FINITE-ELEMENT ANALYSIS OF ELECTROMAGNETIC WAVE SCATTERING BY A PLANE GRATING IN CASE OF OBLIQUE INCIDENCE AND ARBITRARY POLARIZATION

Yasunori NAKATA Masanori KOSHIBA

Department of Electronic Engineering, Faculty of Engineering, Hokkaido University, Sapporo-shi, 060 Japan

1. INTRODUCTION

A problem of electromagnetic wave scattering by a plane grating is one of the basic electromagnetic problems and is of great importance in practical application to spectrum analyzers, reflector antennas, polarization discriminators, and so on. Plane gratings show two kinds of features, that is, frequency selectivity and polarization selectivity. In connection with the latter property, plane gratings have attracted antenna researchers' attention from the practical view point of the efficient use of radio frequency band via orthogonal polarization [1]-[8]. Considering the practical situation, it is important to deal with oblique incidence as well as arbitrary polarization for the design of efficient polarization discriminators [1]-[8].

Recently the finite-element method (FEM) has been presented for various dielectric or metallic gratings [9], [10]. The FEM is a powerful tool for the analysis of arbitrarily shaped gratings consisting of inhomogeneous and/or lossy media. In [9] and [10], however, the incident direction has been assumed to be in the plane perpendicular to the grating axis.

In this paper, a numerical approach based on the FEM is described for the analysis of scattering by a plane grating in case of oblique incidence and arbitrary polarization. The FEM is a domain-type method and, in general, requires a plenty of computer resources. However, the computation in the FEM can be efficiently performed by introducing the substructure method [11]. In addition, the most part of the FEM computation is the manipulation of matrices, and therefore the FEM is very suitable for supercomputers. Numerical examples are calculated for three kinds of metallic strip gratings on a dielectric sheet. The validity of the approach proposed here is confirmed by comparing numerical results with the earlier theoretical results [1]. Also, we check the required computer resources and show that the computation is efficiently performed even if the FEM is used.

2. OUTLINE OF FORMULATION

We consider a periodic structure with period d in the x-direction and with no variation in the z-direction as shown in Fig. 1. Input and output regions consist of lossless and homogeneous media, whose relative permittivities are ε_1 and ε_2, respectively. Here boundaries Γ_1 and Γ_2 are located at $y=y_1$ and $y=y_2$, respectively, so that the grating region is included in the range of $y_1<y<y_2$. And boundaries Γ_3 and Γ_4 are located at $x=0$ and $x=d$, respectively, so that one period is partitioned with those boundaries. The region Ω with the boundaries Γ_1 to Γ_4 completely encloses the grating region.

An incident wave is linearly polarized and the direction of

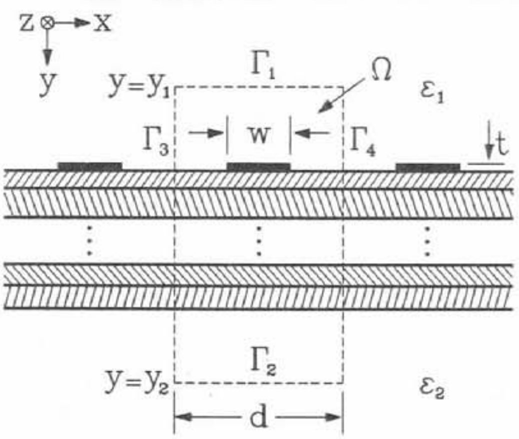

Fig.1 Geometry of problem.

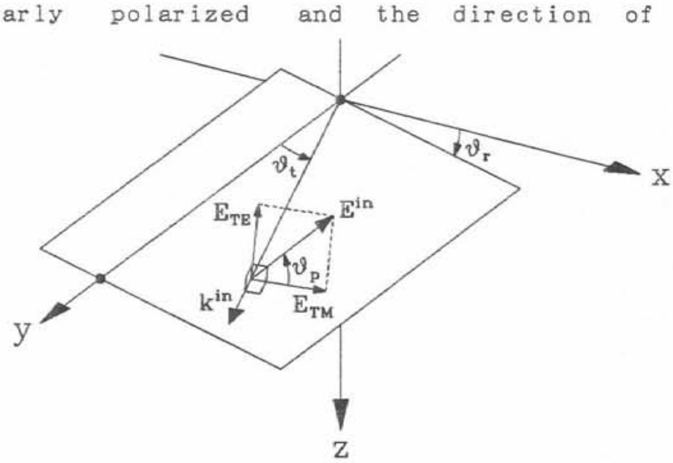

Fig.2 Incident angle and polarization angle.

polarization is shown in Fig. 2. Let the incident plane be the one including incident wave-number vector $\boldsymbol{k}^{in}$ and y-axis, θ_r be the angle between the incident plane and x-axis, and θ_t be the one between y-axis and the $\boldsymbol{k}^{in}$ vector. Then $\boldsymbol{k}^{in}$ is expressed as

$$\boldsymbol{k}^{in}=(k_x^{in},\ k_y^{in},\ k_z^{in})=k_0\sqrt{\varepsilon_1}\,(\sin\theta_t\cos\theta_r,\ \cos\theta_t,\ \sin\theta_t\sin\theta_r) \tag{1}$$

$$k_0=\omega\sqrt{\varepsilon_0\mu_0} \tag{2}$$

where ω is the angular frequency, ε_0 and μ_0 are the permittivity and permeability of free space, respectively. The direction of incident electric field $\boldsymbol{E}^{in}$ is also defined as shown in Fig. 2, where θ_p is the angle between the $\boldsymbol{E}^{in}$ vector and the incident plane. The $\boldsymbol{E}^{inp}$ vector can be decomposed into two components, i.e., E_{TE} and E_{TM} vectors, and is expressed as

$$\boldsymbol{E}^{in}=\boldsymbol{E}_{TE}+\boldsymbol{E}_{TM}=|\boldsymbol{E}^{in}|\sin\theta_p\,\boldsymbol{e}^{TE}+|\boldsymbol{E}^{in}|\cos\theta_p\,\boldsymbol{e}^{TM} \tag{3}$$

$$\boldsymbol{e}^{TE}=(\sin\theta_r,\ 0,\ -\cos\theta_r),\ \boldsymbol{e}^{TM}=(\cos\theta_r\cos\theta_t,\ -\sin\theta_t,\ \sin\theta_r\cos\theta_t) \tag{4}$$

Now, in actual application, another angle on the projected plane which is normal to the incident direction $\boldsymbol{k}^{in}$ is more important. Let the angle $\theta_{p'}$ be the one between the projections of the electric field $\boldsymbol{E}^{in}$ and the unit vector in the z-direction $\boldsymbol{i}_z$. Then the $\boldsymbol{E}^{in}$ vector is also expressed as

$$\boldsymbol{E}^{in}=|\boldsymbol{E}^{in}|\sin\theta_{p'}\,\boldsymbol{e}^{\perp}+|\boldsymbol{E}^{in}|\cos\theta_{p'}\,\boldsymbol{e}' \tag{5}$$

$$\boldsymbol{e}^{\perp}=(\cos\theta_t,\ -\sin\theta_t\cos\theta_r,\ 0)/A,\ \boldsymbol{e}'=(\sin^2\theta_t\sin\theta_r\cos\theta_r,\ \sin\theta_t\cos\theta_t\sin\theta_r,\ \sin^2\theta_t\sin^2\theta_r-1)/A \tag{6}$$

$$A=\sqrt{1-\sin^2\theta_t\sin^2\theta_r} \tag{7}$$

Here the relationship between the angles θ_p and $\theta_{p'}$ is given by

$$\theta_p+\theta_{p'}=90^\circ+\theta_{p0},\ \tan\theta_{p0}=\cos\theta_t\tan\theta_r \tag{8}$$

Dividing the region Ω into a number of quadratic triangular elements, using the FEM based on a Galerkin procedure, and considering the contributions of all elements, we obtain the following equation:

$$\begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{bmatrix}\{E_z\}\\ \{\eta_0H_z\}\end{bmatrix}=\begin{bmatrix} jk_0\eta_0\sum_e'\left[\int_e\{N\}_1H_x|_{\Gamma_1}dx-\int_e\{N\}_2H_x|_{\Gamma_2}dx-\int_e\{N\}_3H_y|_{\Gamma_3}dy+\int_e\{N\}_4H_y|_{\Gamma_4}dy\right]\\ -jk_0\sum_e'\left[\int_e\{N\}_1E_x|_{\Gamma_1}dx-\int_e\{N\}_2E_x|_{\Gamma_2}dx-\int_e\{N\}_3E_y|_{\Gamma_3}dy+\int_e\{N\}_4E_y|_{\Gamma_4}dy\right]\end{bmatrix} \tag{9}$$

where $[A]$, $[B]$, $[C]$, and $[D]$ are the finite-element matrices, $\eta_0=\sqrt{\mu_0/\varepsilon_0}$, the components of $\{E_z\}$ and $\{\eta_0H_z\}$ vectors are the values of E_z and η_0H_z at all nodes, respectively, $\{N\}$ is the shape function vector in the region Ω, $\{N\}_i$ ($i=1,2,3,4$) is the shape function vector on boundary Γ_i, T denotes a transpose, $\int_e dx$ and $\int_e dy$ are carried over the element contour Γ_e on Γ_i, and $\sum_e'$ extends over the elements related to the boundary Γ_i. In case of $\theta_r=0$, the matrices $[B]$ and $[C]$ become zero matrices and (9) is reduced to the equations derived by Nakata, Koshiba and Suzuki [9], and Nakata and Koshiba [10].

Considering the analytical solutions for the boundaries Γ_1 and Γ_2, and the periodic conditions for the boundaries Γ_3 and Γ_4, the final matrix equation is completed. Once the electromagnetic field on the boundary Γ_i ($i=1,2$) is obtained, normalized reflected and transmitted powers are calculated.

3. NUMERICAL EXAMPLES

In this section, we deal with three kinds of metallic strip gratings as shown in Fig. 3. The grating (A) is backed with a dielectric sheet and is optimized by varying the thickness of the dielectric sheet t_A. The grating (B) consists of a metallic strip on a dielectric sheet and another dielectric sheet and is optimized by varying the thickness of lower dielectric sheet t_{B2} and the space between the upper and lower dielectric sheets t_{B3} under the fixed value of the thickness of the upper dielectric sheet t_{B1}. The grating (C) consists of two opposite metallic strips on a dielectric sheet and is optimized by varying the space of the upper and lower dielectric sheets t_{C3} under the fixed values of the thicknesses of the two sheets t_{C1} and t_{C2}. These gratings are designed and operated with the following parameters:

$$\left.\begin{array}{l}\dfrac{t_A}{\lambda_0}=0.2725,\ \dfrac{t_{B1}}{\lambda_0}=0.015,\ \dfrac{t_{B2}}{\lambda_0}=0.035,\ \dfrac{t_{B3}}{\lambda_0}=0.27665,\ \dfrac{t_{C1}}{\lambda_0}=\dfrac{t_{C2}}{\lambda_0}=0.015,\ \dfrac{t_{C3}}{\lambda_0}=0.29\\ \dfrac{\varepsilon_r}{\varepsilon_0}=3.5,\ \dfrac{w}{d}=0.375,\ \dfrac{d}{\lambda_0}=0.1,\ \theta_t=45^\circ,\ \theta_r=0^\circ,\ \theta_{p'}=0^\circ\text{ or }90^\circ,\ \lambda_0=\dfrac{c}{f_0}\end{array}\right\} \tag{10}$$

where f_0 is the design frequency and c is the light velocity. Here the strips are made of perfectly conducting metal and the strip thickness t is assumed to be negligibly small. For these gratings, the reflected and transmitted powers near the above parameters are expected to be $P_0^r \approx 0$ and $P_0^t \approx 1$ for $\theta_{p'}=90°$, and $P_0^r \approx 1$ and $P_0^t \approx 0$ for $\theta_{p'}=0°$. Numerical examples are calculated from the view point of the fluctuation of each parameter.

First, the characteristics for incident conditions are investigated. Fig. 4(a) shows the frequency dependence at $\theta_{p'}=90°$. The grating (A) has a simple structure and inevitably has a narrow band-width. There is not much difference between the gratings (B) and (C). Fig. 4(b) shows the frequency dependence at $\theta_{p'}=0°$. The curve for the grating (C) is almost on the frame line of $P_0^r=0$(dB). The reflected power of the grating (B) does not much change near $P_0^r \approx -0.025$(dB), but that of the grating (A) becomes small slowly as the frequency departs from f_0. Fig. 4(c) shows the dependence of reflection on the angle θ_t between y-axis and $\boldsymbol{k}^{in}$. For the grating (C), the reflected power P_0^r changes rapidly at $\theta_t \approx 45°$ and 66°. But this phenomenon is not much interesting because, in actual application, it is enough to be $P_0^t > -0.025$(dB) and $P_0^r < -22.41$(dB) [1]. Fig. 4(d) shows the dependence of reflection on the angle θ_r between the incident plane and x-axis. The θ_r-dependence changes more slowly than the θ_t-dependence. In the above

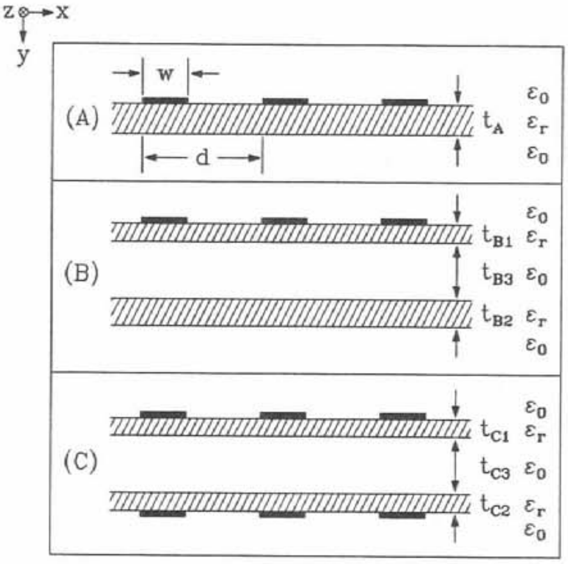

Fig.3 Three kinds of gratings:(A),(B) and (C).

Table 1 Element division and required computer resources (processor:HITAC S-810/10).

grating	elements	nodes	processing time for one iteration		required storage (MB)
			CPU-time (sec)	VPU-time (sec)	
(A)	55296	112201	22.0	21.6	15.4
(B)	58752	119209	30.8	30.3	17.9
(C)	57600	116873	30.2	29.7	17.9

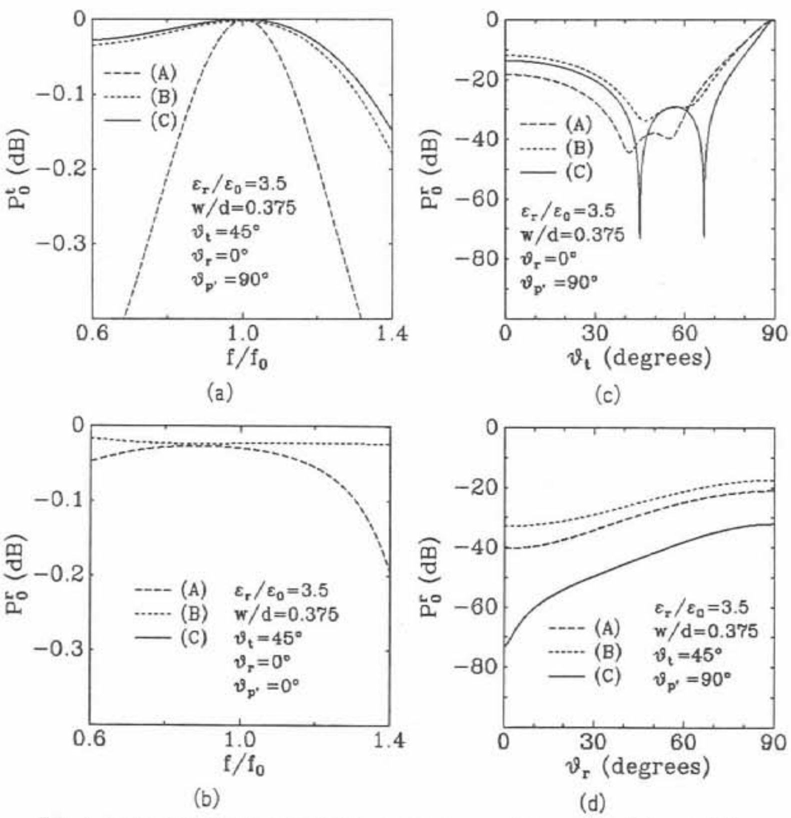

Fig.4 Reflected and transmitted power for gratings (A),(B) and (C).

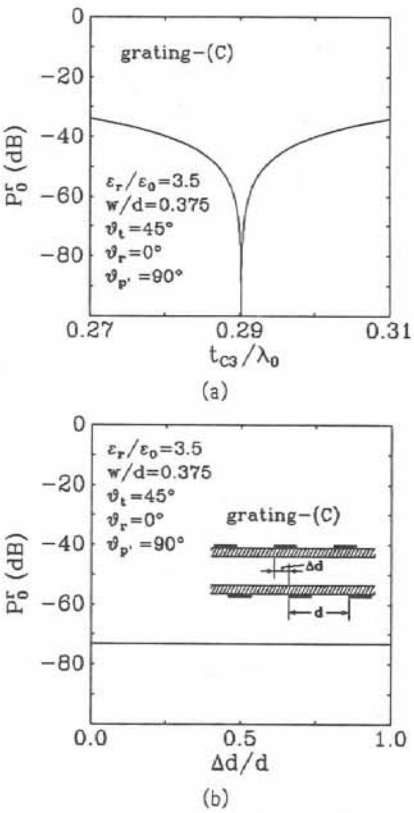

Fig.5 Reflected power changes due to structural displacement in the grating (C)

analysis, our results agree well with those by the point-matching method [1].

Next, the characteristics for structural displacements of the grating (C) are investigated. Fig. 5(a) shows the dependence of reflection on the space between two dielectric sheets. The reflected power P_0^r changes rapidly at $t_{C3}/\lambda \approx 0.29$, but that is not much interesting because of the above mentioned reason. Fig. 5(b) shows the dependence of reflection on the displacement Δd of dielectric sheet in the x-direction. The reflected power P_0^r does not change at all. So, it is unnecessary to set two sheets precisely in the x-direction.

Through all examples, although the energy error of each diffracted wave seems to be less than about 10^{-3}, the energy error of conservation is less than 10^{-11}.

Finally, Table 1 shows the element division and required computer resources. Numerical computation was performed using the supercomputer HITAC S-810/10 made in HITACHI Ltd., Japan. The number of nodes is much large, about 110000 to 120000. But the computation is carried out using the supercomputer and the substructure method [11] is introduced, so the computation does not require much resources even if the FEM is used. In addition, the ratio of VPU(vector processing unit)-time to CPU-time is almost equal to one. It is because the vectorization ratio for these examples is almost 100%, and is not less than 99.5%.

4. CONCLUSIONS

A numerical approach based on the FEM was described for the analysis of electromagnetic wave scattering by a plane grating in case of oblique incidence and arbitrary polarization. In order to perform the FEM computation efficiently, the substructure method was introduced and supercomputer was used. To show the validity of this approach, numerical examples were calculated for three kinds of metallic strip gratings. In addition, required computer resources were also checked.

This approach may be useful for the optimum design of efficient polarization discriminators.

REFERENCES

[1] M. Murota, M. Ando and N. Goto: "Design of a low-loss grating on a dielectric sheet", Trans. IECE Japan, E69, 4, pp. 321-322, April 1986.

[2] M. Ando and M. Murota: "Reflection and transmission coefficients of a thin strip grating on a dielectric sheet", Trans. IECE Japan, E69, 11, pp. 1189-1198, Nov. 1986.

[3] S. Kanazawa and T. Itakura: "Three-dimensional scattering of electromagnetic wave by an infinite plane grating", Trans. IEICE Japan, J70-B, 1, pp. 172-174, Jan. 1987 (in Japanese).

[4] K. Uchida, T. Noda and T. Matsunaga: "Electromagnetic wave scattering by a plane metallic grating loaded with three dielectric layers", Trans. IEICE Japan, J70-B, 3, pp. 375-384, March 1987 (in Japanese).

[5] T. Noda, K. Uchida and T. Matsunaga: "Scattering of electromagnetic plane wave by an infinite plane grating on a dielectric slab in case of oblique incidence", Trans. IEICE Japan, J71-B, 2, pp. 263-270, Feb. 1988 (in Japanese).

[6] K. Uchida, T. Noda and T. Matsunaga: "Electromagnetic wave scattering by an infinite plane metallic grating in case of oblique incidence and arbitrary polarization", IEEE Trans. Antennas & Propag., AP-36, 3, pp. 415-422, March 1988.

[7] T. Noda, T. Matsunaga and K. Uchida: "Approximation for scattering by an infinite plane grating in case of oblique incidence and experiment of polarization-selectivity", Trans. IEICE Japan, J71-B, 8, pp. 998-1000, Aug. 1988 (in Japanese).

[8] A. Kondo and K. Kagoshima: "Design and characteristics of low-loss polarization grid", Trans. IEICE Japan, J71-B, 12, pp. 1640-1647, Dec. 1988 (in Japanese).

[9] Y. Nakata, M. Koshiba and M. Suzuki: "Finite-element analysis of plane wave diffraction from dielectric gratings", Trans. IECE Japan, J69-C, 12, pp. 1503-1511, Dec. 1986 (in Japanese).

[10] Y. Nakata and M. Koshiba: "Finite-element analysis of plane wave diffraction from metallic gratings with arbitrary complex permittivity", Trans. IEICE Japan, J70-C, 11, pp. 1513-1522, Nov. 1987 (in Japanese).

[11] O. C. Zienkiewicz: "The Finite Element Method", 3rd ed., McGraw-Hill, London, 1977.