New Diffraction Analysis for Physical Optics Scattered Field from Perfectly Conducting Structure

Suomin Cui, Ken-ichi Sakina and Makoto Ando
Department of Electrical and Electronic Engineering, Tokyo Institute of Technology
cuism@antenna.pe.titech.ac.jp

1. Introduction

The problem of converting the physical optics (PO) surface integral to line integral has had a very long history. More recently, there are two leading approaches to convert PO surface integral into line one in the field of electromagnetics including exact analysis [1-3] and method of equivalent currents(MEC) [4-8]. As is expected, the exact solution is too complicated, moreover, it just can be applied for limited structures and sources. The MEC, on the other hand, generally suffers from false singularities out of *Keller* cone which depend upon the direction of the coordinate of inner integration on the surface to edge integral reduction [5,7]. In order to overcome the difficulty, several approaches have been proposed which include introducing Fresnel integral [6], optimum direction [7] and modified edge representation (MER)[8]. In this paper, an alternative diffraction analysis is presented to convert the PO surface integral to line one for planar scatterer. It is based upon field equivalence principle and integration is analytically conducted provided that the incidence on the edge behaves as the local spherical wave.

2. PO diffraction field in terms of line integral

The total field by PO from a planar scatterer S shown in Fig.1 is given as follows

$$\vec{E}^{PO}(\vec{r}_P) = \vec{E}^{i} + \vec{E}^{Sca}$$

$$\vec{E}^{Sca} = -\frac{j\eta}{k}\nabla\times\nabla\times\vec{A}^{PO}, \vec{A}^{PO} = \iint_S \vec{I}^{PO} G(\vec{r}_P, \vec{r}_S)dS \quad , \quad \vec{I}^{PO} = 2\hat{n}\times\vec{H}^{i}. \tag{1}$$

where $\vec{E}^{i}$ is the direct field from the source.

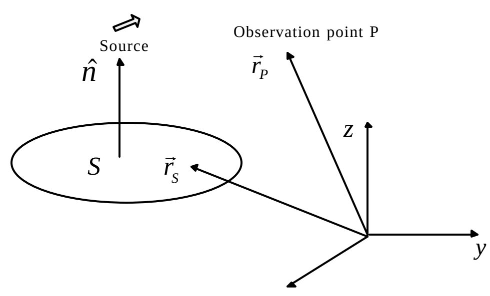


Fig.1 Scattering geometry for planar scatterer

$G(\vec{r}_P - \vec{r}_S) = \frac{\exp(-jk|\vec{r}_P - \vec{r}_S|)}{4\pi|\vec{r}_P - \vec{r}_S|}$ is the Green function, k and η are the wave number and the intrinsic impedance of free space, respectively. We define $\vec{E}^{\beta}_{\alpha}(\vec{r}_P)$ which is the field at $\vec{r}_P$ radiated from the equivalent currents $\vec{I}^{\beta} = \hat{n}\times\vec{H}^{\beta}$ and $\vec{M}^{\beta} = \vec{E}^{\beta}\times\hat{n}$ produced by source β on the surface α with outward unit normal vector $\hat{n}$ as,

$$\vec{E}^{\beta}_{\alpha}(\vec{r}_P) = -\frac{j\eta}{k}\nabla\times\nabla\times\vec{A}^{\beta} - \nabla\times\vec{B}^{\beta},$$

$$\vec{A}^{\beta} = \iint_{\alpha}\vec{I}^{\beta}G(\vec{r}_P - \vec{r}_a)d\alpha \ , \vec{B}^{\beta} = \iint_{\alpha}\vec{M}^{\beta}G(\vec{r}_P - \vec{r}_a)d\alpha. \tag{2}$$

where $\vec{r}_a$ is the position vector on the surface α. The superscript $\beta = \underline{inc}(\underline{ref})$ indicates the component associated with the real source (image) .

In order to reduce the surface integral to line one, we construct the closed surface $S_1^{'}$ by adding the scatterer S with two complimentary surfaces S_1 and $S_{1,\infty}$. S_1 is on the shadow boundary of the source whose outward unit normal vector is denoted by $\hat{n}_1$ shown in Fig.2(a). $S_{1,\infty}$ caps it at infinity. Applying field equivalence principle (FEP) for the source and the surface $S_1^{'}$ and using the radiation condition for $S_{1,\infty}$ ($\vec{E}^{inc}_{S_{1,\infty}}(\vec{r}_P) = 0$), we obtain

$$\vec{E}^{inc}_{S_1}(\vec{r}_P) + \vec{E}^{inc}_{S}(\vec{r}_P) = \begin{cases} 0 & \text{P outside of } S_1 \\ -\vec{E}^{i} & \text{P inside of } S_1 \end{cases} \tag{3}$$

Similarly, by using FEP for the image and the surface $S_2 + S + S_{2,\infty}$, where S_2 is on the reflection boundary shown in Fig.2(b), we can get the following relation.

$$\vec{E}^{ref}_{S_2}(\vec{r}_P) + \vec{E}^{ref}_{S}(\vec{r}_p) = \begin{cases} 0 & \text{P outside of } S_2 \\ -\vec{E}^{GO} & \text{P inside of } S_2 \end{cases} \tag{4}$$

where $\vec{E}^{GO}$ is the electrical field from the image.

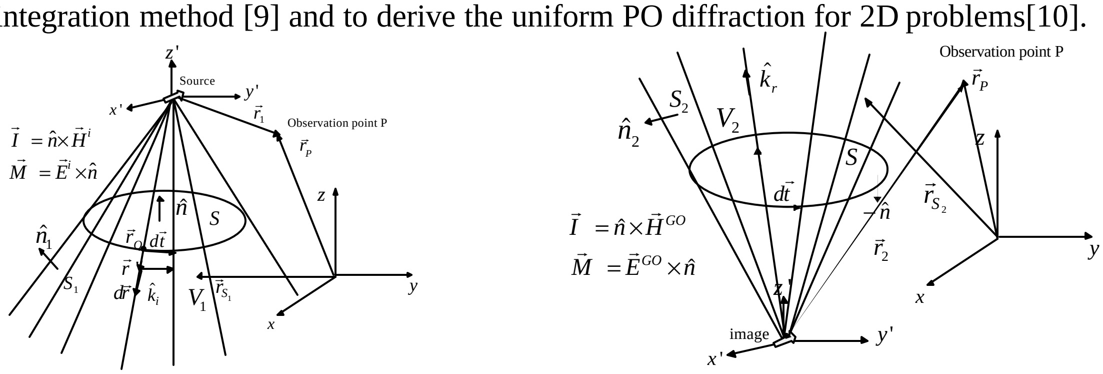
Using $\hat{n}\times(\vec{H}^{i} + \vec{H}^{GO}) = 2\hat{n}\times\vec{H}^{i}$, $\hat{n}\times(\vec{E}^{i} + \vec{E}^{GO}) = 0$ on the *S* and Eqs. (3-4) we obtain

$$\vec{E}^{PO} = -\vec{E}^{inc}_{S_1}(\vec{r}_P) + \vec{E}^{ref}_{S_2}(\vec{r}_P) + \begin{cases} \vec{E}^{GO} + \vec{E}^{i} & \text{P inside of } S_2 \\ \vec{E}^{i} & \text{P inside of } S_1 \\ 0 & \text{others} \end{cases} \tag{5}$$

then PO diffraction field can be written as

$$\vec{E}^{d} = -\vec{E}^{inc}_{S_1}(\vec{r}_P) + \vec{E}^{ref}_{S_2}(\vec{r}_P) \tag{6}$$

Consequently, the original PO surface integration is rigorously deformed in those on shadow and reflection boundaries. Similar techniques were successfully used to evaluate the aperture field integration method [9] and to derive the uniform PO diffraction for 2D problems[10].



(a) The surface for the source (b) The surface for the Image

Fig.2 Integration surfaces for PO diffraction field

Now, the field from the source incident on S_1 is assumed to be local spherical wave, $\hat{k}_i \cdot \vec{E}^{i} = \hat{k}_i \cdot \vec{H}^{i} = 0$, where $\hat{k}_i$ is the incident direction. The radiation fields are assumed as

$$\vec{E}^{i}(\vec{r}\,') = \vec{E}^{i}_{0}\exp(-jkr')/r' \quad , \quad \vec{H}^{i}(\vec{r}\,') = \vec{H}^{i}_{0}\exp(-jkr')/r'. \tag{7}$$

where r' is the distance between the source and the point of integration.

The source locates at the origin of the local coordinate system shown in Fig.2(a). The field from the currents on the surface S_1 can be written as

$$\vec{E}^{inc}_{S_1}(\vec{r}_1) = jk(\boldsymbol{h}\,\hat{r}_1\times\hat{r}_1\times\vec{A}^{S_1} + \hat{r}_1\times\vec{B}^{S_1})\frac{\exp(-jkr_1)}{4\boldsymbol{p}\,r_1} \tag{8}$$

$$\vec{A}^{S_1} = \iint_{S_1}\vec{I}^{\,inc}\exp(jk\hat{r}_1\cdot\vec{r}\,')dS_1 \quad , \quad \vec{B}^{S_1} = \iint_{S_1}\vec{M}^{\,inc}\exp(jk\hat{r}_1\cdot\vec{r}\,')dS_1 \quad . \tag{9}$$

The above surface integration is conducted along r' analytically while numerically along *t*.

$$\vec{A}^{S_1} = -\int \hat{k}_i \vec{H}^i \cdot d\vec{t}\, \frac{\exp(jkr_Q \cos\beta_{inc})}{jk(1-\cos\beta_{inc})} \qquad \vec{B}^{S_1} = \int \hat{k}_i \vec{E}^i \cdot d\vec{t}\, \frac{\exp(jkr_Q \cos\beta_{inc})}{jk(1-\cos\beta_{inc})} \quad (10)$$

where $\cos\beta_{inc} = \hat{k}_i \cdot \hat{r}_1$, $\hat{k}_i = \frac{\vec{r}\,'}{r'}$.

Taking the same procedures as those for the currents on the source S_1 from the source, we can obtain the field associated with the currents on the surface S_2 from the image as ;

$$\vec{E}^{ref}_{S_2}(\vec{r}_2) = jk(\eta\,\hat{r}_2 \times \hat{r}_2 \times \vec{A}^{S_2} + \hat{r}_2 \times \vec{B}^{S_2}) \frac{\exp(-jkr_2)}{4\pi r_2} \quad (11)$$

$$\vec{A}^{S_2} = \int \hat{k}_r \vec{H}^{GO} \cdot d\vec{t}\, \frac{\exp(jkr_w \cos\beta_{ref})}{jk(1-\cos\beta_{ref})}, \vec{B}^{S_2} = -\int \hat{k}_r \vec{E}^{GO} \cdot d\vec{t}\, \frac{\exp(jkr_W \cos\beta_{ref})}{jk(1-\cos\beta_{ref})} \quad (12)$$

where $\cos\beta_{ref} = \hat{k}_r \cdot \hat{r}_2$, $\hat{k}_r$ is the propagation direction of GO ray.

For far field observer, $\hat{r}_1 = \hat{r}_2 = \hat{r}_0$, where $\hat{r}_0 = (\vec{r} - \vec{r}\,')/R$, $R = |\vec{r} - \vec{r}\,'|$, $\vec{r}$ and $\vec{r}\,'$ are the position vectors of the point of observation and of a point on the rim of the planar scatterer. The PO diffraction field in (6) can be expressed in terms of the line integral as

$$\vec{E}^d = jk\int_l [\eta\,\hat{r}_0 \times \hat{r}_0 \times \vec{I}_e + \hat{r}_0 \times \vec{M}_e] \frac{\exp(-jkR)}{4\pi R} dt \quad (13)$$

$$\vec{I}_e = \frac{\vec{H}^i \cdot \hat{t}}{jk(1-\cos\beta_{inc})}\hat{k}_i + \frac{\vec{H}^{GO} \cdot \hat{t}}{jk(1-\cos\beta_{ref})}\hat{k}_r, \vec{M}_e = -\frac{\vec{E}^i \cdot \hat{t}}{jk(1-\cos\beta_{inc})}\hat{k}_i - \frac{\vec{E}^{GO} \cdot \hat{t}}{jk(1-\cos\beta_{ref})}\hat{k}_r . \quad (14)$$

By using the relations of $\hat{t} \cdot \vec{H}^i = \hat{t} \cdot \vec{H}^{GO}, \hat{t} \cdot \vec{E}^i = -\hat{t} \cdot \vec{E}^{GO}$ for perfectly conducting plates , $\vec{I}_e$ and $\vec{M}_e$ can be written as

$$\vec{I}_e = \left[\frac{\hat{k}_i}{1-\cos\beta_{inc}} + \frac{\hat{k}_r}{1-\cos\beta_{ref}}\right]\frac{\vec{H}^i \cdot \hat{t}}{jk}, \vec{M}_e = \left[-\frac{\hat{k}_i}{1-\cos\beta_{inc}} + \frac{\hat{k}_r}{1-\cos\beta_{ref}}\right]\frac{\vec{E}^i \cdot \hat{t}}{jk} \quad (15)$$

The procedures for deriving the new EECs are mathematical; we do not need any asymptotic methods such as the stationary phase method which is popularly used in the high frequency diffraction analysis. The expression of the PO diffraction field is very similar to that of the MEC[4-8], so $\vec{I}_e$ and $\vec{M}_e$ shown in equation (15) are defined as the new EECs, respectively . The coefficients in Eq.(15) are determined by incident and reflection directions, which is independent of $\hat{t}$. Their orientation is not along the tangent unit vector $\hat{t}$ while that for the other EECs is identical with $\hat{t}$. It is true that the EECs can be cast into the vector along $\hat{t}$, so that we can compare them with available ones. The most important advantage of the new EECs is that $\vec{I}_e$ and $\vec{M}_e$ have no fictitious singularities except the real ones at shadow and reflection boundaries. The singularities are *keller* style which appear in all diffraction analysis addressing this work. The expressions of the new EECs are much simpler in comparison with other works, only two cosine functions are used, it is believed that the solution costs much smaller CUP time.

3. Numerical check

The new solution predicts very accurate results like the POEECs[7] and MER[8] in the case that incident source is much far away from the scatterer including plane wave. We focus on the comparison when the source is not far from the scatterer. PO field can be numerically obtained by directly integrating the PO currents on the scatterer as the reference

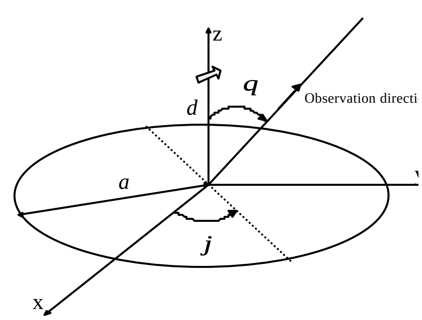
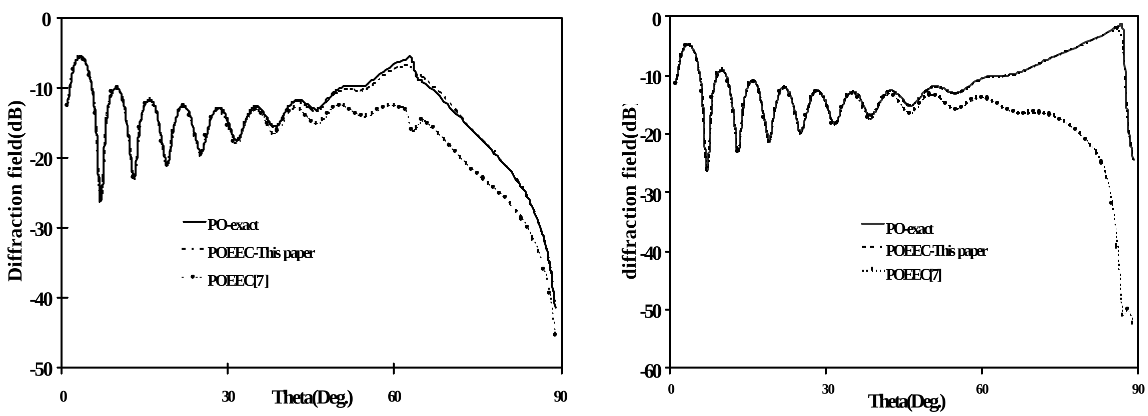


Fig.3 Scattering Geometry of disk

"PO-exact", hereafter. Let us consider the radiation field of electric dipole waves from a flat disk depicted in Fig.3, in which a is the radius of the disk, the dipole locates on z-axis and is in the distance d from the disk, θ and φ represent the observation direction. In the following calculation, we set $a = 5\lambda$ and $\varphi = 0°$, respectively. Figures 4 show the results obtained by different methods for a z-directed dipole source with different values of d. When $d = 2.5\lambda$, $d = 0.25\lambda$, better agreement between the new analysis discussed in this paper and the exact solution is observed except small discrepancies around the reflection boundary, while the POEECs [7] is deviating from the exact one in wide observation regions. MER, empirically developed, predicts very accurate results which are not included in the figures. Extensive numerical results show the POEECs discussed in this paper is more accurate than the POEECs [7] which was discussed in [11] in more details, but it is worse than MER[8]. The reason and the relationship between the new analysis and available works will be investigated.



(a) $a = 5\lambda$ and $d = 2.5\lambda$ (b) $a = 5\lambda$ and $d = 0.25\lambda$

Fig.4 Accuracy check of the new diffraction analysis for a disk (z-directed dipole)

4. Conclusion

A novel diffraction analysis has been presented to convert PO surface integral into line one in order to calculate PO field efficiently. We first apply for field equivalence principle to extract the PO diffraction field from total PO field in the form of the surfaces integration on the RB and SB, then convert the two surface integrals into line ones by using simple mathematics treatments. Finally the PO diffraction field is expressed by local parameters at the edge of the scatterer . The new diffraction analysis only suffers from the real singularities on SB and RB, its accuracy is checked by numerical results, and its superiority is demonstrated. The analysis can be applied to predict the diffraction field from curve surfaces.

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