

J. Paul Shelton
 Naval Research Laboratory
 Washington, D. C. 20375, U.S.A.

Introduction

The Butler matrix was introduced in the early 1960's as a technique to obtain multiple directive beams from linear antenna arrays. In general, a Butler matrix of $2N$ ports feeds an array of N elements, where $N = 2^p$ and p is integer. Shelton and Kelleher pointed out that a symmetric six-port junction could be used as a building block to generate networks with $N = 2^p 3^q$ elements. [1] Shelton described a six-port network with $N = 3$ comprised of directional couplers. [2] Nolen derived but did not publish a procedure for synthesizing networks for all N . [3] DuHamel et al described a network for $N = 5$. [4]

Useful applications for multibeam networks are found in antenna configurations other than linear arrays. Chadwick and Shelton described monopulse arrays and multimode spiral radiators which must be fed by such networks. [5] Spiral radiators with $2(2N+1)$ arms also have desirable monopulse properties. [6] Shelton described hexagonal planar arrays using feed networks with $N = 3$ and $N = 7$. [2,7] Kajfez and co-workers studied the tripole antenna for several years but as recently as 1974 were unaware of the existence of $N = 3$ networks, which are the appropriate feed systems. [8,9] Circular arrays can be transformed so as to be equivalent to linear arrays by means of a multibeam network. [10]

Therefore, one purpose of this paper is to present results which have been previously unpublished or obscurely published. Methods for designing basic networks for arbitrary N are described and applied. The body of the paper is divided into three sections. The first discusses the required characteristics of the networks which are to be synthesized; the second describes the known network synthesis procedures; and the third presents network diagrams for $N = 3, 4, 5, 6,$ and 7 .

Required Network Characteristics

The characteristics of multibeam networks are defined in terms of their scattering matrices. The networks have $2N$ matched ports, with two separate sets of N isolated ports. Thus, the scattering matrix is completely described in terms of the transfer coefficients from one set of N ports to the other. Finally, the networks must be lossless, and we restrict ourselves to conventional four-port directional couplers and transmission lines.

Therefore, the first task is the specification of the network transfer coefficients. It is possible to define the transfer coefficient of a $2N$ port network based on the orthogonal aperture distributions for an N -element multibeam array. One expression for the excitation of the n th element for the m th beam is

$$e_{mn} = \frac{1}{\sqrt{N}} e^{j2\pi(m-1)(n-(N+1)/2)/N} \quad (1)$$

Although the coefficients of Equation (1) are the ones required to form the multiple beams, they may be difficult to synthesize directly. An alternative approach is to transform these coefficients into distributions which are more easily generated. Such a set of distributions is obtained by taking sums and differences of the complex conjugate distributions to produce

$$e'_{mn} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ \sqrt{2}\cos(4\pi/5) & \sqrt{2}\cos(2\pi/5) & \sqrt{2} & \sqrt{2}\cos(2\pi/5) & \sqrt{2}\cos(4\pi/5) \\ \sqrt{2}j\sin(4\pi/5) & \sqrt{2}j\sin(2\pi/5) & 0 & -\sqrt{2}j\sin(2\pi/5) & -\sqrt{2}j\sin(4\pi/5) \\ \sqrt{2}\cos(2\pi/5) & \sqrt{2}\cos(4\pi/5) & \sqrt{2} & \sqrt{2}\cos(4\pi/5) & \sqrt{2}\cos(2\pi/5) \\ \sqrt{2}j\sin(2\pi/5) & -\sqrt{2}j\sin(4\pi/5) & 0 & \sqrt{2}j\sin(4\pi/5) & -\sqrt{2}j\sin(2\pi/5) \end{pmatrix} \quad (2)$$

The result of this transformation is a set of in-phase distributions which exhibit even and odd symmetry about the centers of the rows. Once the distributions of Equation (2) have been synthesized, it is straightforward to combine corresponding sin and cos distributions through 3-dB directional couplers to form the distributions of Equation (1).

Note that Equation (1) is arranged so that the element excitations for each beam are symmetric about the center of the array, by using the factor, $n - (N+1)/2$. Furthermore, the factor $2\pi(m-1)/N$ guarantees that there will always be one distribution with all elements in phase. That is, there will always be a beam with its maximum normal to the array.

Synthesis Techniques

One method for synthesizing multibeam networks can be referred to as a "building block" process and is illustrated with an example. If the overall network will feed an array of N_0 elements, such that $N_0 = 2^p 3^q$, the network consists of p rows of networks, or building blocks, for which $N = 2$ and q rows of building blocks for which $N = 3$. Thus, the building block process requires that N_0 be highly composite and furthermore that the building block network be known. If all the building blocks have $N = 2$, the multibeam network is a Butler matrix. This process will not be discussed further because the objective of this paper is to synthesize networks for small N .

A second method for synthesizing networks is to separate the sin-cos distributions of Equation (2), which greatly simplifies the synthesis, then combine the distributions in quadrature to form the required multibeam distributions. This procedure works well for networks as large as $N = 5$. Figure 1 illustrates the result for $N = 5$. This network contains the minimum number of four-port junctions to realize a network for $N = 5$.

A third synthesis technique, due to Nolen, has the advantages that it can be straightforwardly applied for any N and it can generate planar networks with one or both sets of N ports external and adjacent.[5] The concept is elegantly simple, utilizing the triangular configuration of directional couplers shown in Figure 2 for $N = 4$. Initially, the order of input ports, m , and output ports, n , is selected. The synthesis consists of computing the coupling values and output phase shifts of the couplers in the sequence indicated. For the leftmost input, couplers 1, 2, and 3 are determined. Then, for the next port to the right, couplers 4 and 5 are determined. Finally, for the second port to the right, coupler 6 is determined. The Nolen network is similar to the Blass matrix described by Kahn and Shubel.[11]

Results

In this section, several examples of networks are presented, all of which have been synthesized by the Nolen procedure. Figure 3 illustrates the convention used in all networks to define the directional coupler. Oval blocks represent phase shifts. All values inside blocks are angles in degrees.

All networks have one feature in common: The leftmost input port produces the in-phase, uniform-amplitude distribution. The networks of Figure 4 have been con-

strained to be symmetric and to have the output ports in order. Thus, they are most closely analogous to an optical lens. Furthermore, the front-back symmetry allows them to be used in symmetric and reflective Butler matrices. [12]

Figure 5 shows a symmetric network for $N = 4$ in which the ports are not in sequence. This network is simpler than that of Figure 4(b) and has the interesting property of changing from a linear array multibeam network to a square array (monopulse) network by the insertion of a 90-degree phase shift.

It is important to avoid crossing lines if the network is to be realized in strip or microstrip transmission line. Furthermore, if a building block synthesis is to be performed and the objective is to place as large a network as possible on a single planar transmission line region without crossing lines, then it is necessary to have as many ports as possible at the edge of the small networks. It can be shown that, for one set of N ports external, a network with $N_o = 2N$ can be synthesized. For both sets of N ports external and also adjacent, a network with $N_o = 4N$ can be synthesized. For Butler matrices, this means that the largest network that can be placed on a single transmission line region is $N = 8$. Therefore, all the networks in Figures 4 and 5 allow layout on one transmission line layer of a larger network with $N_o = 4N$. Thus, the network of Figure 5 allows a network with $N = 16$ to be put on one layer. Figure 6 shows networks for $N = 5, 6,$ and 7 . These networks were synthesized by first generating sin-cos distributions, then combining these distributions through hybrid couplers. The networks lack symmetry and have some internal ports, but they require phase shifts of only 90 or 180 degrees.

References

1. J. P. Shelton and K. S. Kelleher, "Multiple Beam from Linear Arrays," IRE Trans, Vol. AP-9, No. 2, pp 154-161, Mar 1961.
2. Paul Shelton, "Multibeam, Hexagonal, Triangular-Grid, Planar Arrays," 1965, G-AP Symposium.
3. J. C. Nolen, "Synthesis of Multiple Beam Networks for Arbitrary Illuminations," 21 Apr 1965 (unpublished).
4. R. H. DuHamel, W. R. Jones, G. F. Van Blaricum, Jr., "Synthesis of Hybrid Networks," Final Engineering Report, Contract N60530-12660, submitted to Naval Ordnance Test Station, 25 Jan 1967.
5. G. G. Chadwick and J. P. Shelton, "Two-Channel Monopulse Techniques - Theory and Practice," 1965 IEEE MIL-E-CON Record, pp 177-181.
6. G. G. Chadwick and J. P. Shelton, "Sum-Difference Feed Network," U.S. Patent No. 3,346,861, Oct 10, 1967.
7. J. P. Shelton, "Multibeam Planar Arrays," Proc. IEEE, Vol. 56, No. 11, pp 1818-1821, Nov 1968.
8. D. Kajfez, "Three-Phase Separator for Circular Polarization," Trans. IEEE, Vol. MTT-17, No. 9, pp 726-727, Sep 1969.
9. D. Kajfez, M. G. Harrison, C. E. Sterling, "Electric Tripole Antenna for Circular Polarization," Trans IEEE, Vol. AP-22, No. 5, pp 647-650, Sep 1974.
10. B. Sheleg, "A Matrix-Fed Circular Array for Continuous Scanning," Proc IEEE, Vol. 56, pp 2016-2027, Nov 1968.
11. W. K. Kahn and E. J. Shubel, "Passive, Series Fed, Multibeam Antenna," 1965 G-AP Symposium.
12. J. P. Shelton and J. K. Hsiao, "Reflective Butler Matrix," 1976 AP-S Symposium.

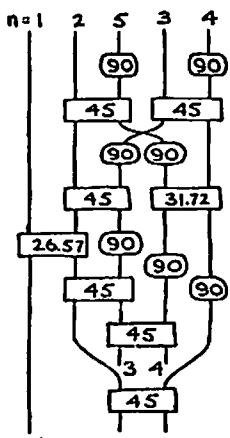


FIG. 1. MULTIBEAM NETWORK FOR N=5

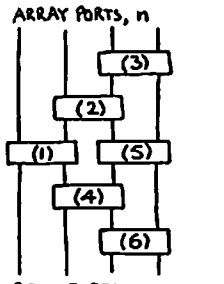


FIG. 2. NOLEN NETWORK CONFIGURATION FOR N=4

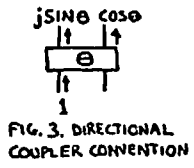


FIG. 3. DIRECTIONAL COUPLER CONVENTION

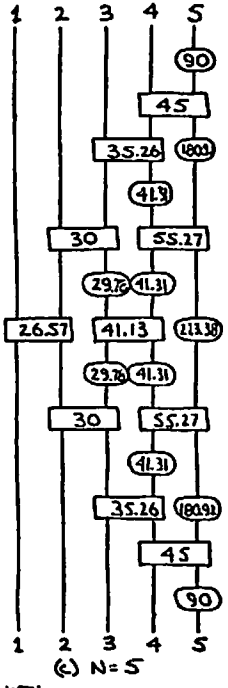
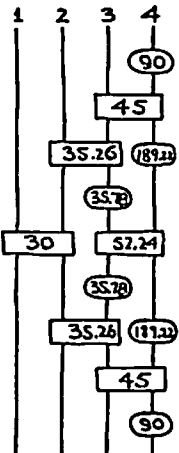
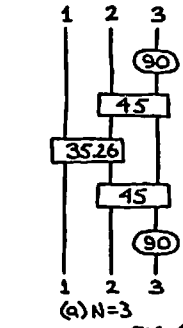


FIG. 4. SYMMETRIC NETWORKS WITH PORTS IN SEQUENCE

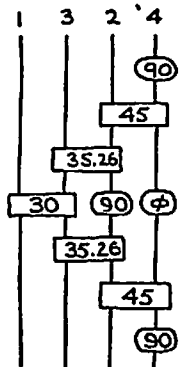


FIG. 5. ALTERNATE NETWORK FOR N=4
 $\phi = 0$ FOR LINEAR ARRAY
 $\phi = 90^\circ$ FOR SQUARE MONOPULSE ARRAY

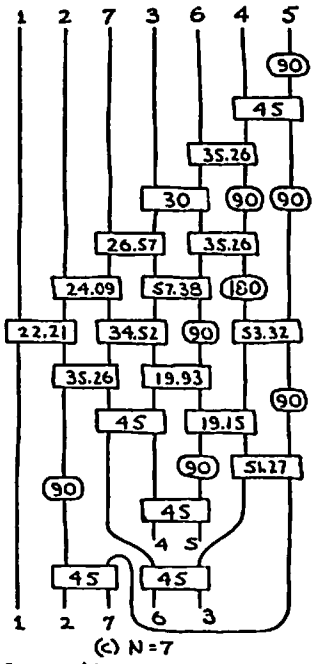
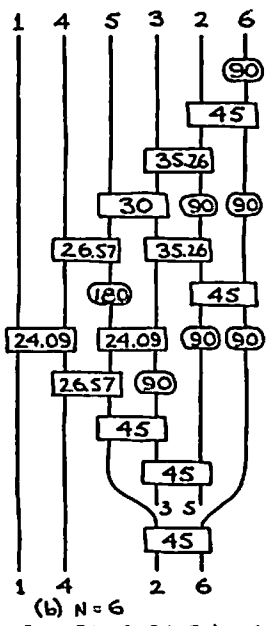
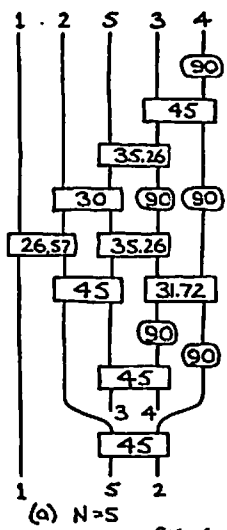


FIG. 6. NETWORKS BASED ON SIN-COS DISTRIBUTIONS