Path Loss and Matched Filter Gain for UWB Radio System

P. SUPANAKOON[†], P. TANGTISANON[†], S. PROMWONG[††] and J. TAKADA[††]

[†] Dept. of Information Engineering, Faculty of Engineering,
King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand.
Email: kspichay@kmitl.ac.th
[††] Graduate School of Science and Engineering, Tokyo Institute of Technology
O-okayama Minami 6 Bldg., 2–12–1, O-okayama, Meguro-ku, 152–8552, Tokyo, JAPAN
Email: ken@ap.ide.titech.ac.jp

1 Introduction

The ultra wideband (UWB) radio technology is a candidate that can be utilized for short-range, low power, low cost indoor communication such as wireless personal area network (WPAN) [1]. Federal Communication Commission (FCC) [2], in US specifies that the UWB signal have a frequency spectrum ranging from 3.1 to 10.6 GHz. The FCC defined UWB signals as those which have a fractional bandwidth greater than 0.20 or a bandwidth greater than 500 MHz measured at −10 dB points.

Frris' transmission formula is widely used to calculate the free space path loss for narrow band system [3]. Complex form Friis' transmission formula and the use of matched filter is developed for UWB system [4]. In this paper, the complex form Friis' transmission formula is used to derive the close form expression of free space path loss and matched filter gain for UWB system.

2 Expression of Free Space Path Loss and Matched Filter Gain

The UWB transmitted signal is as a passband rectangular pulse which have an expression in time domain $v_\mathrm{t}(t)$ and its spectral density function $V_\mathrm{t}(f)$ are

$$v_\mathrm{t}(t) = \frac{1}{f_\mathrm{b}}[f_\mathrm{max}\mathrm{sinc}(2f_\mathrm{max}t) - f_\mathrm{min}\mathrm{sinc}(2f_\mathrm{min}t)], \tag{1}$$

$$V_\mathrm{t}(f) = \begin{cases} \frac{1}{2f_\mathrm{b}} & |f-f_\mathrm{c}| \le \frac{f_\mathrm{b}}{2} \\ 0 & |f-f_\mathrm{c}| > \frac{f_\mathrm{b}}{2} \end{cases}, \tag{2}$$

where f_min and f_max are the minimum and maximum frequencies respectively, $f_\mathrm{b} = f_\mathrm{max} - f_\mathrm{min}$ is the spectral bandwidth, $f_\mathrm{c} = (f_\mathrm{max} - f_\mathrm{min})/2$ is the center frequency and $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$.

The transfer function of free space channel can be expressed by the complex form Friis' transmission formula H_f [4]

$$H_\mathrm{f}(f,d) = \frac{c}{4\pi|f|d}\exp(-j2\pi fd/c), \tag{3}$$

where c is the velocity of light and d is the transmitter-receiver (TR) separation distance.

The spectral density function and the waveform in the time domain of the received signal can be found from

$$V_\mathrm{r}(f,d) = V_\mathrm{t}(f)\cdot H_\mathrm{f}(f,d) = \begin{cases} \frac{c}{8\pi f_\mathrm{b}|f|d}\exp(-j2\pi fd/c) & |f-f_\mathrm{c}| \le \frac{f_\mathrm{b}}{2} \\ 0 & |f-f_\mathrm{c}| > \frac{f_\mathrm{b}}{2} \end{cases}, \tag{4}$$

$$v_{\mathrm{r}}(t,d)=\mathcal{F}^{-1}\{V_{\mathrm{r}}(f,d)\}=\begin{cases}\frac{c}{4\pi f_{\mathrm{b}}d}\ln(\frac{f_{\mathrm{max}}}{f_{\mathrm{min}}}) & t=\frac{d}{c}\\ \frac{c}{4\pi f_{\mathrm{b}}d}\left\{\ \mathrm{C_i}(2\pi f_{\mathrm{max}}|t-\frac{d}{c}|)-\mathrm{C_i}(2\pi f_{\mathrm{min}}|t-\frac{d}{c}|)\ \right\} & t\neq\frac{d}{c}\end{cases}, \tag{5}$$

where $\mathcal{F}^{-1}\{\cdot\}$ is the inverse Fourier transform operator and $\mathrm{C_i}=\int_{\infty}^{x}\frac{\cos(\tau)}{\tau}d\tau$ is the cosine integrals.

At the receiver, the matched filter detection is introduced. Its frequency transfer function H_{MF} satisties the following constant noise power condition between the input and output,

$$\int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}}|H_{\mathrm{MF}}(f,d)|^2df=f_{\mathrm{b}}. \tag{6}$$

For satisfying that condition, H_{MF} is given by

$$H_{\mathrm{MF}}(f,d)=V_{\mathrm{r}}^{*}(f,d)\sqrt{\frac{f_{\mathrm{b}}}{\int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}}|V_{\mathrm{r}}^{2}(f,d)|^2df}}=\begin{cases}\frac{\sqrt{f_{\mathrm{min}}f_{\mathrm{max}}}}{|f|}\exp(j2\pi fd/c) & |f-f_{\mathrm{c}}|\leq\frac{f_{\mathrm{b}}}{2}\\ 0 & |f-f_{\mathrm{c}}|>\frac{f_{\mathrm{b}}}{2}\end{cases}. \tag{7}$$

The spectral density function of the signal and time domain waveform at the matched filter output can be written as

$$V_{\mathrm{MF}}(f,d)\ =\ V_{\mathrm{r}}(f,d)\cdot H_{\mathrm{MF}}(f,d)=\begin{cases}\frac{c\sqrt{f_{\mathrm{min}}\cdot f_{\mathrm{max}}}}{8\pi f_{\mathrm{b}}f^2d} & |f-f_{\mathrm{c}}|\leq\frac{f_{\mathrm{b}}}{2}\\ 0 & |f-f_{\mathrm{c}}|>\frac{f_{\mathrm{b}}}{2}\end{cases}, \tag{8}$$

$$v_{\mathrm{MF}}(t,d)\ =\ \mathcal{F}^{-1}\{V_{\mathrm{MF}}(f,d)\} \tag{9}$$

$$=\begin{cases}\frac{c}{4\pi d\sqrt{f_{\mathrm{min}}f_{\mathrm{max}}}} & t=0\\ \frac{c\sqrt{f_{\mathrm{min}}f_{\mathrm{max}}}}{4\pi f_{\mathrm{b}}d}\left\{\begin{array}{c}\frac{\cos(2\pi f_{\mathrm{min}}t)}{f_{\mathrm{min}}}-\frac{\cos(2\pi f_{\mathrm{max}}t)}{f_{\mathrm{max}}}\\ +2\pi t\mathrm{S_i}(2\pi f_{\mathrm{min}}t)-2\pi t\mathrm{S_i}(2\pi f_{\mathrm{max}}t)\end{array}\right\} & t\neq0\end{cases}, \tag{10}$$

where $\mathrm{S_i}(x)=\int_{0}^{x}\frac{\sin(\tau)}{\tau}d\tau$ is the sine integrals.

Defining the UWB path loss PL_{UWB} by the ratio between the maximum amplitude of the transmitted and received signal waveforms. Therefore, the UWB free space loss in dB can be derived as shown below

$$PL_{\mathrm{UWB}}(d)[dB]=20\log\left[\frac{v_{\mathrm{t}}(0)}{v_{\mathrm{r}}(\frac{d}{c})}\right]=20\log\left[\frac{4\pi f_{\mathrm{b}}d}{c\ln\left(\frac{f_{\mathrm{max}}}{f_{\mathrm{min}}}\right)}\right]. \tag{11}$$

Finally, the UWB matched filter gain G_{MF} is defined as the ratio between the maximum amplitude of the signal waveform at the output of the matched filter and the received signal waveform. It can be expressed as

$$G_{\mathrm{MF}}[dB]=20\log\left[\frac{v_{\mathrm{MF}}(0)}{v_{\mathrm{r}}(\frac{d}{c})}\right]=20\log\left[\frac{f_{\mathrm{b}}}{\sqrt{f_{\mathrm{min}}f_{\mathrm{max}}}\ln\left(\frac{f_{\mathrm{max}}}{f_{\mathrm{min}}}\right)}\right]. \tag{12}$$

3 Results

In order to study the distortion of the UWB signal waveform, the UWB transmitted signal is set in the full UWB spectrum bandwidth. The maximum and minimum frequencies are $f_{\mathrm{min}}=3.1$

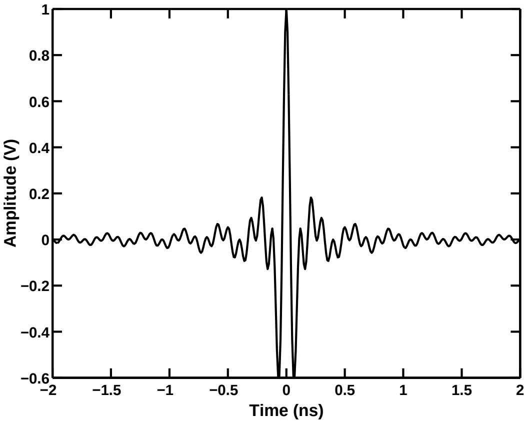

Figure 1: UWB transmitted signal.

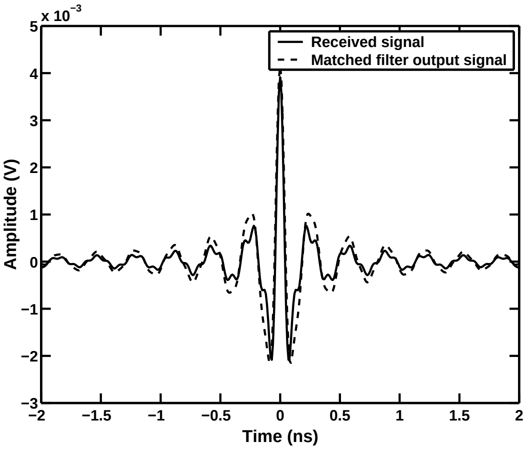

Figure 2: Received signal and signal at the output of matched filter.

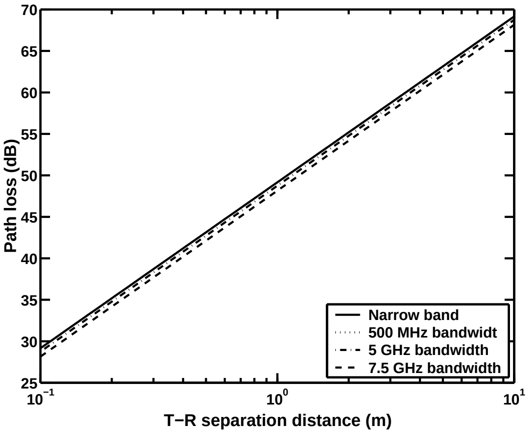

Figure 3: Path loss of the 500 MHz, 5 and 7.5 GHz signals compared with the narrow band signal with 6.85 GHz center frequency.

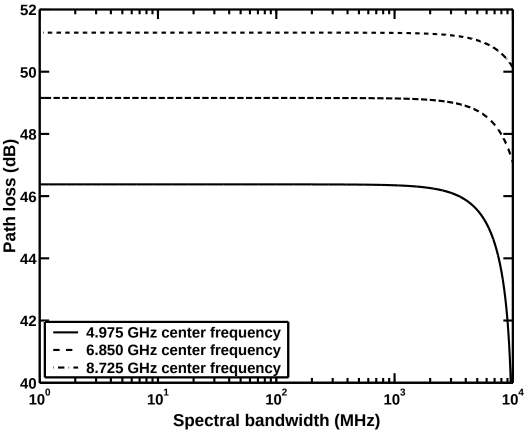

Figure 4: Path loss of the 4.945, 6.850 and 8.725 GHz center frequency signals at the 1 m TR separation distance.

GHz and $f_{\max} = 10.6$ GHz, respectively. Then, the spectral bandwidth is $f_{\rm b} = 7.5GHz$. The UWB transmitted signal is shown in Fig. 1. Figure 2 shows the received signal at 1 m T-R separation distance as compared with the signal waveform output of the matched filter. Time delay of the received signal is normalized to zero for easy comparison. We can observe that the received signal has improved in the matched filter. On the other hand, the amplitude of the signal based on the output of the matched filter has increased. For this case, the UWB matched filter gain is 0.54 dB.

For the path loss consideration, the path loss of the three UWB signals with 500 MHz, 5 and 7.5 GHz bandwidth are investigated and compared with that of the narrow band signal obtained form the Friis' transmission formula along the T-R separation distance about 0.1 to 10 m. The center frequency $f_{\rm c}$ of these signals are 6.85 GHz. The results are shown in Fig. 3. From the figure, we can observe that the path loss of 500 MHz bandwidth signal is very close to that of the narrow band signal. If the bandwidth of the signal is increased, the path loss is increased and it is less than that of the narrow band signal. And for all cases, of the free space path loss have the same path loss exponent which is equal to 2. In addition, the effect of signal bandwidth on the free space path loss is considered. Figure 4 shows the path loss in the 4.975, 6.850 and 8.725 GHz center frequency signals along the possible bandwidth. The path loss is nearly constant at a frequency prier to 2 GHz and beyond this the frequency decreased gradually.

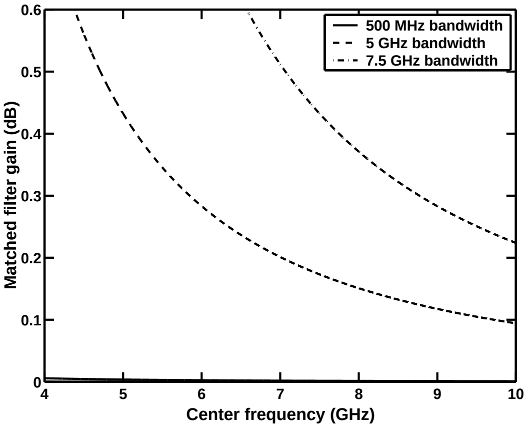

Figure 5: Matched filter gain of the 500 MHz, 5 and 7.5 GHz bandwidth signals.

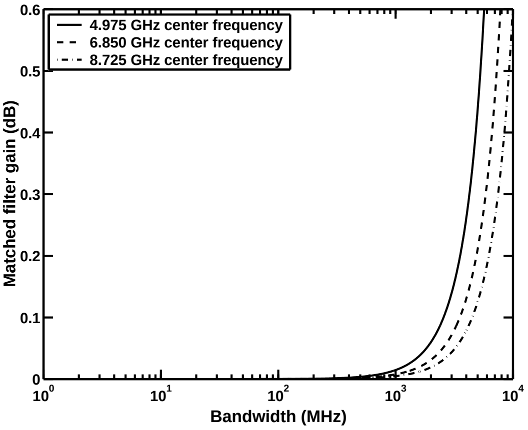

Figure 6: Matched filter gain of the 4.945, 6.850 and 8.725 GHz center frequency signals.

Furthermore, also the ffect of center frequency and bandwidth of the signal on the matched filter gain is studied. Figure 5 shows the matched filter gain of the 500 MHz, 5 and 7.5 GHz bandwidth signals. The path loss of the 4.945, 6.850 and 8.725 GHz center frequency signals are shown in Fig. 6. From these figures, we can observe that the matched filter gain has increased when the center frequency of the signal is decreased or the bandwidth of the signal is increased.

4 Conclusion

In this paper, the path loss and matched filter gain of the free space for UWB system are derived. From these derived expression, we can directly evaluate the signal waveform, the free space path loss and matched filter gain along the TR separation distance with arbitrary spectral bandwidth and center frequency signal. For future studies, we will derived the path loss and matched filter gain expressions for ground reflection channel.

References

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