Asymptotic Analysis of Edge-Excited Currents on a Convex Face of an Impedance Wedge

Kohei Hongo † and Hirokazu Kobayashi††

† Department of Information Science, Toho University, 2-2-1, Miyama, Funabashi City, 274-8510, Japan
e-mail: hongo@is.sci.toho-u.ac.jp
†† Fujitsu System Integration Laboratories. Kamiodanaka 4-1, Nakaharaku, Kawasaki City,Japan
e-mail: kobayasi@fsi.se.fujitsu.co.jp

1. Introduction

Asymptotic expressions of the edge-excited surface currents on a convex face of an impedance wedge are derived by using Maliuzhinets' and Fock' theories according to the method of the synthesis developed by Michaeli [1]. The edge of the wedge is assumed straight, and the incident electromagnetic wave is locally plane and normal to the edge. The angle of incidence includes the case where the penumbra regions of the edge and surface diffraction overlap.

An asymptotic description of electromagnetic field diffracted by an impedance curved wedge is important since it may provide a canonical problem of high frequency diffraction theory. Michaeli derived a relatively simple form of solution for the edge-diffracted surface currents of perfectly conducting wedge by using the concept of spectral theory of diffraction. His discussion is focused on the case where the penumbra regions of the edge and surface diffraction overlap, where ray interpretation of the diffraction is difficult. When an angle of incidence is far from the shadow boundaries, it is possible to obtain the solution by using the equivalent current method used in the area of the GTD. Molinet [2] inferred the solution for impedance curved wedge from the Michaeli's results, but his result seems to be incomplete. He used the diffraction coefficients of perfectly conducting wedge and effect of surface wave is missing, which is excited under some condition. In this paper, the expression of the edge-diffracted surface field on an impedance curved wedge are derived by applying the Maliuzhinets's theory for straight impedance wedge combined with the Fock's theory for curved surface. The synthesis of the solution follows Michaeli's procedure. The results can be used as the canonical problem of physical theory of diffraction with transition currents, which we are developing. The edge diffracted field is interpreted as a spectrum of inhomogeneous plane waves, and the surface field excited by each spectral plane wave is obtained by analytic continuation of the Fock function into complex space. In contrast to the case of perfectly conducting wedge, the contribution due to the surface wave excited at the edge must be added to the edge diffracted field. The results reduce to those of Michaeli for perfectly conducting wedge when the surface impedance approach zero.

2 Derivation of Solution

Consider plane wave incident to the curved impedance wedge, as described in Fig.1. The $z-$ components of electromagnetic fields of an incident plane wave is given by

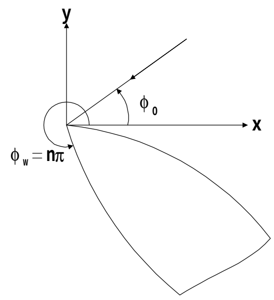

Fig. 1 Curved Wedge

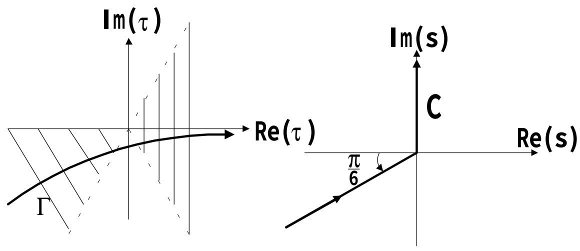

Fig.2 Contour Γ and C

$$\begin{bmatrix} H_z^i \\ E_z^i \end{bmatrix} = \begin{bmatrix} H_z^i(O) \\ E_z^i(O) \end{bmatrix} \exp[jk(x\cos\phi_0 + y\sin\phi_0)] = \begin{bmatrix} H_z^i(O) \\ E_z^i(O) \end{bmatrix} \exp[jk\rho\cos(\phi - \phi_0)] \tag{1}$$

where (ρ, ϕ) are the cylindrical coordinates of the observation point (x, y). This wave strikes the edge and produces the surface wave and diffracted field. The interaction of the direct incident wave with the curved face may be treated separately and is omitted here. The surface field excited at the edge of an impedance wedge is given by [3],[4]

$$F_z^{sw} = U^{sw} \exp[-jk\rho\cos(\theta_+ - \phi)]H[A], \qquad U^{sw} = -\frac{\Psi(\Phi - \pi - \theta_+)}{\Psi(\Phi - \phi_0)} \frac{2p\tan\theta_+ \sin p\phi_0}{\cos p(\pi + \theta_+) - \cos p\phi_0} \tag{2}$$

where F_z^{sw} denotes E_z^{sw} for $\sin\theta_+ = \dfrac{1}{\zeta}$ and H_z^{sw} for $\sin\theta_+ = \zeta$, ζ is the normalized surface impedance of the wedge face. $H(x) =$ is the Heaviside's unit step function, and $H(x) = 1$ for $x > 0$ and $H(x) = 0$ for $x < 0$. $2\Phi = \phi_w$ is the exterior wedge angle and $p = \dfrac{1}{n} = \dfrac{\pi}{2\Phi}$. $\Psi(z)$ is given by the product of four Maliuzhinets functions [3]. If the surface pole of the wedge function representation [3],[4] is denoted by $\alpha^{sw} = \pi + \theta_+ + \phi = \alpha_r^{sw} + j\alpha_i^{sw}$, $A > 0$ provided that the condition

$$-\pi + \mathrm{sign}(\alpha_i^{sw})\cosh^{-1}\left(\frac{1}{\cosh\alpha_i^{sw}}\right) < \alpha_r^{sw} < \pi + \mathrm{sign}(\alpha_i^{sw})\cosh^{-1}\left(\frac{1}{\cosh\alpha_i^{sw}}\right) \tag{3}$$

is satisfied and otherwise $A < 0$. When plane wave $\exp[-jk(x\cos\xi + y\sin\xi)]$ is incident to the curved impedance surface, the surface magnetic fields are obtained [5] by using the Fock's theory. Since the surface field is intepreted as an inhomogeneous plane wave with complex wavenumber the field on the curved face excited by the surface wave is given by

$$H_z = U^{sw1} \exp[jka\sin\theta_+ - jk(\ell + a\theta_+)]f(\sigma' + m\theta_+, q_H), \qquad q_H = -jm\zeta \tag{4a}$$

$$H_\ell = \frac{q_E U^{sw1}}{jm} \exp[jka\sin\theta_+ - jk(\ell + a\theta_+)]f(\sigma' + m\theta_+, q_E), \qquad q_E = -\frac{jm}{\zeta} \tag{4b}$$

$$m = \left(\frac{ka}{2}\right)^{\frac{1}{3}}, \qquad \sigma' = m\frac{\ell}{a}, \qquad f(x, q) = \frac{1}{\sqrt{\pi}} \int_\Gamma \frac{\exp(-jx\tau)}{w_2'(\tau) - qw_2(\tau)} d\tau \tag{4c}$$

where a is the radius of the curvature of the curved face, ℓ is the distance from the edge along the face directed normal to the edge, $w_2(\tau)$ is the Airy function of Fock type, $w_2'(\tau)$ is its derivative with respect to τ. The contour Γ is shwon in Fig. 2.

The contribution of the edge-diffracted wave to the surface field on the convex impedance face is obtained by using the spectral theory of diffraction. The result is written as

$$\begin{aligned}\begin{bmatrix} H_z^d \\ H_\ell^d \end{bmatrix} &= \begin{bmatrix} H_z^i(O) \\ Y_0 E_z^i \end{bmatrix} \exp(-jk\ell) \int_\Gamma G_1(\tau, \phi) \exp(-j\sigma'\tau) \begin{bmatrix} [w_2'(\tau) - q_H w_2(\tau)]^{-1} \\ (q_E/jm)[w_2'(\tau) - q_E w_2(\tau)]^{-1} \end{bmatrix} d\tau \\ &+ \begin{bmatrix} H_z^i(O) \\ Y_0 E_z^i \end{bmatrix} \exp(-jk\ell) \int_\Gamma G_2(\tau, \phi) \exp(-j\sigma'\tau) \begin{bmatrix} [w_2'(\tau) - q_H w_2(\tau)]^{-1} \\ (q_E/jm)[w_2'(\tau) - q_E w_2(\tau)]^{-1} \end{bmatrix} d\tau \end{aligned} \tag{5}$$

The contour C is shown in F.2. where

$$\begin{aligned} G_1(\tau, \phi) = \frac{1}{m\sqrt{\pi}} \int_C &\left\{ \frac{\Psi[\Phi - (s/m) - \pi]}{\Psi(\Phi - \phi_0)} \cot\frac{\pi + (s/m) - \phi_0}{2n} - \frac{\Psi[\Phi + (s/m) - \pi]}{\Psi(\Phi - \phi_0)} \cot\frac{\pi - (s/m) + \phi_0}{2n} \right\} \\ &\times \exp\left[-j\frac{s^3}{3} - j\tau s\right] ds \end{aligned} \tag{6a}$$

$$G_2(\tau, \phi) = \frac{1}{\sqrt{\pi}} \int_C \frac{\Psi[\Phi + (s/m) - \pi]}{\Psi(\Phi - \phi_0)} \frac{2\sin\theta_+}{\sin\xi + \sin\theta_+} \cot\frac{\pi - (s/m) + \phi_0}{2n} \exp\left[-j\frac{s^3}{3} - j\tau s\right] ds \tag{6b}$$

(i) When $|\pi - \phi_0| > \dfrac{1}{m}$

The integrand of $G_1(\tau, \phi)$ is expanded by the Taylor series and truncated it by the second term. Then $G_1(\tau, \phi)$ becomes linear combination of the integral representation of the Airy integral $w_2(\tau)$ and its derivative $w_2'(\tau)$. Then the tangential components of the magnetic field is given by

$$\begin{aligned} H_z^d &= \frac{j}{m} H_z^i(O)\exp(-jk\ell)[jD_0(\phi_0)V_0(\sigma', q_H) + D_1(\phi_0)U_0(\sigma', q_H)] \\ &\quad - H_z^i(O)\frac{\Psi(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\sin\theta_+ \cot\frac{\pi+\phi_0}{2n}\exp(-jk\ell)W_0(\sigma', q_H, -m\sin\theta_+) \qquad (7a) \\ H_\ell^d &= \frac{q_E}{jm^2} Y_0 E_z^i(O)\exp(-jk\ell)[jD_0(\phi_0)V_0(\sigma', q_E) + D_1(\phi_0)U_0(\sigma', q_E)] \\ &\quad - \frac{q_E}{jm} Y_0 E_z^i(O)\frac{\Psi(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\sin\theta_+ \cot\frac{\pi+\phi_0}{2n}\exp(-jk\ell)W_0(\sigma', q_E, -m\sin\theta_+) \qquad (7b) \end{aligned}$$

where

$$\begin{aligned} D_0(\phi_0) &= \frac{\Psi(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\left[\cot\frac{\pi-\phi_0}{2n} - \cot\frac{\pi+\phi_0}{2n}\right] \\ D_1(\phi_0) &= \frac{1}{m}\frac{\Psi'(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\left[\cot\frac{\pi-\phi_0}{2n} + \cot\frac{\pi+\phi_0}{2n}\right] + \frac{1}{2mn}\frac{\Psi(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\left[\operatorname{cosec}^2\frac{\pi-\phi_0}{2n} + \operatorname{cosec}^2\frac{\pi+\phi_0}{2n}\right] \\ V_0(\sigma, q) &= \int_C \frac{w_2(\tau)}{w_2'(\tau) - qw_2(\tau)}\exp(-j\sigma\tau)d\tau, \qquad U_0(\sigma, q) = \int_C \frac{w_2'(\tau)}{w_2'(\tau) - qw_2(\tau)}\exp(-j\sigma\tau)d\tau \\ W_0(\sigma, q, p) &= \int_C \frac{W(\tau, p)}{w_2'(\tau) - qw_2(\tau)}\exp(-j\sigma\tau)d\tau, \qquad W(\tau, p) = -\frac{1}{\sqrt{\pi}}\int_C \frac{1}{s-p}\exp\left[-j\frac{s^3}{3} - j\tau s\right]ds \quad (7c) \end{aligned}$$

It is noted that $W(\tau, p) \approx -\frac{j}{p}w_2(\tau)$ and $W_0(\tau, q, p) \approx -\frac{j}{p}V_0(\tau, q)$ for large value of p. Thus the solution near the shadow boundary can be recovered from the ray representation for the region far from the shadow boundary by substituting $jpW_(\tau, q, p)$ for $V_0(\tau, q)$.

(ii) When $|\phi_0 - \pi| < \frac{1}{m}$:

This is the case where the penumbra regions of the edge and surface diffraction overlap. Evaluation similar to the case (i) leads to the results of the magnetic field.

$$\begin{aligned} H_z^d &= \frac{1}{m} H_z^i(O)\exp(-jk\ell)[-jD_2(\phi_0)V_0(\sigma', q_H) + D_3(\phi_0)U_0(\sigma', q_H) - D_4(\phi_0)W_0(\sigma', q_H, -u)] \\ &\quad - H_z^i(O)\frac{\Psi(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\sin\theta_+ \cot\frac{\pi+\phi_0}{2n}\exp(-jk\ell)W_0(\sigma', q_H, -m\sin\theta_+) \qquad (8a) \\ H_\ell^d &= \frac{q_E}{jm^2} Y_0 E_z^i(O)\exp(-jk\ell)[-jD_2(\phi_0)V_0(\sigma', q_E) + D_3(\phi_0)U_0(\sigma', q_E) - D_4(\phi_0)W_0(\sigma', q_E, -u)] \\ &\quad - \frac{q_E}{jm} Y_0 E_z^i(O)\frac{\Psi(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\sin\theta_+ \cot\frac{\pi+\phi_0}{2n}\exp(-jk\ell)W_0(\sigma', q_E, -m\sin\theta_+) \qquad (8b) \end{aligned}$$

where

$$\begin{aligned} D_2(\phi_0) &= \left[\cot\frac{\pi}{n} + 2n\frac{\Psi'(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\right], \qquad D_3(\phi_0) = \frac{1}{m}\left[\frac{1}{2n} + \operatorname{cosec}^2\frac{\pi}{n} + \frac{\Psi'(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\cot\frac{\pi}{n}\right] \\ D_4(\phi_0) &= 2mn\left[1 + \frac{\Psi'(\Phi-\pi)}{\Psi(\Phi-\phi_0)}\frac{u}{m}\right], \qquad u = m\tan\frac{\pi-\phi_0}{2n} \qquad (8c) \end{aligned}$$

3 Some Remarks on Numerical Computation

Thus the problem reduces to compute three kinds of functions given by $V_0(x, q)$, $U_0(x, q)$ and W_0 (x, q, u). Logan[6] derived the approximate expressions $V_a(x, q)$ and $U_a(x, q)$ for the functions $V_0(x, q)$ and $U_0(x, q)$. The corrections $V_c(x, q) = V_0(x, q) - V_a(x, q)$ and $U_c(x, q) = U_0(x, q) - U_a(x, q)$ can be transformed into a form convenient for numerical computation. The final forms for the functions $V_0(x, q)$ and $U_0(x, q)$ are given by

$$V_0(x, q) = -2\sqrt{\pi}\exp\left(-j\frac{\pi}{6}\right)\left\{\frac{1}{\sqrt{\pi z}} - w\left[j\sqrt{z}Q\right]\right\} - \frac{1}{\sqrt{\pi}}\exp\left(-j\frac{\pi}{6}\right)\frac{\sin(zT)}{2zT^2}$$

$$+\frac{2}{\sqrt{\pi}}\exp\left(-j\frac{\pi}{6}\right)\Re\int_0^T\left[\frac{Ai(j\alpha)}{Ai'(j\alpha)-QAi(j\alpha)}+\frac{1}{\sqrt{j\alpha}+Q}\right]\exp[jz\alpha]d\alpha+O(T^{-3}) \tag{9}$$

$$U_0(x,q)=-2\sqrt{\pi}Q\exp\left(-j\frac{\pi}{6}\right)\left\{\frac{1}{\sqrt{\pi z}}-Qw\left[j\sqrt{z}Q\right]\right\}-\frac{Q}{\sqrt{\pi}}\exp\left(-j\frac{\pi}{6}\right)\frac{\sin(zT)}{2zT^2}$$

$$+\frac{2}{\sqrt{\pi}}\exp\left(-j\frac{\pi}{6}\right)\Re\int_0^T\left[\frac{Ai'(j\alpha)}{Ai'(j\alpha)-QAi(j\alpha)}-\frac{\sqrt{j\alpha}}{\sqrt{j\alpha}+Q}\right]\exp[jz\alpha]d\alpha+O(T^{-3}) \tag{10}$$

where T is an aribtrary large value, $\Re$ denotes the real part of, $z=\frac{\sqrt{3}+j}{2}x$, $Q=q\exp\left(-j\frac{2\pi}{3}\right)$ and $w(z)$ is the error function defined by

$$w(z)=-\frac{j2z}{\pi}\int_0^\infty\frac{\exp(-u^2)}{u^2-z^2}du \tag{11}$$

The Airy functions $Ai(z)$ and its derivatives of the complex arguments can be computed by using the relations between the Airy and the modified Bessel functions.

According to the Michaeli's procedure, the function $W_0(x,q,u)$ can be transformed into

$$W_0(\sigma,q,\sigma')=-j\exp\left(-j\frac{\sigma^3}{3}\right)\int_\Gamma\frac{w_2(t)}{w_2'(t)-qw_2(t)}\left[qI_2(t,-\sigma)-\frac{\partial}{\partial t}I_2(t,-\sigma)\right]\exp[-j(\sigma'-\sigma)t]dt$$

$$-j2\sqrt{\pi}\exp\left(-j\frac{\sigma^3}{3}\right)H(-\sigma)\int_\Gamma\frac{\exp[-j(\sigma'+\sigma')t]}{w_2'(t)-qw_2(t)}dt \tag{12}$$

where $I_2(x,y)$ is the incomplete Airy function. The definition of $I_2(x,y)$ and its asymptotic expression for large value of y are given by

$$I_2(x,y)=\int_y^{\infty\exp(-j\psi)}\exp\left[-j\frac{s^3}{3}-jxs\right]ds,\qquad\frac{2\pi}{3}\le\psi\le\pi,\quad|y|\gg1$$

$$\approx\left[-\frac{j}{(y^2+x)}+\frac{2y}{(y^2+x)^3}+j\frac{10y^2-2x}{(y^2+x)^5}\right]\exp\left[-j\left(\frac{y^3}{3}+xy\right)\right] \tag{13}$$

Using the properties of $I_2(x,y)$ described in [7] and the above asymptotic expansion, $I_2(x,y)$ can be computed numerically for any values of x and y.

4 Conclusion

We derived the expressions for the surface fields of an impedance curved wedge by applying the theories of Fock for curved surface and Maliuzhinets for impedance wedge. To construct the solution, we use the concept of the spectral theory of diffraction according to Michaeli. The surface fields are expressed as the combination of three kinds of functions $U_0(x,q)$, $V_0(x,q)$ and $W_0(x,q,u)$ and these are transformed into to the forms convenient for numerical computation. The development of the numerical codes for these functions are under study.

References

[1] A. Michaeli, Asymptotic analysis of edge-excited currents on a convex face of a perfectly conducting wedge under overlapping penumbra region condition, IEEE Trans. on Antennas and Propagation, vol. 44, no.9, pp. 97-101, 1996

[2] F. Molinet, Asymptotic currents on perfectly conducting and coated convex and concave 2D surfaces with edges, URSI Tessalonich, 25-28, 1998.

[3] T. B. A Senior and J. L. Volakis, Approximate Boudary Conditons in Electromagnetics, The IEE, London, 1995.

[4] A. Osipov, K. Hongo and H. Kobayashi, High frequency approximation for electromagnetic field near a face of an impedacne wedge, submitted to the IEEE Trans. on Antennas and Propagation.

[5] A. V. Osipov, H. Kobayashi and H. Hongo, Shadow boundary currents in the problem of high frequency electromagnetic diffraction by a circular impedance cylinder, The Inst. EICE in Japan, vol. E81-C, no. 10, pp. 1655-1666, 1998.

[6] N. A. Logan, General research in diffraction theory, vol. I and II, LMSD Rep.288087 and 288088, Lockheed Aircraft Co., 1959

[7] L. Levey and L. B. Felsen, On incomplete Airy functions and their application to diffraction problem, Radio Science, vol.4, no. 10, pp.959-969, 1969.