microstrip coupler

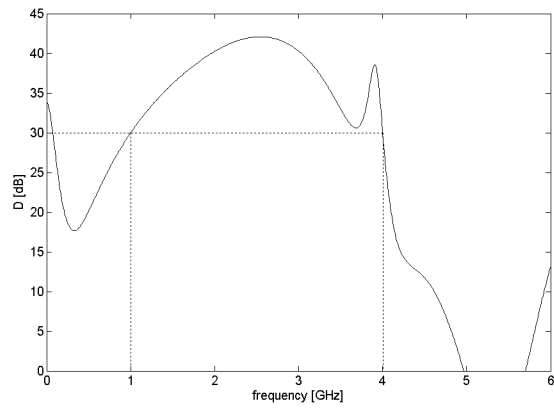Figure 5. The coupling factor of optimum designed nonuniform coupler

Figure 6. The directivity of optimum designed nonuniform coupler

5. Conclusions

The idea of using optimum designed nonuniform microstrip couplers was proposed to design efficient wideband couplers. A nonuniform microstrip coupler is considered and the width and gap functions are written as two truncated Fourier series. Then, the coefficients of the series are determined optimally using a proposed least mean square method. Consequently, the optimum values of the length of coupler, the strip widths and the distance between them are determined. Using an example, the usefulness of the proposed method was verified and it was concluded that nonuniform couplers can be designed to have higher efficiency than the conventional uniform couplers. Of course, the fabrication process of nonuniform couplers is more difficult than that of uniform ones. The proposed method may be used to design directional filters, also.

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